Board Paper of Class 12-Science 2023 Math Delhi(Set 1) - Solutions

Total Time: 180

Total Marks: 80.0

Section A

Solution 1

Given: A = {3, 5}
The number of reflexive relations on a set with the '*n*' number of elements is given by $2^{(n^2-n)}$.

Here, *n* = 2
$\therefore$ The number of reflexive relations on a set A = $2^{(2^2-2)}=2^{(4-2)}=2^2=4$

Hence, the correct answer is option (b).

Solution 2

$\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$

$=\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right)\right]$

$=\sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]$

$=\sin\left[\frac{2\pi+\pi}{6}\right]$

$=\sin\left[\frac{3\pi}{6}\right]$

$=\sin\left[\frac{\pi}{2}\right]$
$=1$

Hence, the correct answer is option (a).

Solution 3

Given that, $\mathrm{A}^2-\mathrm{A}+\mathrm{I}=\mathrm{O}$

Multiplying the given equation with A^{-1}, we get

$$\Rightarrow A^2\left(A^{-1}\right) - A\left(A^{-1}\right) + A^{-1} = O$$

$$\Rightarrow A - I + A^{-1} = O \qquad \left(\because AA^{-1} = I\right)$$

$$\Rightarrow A^{-1} = I - A$$

Hence, the correct answer is an option (c).

Solution 4

Given that,

A = B^2

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$$

Equating the LHS with the RHS, we get

$$\Rightarrow x + 1 = 2$$

$$\Rightarrow x = 1$$

Hence, the correct answer is an option (c).

Solution 5

$$\alpha(2-4) - 3(1-1) + 4(4-2) = 0$$

$$\Rightarrow -2\alpha + 8 = 0$$

$$\Rightarrow 2\alpha = 8$$

$$\Rightarrow \alpha = 4$$

Hence, the correct answer is option (d).

Solution 6

$y = x^{2x}$

Taking log on both sides, we get:

$\log y = \log x^{2x}$

$\log y = 2x \cdot \log x$

Differentiating both sides w.r.t. x, we get:

$$\frac{1}{y} \times \frac{dy}{dx} = 2x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(2x)$$
$$\frac{dy}{dx} = y\left[2x\left(\frac{1}{x}\right) + \log x\,(2)\right]$$
$$\frac{dy}{dx} = x^{2x}(2 + 2\log x)$$
$$\frac{dy}{dx} = 2x^{2x}(1 + \log x)$$

Hence, the correct answer is option (c).

Solution 7

Greatest integer function is continuous at non-integral points.
So, [x] is continuous at x = 1.5 as
L.H.L at x = 1.5 is 1.
R.H.L at x = 1.5 is 1.
Also, the value at x = 1.5 is also 1.

Hence, the correct answer is option (b).

Solution 8

x = A cos 4t + B sin 4t
Differentiating both sides w.r.t. t, we get:

$$\frac{dx}{dt} = -\text{A}\sin\ 4t\,(4) + \text{B}\cos\ 4t\,(4)$$
$$\frac{dx}{dt} = -4\,\text{A}\sin\ 4t + 4\,\text{B}\cos\ 4t$$

Differentiating both sides w.r.t. t, we get:

$$\begin{aligned}\frac{d^2x}{dt^2} &= (-4\text{A})\cos\ 4t\,(4) + 4\text{B}\,(-\sin\ 4t)\,(4)\\ &= -16\,\text{A}\cos\ 4t - 16\,\text{B}\sin\ 4t\\ &= -16\,(\text{A}\cos\ 4t + \text{B}\sin\ 4t)\\ &= -16x\end{aligned}$$

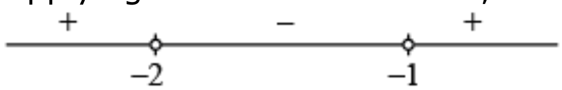
Hence, the correct answer is option (d).

Solution 9

$f(x) = 2x^3 + 9x^2 + 12x - 1$
Differentiating w.r.t. x, we get
$f'(x) = 6x^2 + 18x + 12$
For $f(x)$ to be decreasing, $f'(x) < 0$.
$6x^2 + 18x + 12 < 0$
$6[x^2 + 3x + 2] < 0$

6(x + 2) (x + 1) < 0

Applying number line method, we get:



Since $f'(x) < 0$ when $x \in (-2, -1)$

Hence, $f(x)$ is decreasing in the interval (−2, −1).

Hence, the correct answer is option (b).

Solution 10

$$\int \frac{\sec\ x}{\sec\ x-\tan\ x}\ dx$$

$$= \int \frac{\sec\ x}{(\sec\ x-\tan\ x)} \times \frac{(\sec\ x+\tan\ x)}{(\sec\ x+\tan\ x)}$$

$$= \int \frac{\sec^2 x+\sec\ x\ \tan\ x}{\sec^2 x-\tan^2 x}$$

$$= \int \left(\sec^2 x + \sec\ x\ \tan\ x\right) dx$$

$$= \tan\ x + \sec\ x + \mathrm{c}$$

Hence, the correct answer is option (b).

Solution 11

$$\int_{-1}^{1} \frac{|x-2|}{x-2} dx,\ x \neq 2$$

$$\because -1 < x < 1$$

$$\Rightarrow -3 < x - 2 < -1$$

$$\therefore |x - 2| = -(x - 2)$$

$$\therefore \int_{-1}^{1} \frac{|x-2|}{x-2} dx = \int_{-1}^{1} \frac{-(x-2)}{x-2} dx$$

$$= \int_{-1}^{1} (-1) dx$$

$$= [-x]_{-1}^{1}$$

$$= -[1 - (-1)]$$

$$= -2$$

Hence, the correct answer is option (d).

Solution 12

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 3\left(\frac{dy}{dx}\right)^{3-1} \cdot \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= 3\left(\frac{dy}{dx}\right)^2 \frac{d^2y}{dx^2}$$

$\therefore$ Order = 2
Degree = 1
Required Sum = 2 + 1 = 3

Hence, the correct answer is option (b).

Solution 13

$\overrightarrow{a}$ and $\overrightarrow{b}$ are collinear

if $\overrightarrow{a} = \lambda\hat{b}$

i.e. $a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

Comparing the coefficient of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

$a_1 = \lambda b_1, \quad a_2 = \lambda b_2, \quad a_3 = \lambda b_3$

$\Rightarrow \frac{a_1}{b_1} = \lambda, \quad \frac{a_2}{b_2} = \lambda, \quad \frac{a_3}{b_3} = \lambda$

$\Rightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} \quad \left(\because \text{each} = \lambda\right)$

Hence, the correct answer is option (b).

Solution 14

$Let\ \overrightarrow{a} = 6\hat{i} - 2\hat{j} + 3\hat{k}$

$\therefore \left|\overrightarrow{a}\right| = \sqrt{6^2 + (-2)^2 + 3^2}$

$\Rightarrow \left|\overrightarrow{a}\right| = \sqrt{36 + 4 + 9}$

$\Rightarrow \left|\overrightarrow{a}\right| = \sqrt{49}$

$\Rightarrow \left|\overrightarrow{a}\right| = 7$

Hence, the correct answer is option (c).

Solution 15

The direction cosines are cos 90°, cos 135° and cos 45° i.e., $0, \ -\frac{1}{\sqrt{2}}, \ \frac{1}{\sqrt{2}}$

Hence, the correct answer is option (a).

Solution 16

The direction ratios of line 2x = 3y = −z can be written as $\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{-1}$ are $\left(\frac{1}{2}, \frac{1}{3}, -1\right)$.

The direction ratios of line 6x = −y = −4z can be written as $\frac{x}{\frac{1}{6}} = \frac{y}{-1} = \frac{z}{\frac{-1}{4}}$ are $\left(\frac{1}{6}, -1, \frac{-1}{4}\right)$.

Now,

Dot product of the direction ratios of the line

$$= \frac{1}{2} \times \frac{1}{6} + \frac{1}{3} \times (-1) + (-1) \times \left(\frac{-1}{4}\right)$$
$$= \frac{1}{12} - \frac{1}{3} + \frac{1}{4}$$
$$= 0$$

So, the angle between the lines is 90°.

Hence, the correct answer is option (d).

Solution 17

Given that, $\mathrm{P}(\mathrm{A}) = \frac{4}{5}$ and $\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \frac{7}{10}$

$$\therefore \mathrm{P}(\mathrm{B}/\mathrm{A}) = \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P}(\mathrm{A})}$$
$$= \frac{\frac{7}{10}}{\frac{4}{5}}$$
$$= \frac{7}{8}$$

Hence, the correct answer is an option (c).

Solution 18

Let A be the event that at least one head turns up. Since each coin turns up on either a head or a tail.

Therefore, the sample space consists of $2^5 = 32$ outcomes.

Each outcome has a probability of occurrence as $\frac{1}{32}$.

Then A^c is the event that 'No head turns up'

$$\therefore \mathrm{P}(\mathrm{A}^c) = \frac{1}{32}$$
$$\Rightarrow P(A) = 1 - \frac{1}{32} = \frac{31}{32}$$

Hence, the correct answer is an option (c).

Solution 19

Reason (R): Let E and F be two events with a random experiment, then
$$P(F/E) = \frac{P(E \cap F)}{P(E)}.$$

If A and B are two events associated with the same sample space of a random experiment, the conditional probability of the event A given that B has occurred, i.e. P (A|B) is given by
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$
Thus, Reason (R) is true.

Assertion (A) : Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

When the two coins are tossed the sample space is given by
S = {(H,H), (H,T), (T,H), (T,T)}

Now, let the event of coming up of two heads be named as event A.
∴ A = {(H,H)}
$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$
$$\therefore P(A) = \frac{1}{4}$$

let the event of at least one head coming up, be named as event B.
∴ B = {(H,H), (H,T), (T,H)}
$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$
$$\therefore P(B) = \frac{3}{4}$$

Thus, the probability of getting two heads, if it is known that at least one head comes up = P(A/B)
$$\because P(A/B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$
$$= \frac{1}{3}$$

Thus, Assertion (A) is true.

So, Both (A) and (R) are true and (R) is the correct explanation of (A).
Hence, the correct answer is option (a).

Solution 20

Reason (R): $\int_a^b f(x)\,\mathrm{d}x = \int_a^b f(a+b-x)\,\mathrm{d}x$
This is trivially true according to the properties of the definite integral.
Thus, Reason (R) is true.

Assertion (A): $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,\mathrm{d}x$

$$\text{Let } I=\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,\mathrm{d}x \qquad .....(1)$$

$$=\int_2^8 \frac{\sqrt{10-(8+2-x)}}{\sqrt{8+2-x}+\sqrt{10-(8+2-x)}}\,\mathrm{d}x \qquad \left[\because \int_a^b f(x)\,\mathrm{d}x = \int_a^b f(a+b-x)\,\mathrm{d}x\right]$$

$$=\int_2^8 \frac{\sqrt{10-10+x}}{\sqrt{10-x}+\sqrt{10-10+x}}\,\mathrm{d}x$$

$$=\int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}\,\mathrm{d}x \qquad .....(2)$$

Adding (1) and (2), we get

$$2I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,\mathrm{d}x + \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}\,\mathrm{d}x$$

$$\Rightarrow 2I = \int_2^8 \frac{\sqrt{x}+\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,\mathrm{d}x$$

$$\Rightarrow 2I = \int_2^8 1\,\mathrm{d}x$$

$$\Rightarrow 2I = [x]_2^8$$

$$\Rightarrow 2I = 8-2$$

$$\Rightarrow 2I = 6$$

$$\Rightarrow I = 3$$

Thus, Assertion (A) is true.

So, Both (A) and (R) are true and (R) is the correct explanation of (A).
Hence, the correct answer is option (a).

Section B

Solution 21

Given: $f(x) = \tan^{-1}x$
The domain of function $\tan^{-1}x$ is R.
And the range (principle value branch) of function $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Solution 22

Given: $f\left(x\right)=\begin{cases}x^{2}, & \text{if } x\geq 1\\ x\ , & \text{if } x<1\end{cases}$

$\text{LHD at } x=1$

$=\lim\limits_{x\to 1^{-}}\frac{f(x)-f(1)}{x-1}$

$=\lim\limits_{x\to 1^{-}}\frac{x-(1)^{2}}{x-1}$

$=\lim\limits_{x\to 1^{-}} 1$

$=1$

$\text{RHD at } x=1$

$=\lim\limits_{x\to 1^{+}}\frac{f(x)-f(1)}{x-1}$

$=\lim\limits_{x\to 1^{+}}\frac{x^{2}-(1)^{2}}{x-1}$

$=\lim\limits_{x\to 1^{+}}\frac{(x-1)(x+1)}{x-1}$

$=\lim\limits_{x\to 1^{+}} x+1$

$=2$

(LHD at x = 1) ≠ (RHD at x = 1)

Thus, $f(x)$ is not differentiable at x = 1.

OR

Given: $f\left(x\right)=\begin{cases}\frac{\sin^{2}\lambda x}{x^{2}}, & \text{if } x\neq 0\\ 1\ , & \text{if } x=0\end{cases}$

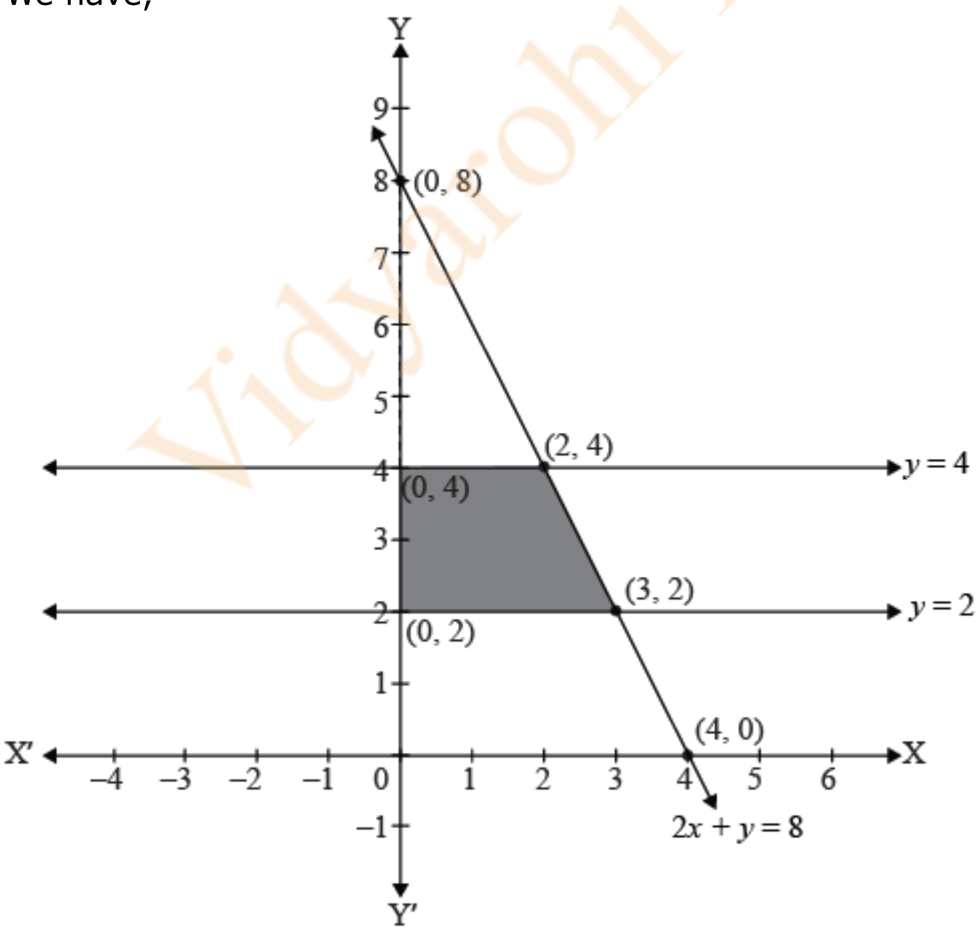
The function $f(x)$ is continuous at x = 0, if and only if

$$\lim_{x \to 0} f(x) = f(0)$$
$$\Rightarrow \lim_{x \to 0} \frac{\sin^2 \lambda x}{x^2} = 1$$
$$\Rightarrow \lim_{x \to 0} \frac{\sin^2 \lambda x}{x^2} \times \frac{\lambda^2}{\lambda^2} = 1$$
$$\Rightarrow \left(\lambda^2\right) \lim_{x \to 0} \frac{\sin^2 \lambda x}{(\lambda x)^2} = 1$$
$$\Rightarrow \left(\lambda^2\right) \lim_{x \to 0} \left(\frac{\sin \lambda x}{\lambda x}\right)^2 = 1$$
$$\Rightarrow \left(\lambda^2\right) \times 1 = 1 \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$
$$\Rightarrow \lambda^2 = 1$$
$$\Rightarrow \lambda = \pm 1$$

Thus, $\lambda = \pm 1$.

Solution 23

We have,



$\therefore$ Area of integration $= \int_2^4 \left(\frac{8-y}{2}\right) \mathrm{d}\, y$

$= \int_2^4 4\, \mathrm{d}\, y - \int_2^4 \frac{y}{2}\, \mathrm{d}\, y$

$= 8 - \frac{1}{4}\left[y^2\right]_2^4$

$= 8 - 3$

$= 5$ sq units

Solution 24

It is given that $\vec{a} \times \vec{b}$ is a unit vector.

$\therefore \left|\vec{a} \times \vec{b}\right| = 1$

Also, $\left|\vec{a} \times \vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right| \sin\theta$

$\Rightarrow \sin\theta = \frac{\left|\vec{a} \times \vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|}$

$\Rightarrow \sin\theta = \frac{1}{3\times\frac{2}{3}}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \sin\frac{\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{6}$

OR

Given: $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{\mathrm{i}} - 7\hat{\mathrm{j}} + \widehat{\mathrm{k}}$

The area of the parallelogram is given by $\left|\vec{a} \times \vec{b}\right|$.

Now, $\overrightarrow{a} \times \overrightarrow{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$

$\Rightarrow \overrightarrow{a} \times \overrightarrow{b} = (-1+21)\hat{i} - (1-6)\hat{j} + (-7+2)\hat{k}$

$\Rightarrow \overrightarrow{a} \times \overrightarrow{b} = 20\hat{i} + 5\hat{j} - 5\hat{k}$

$\Rightarrow \left|\overrightarrow{a} \times \overrightarrow{b}\right| = \sqrt{(20)^2 + (5)^2 + (-5)^2}$

$\Rightarrow \left|\overrightarrow{a} \times \overrightarrow{b}\right| = \sqrt{400 + 25 + 25}$

$\Rightarrow \left|\overrightarrow{a} \times \overrightarrow{b}\right| = \sqrt{450}$

$\Rightarrow \left|\overrightarrow{a} \times \overrightarrow{b}\right| = \sqrt{225 \times 2}$

$\Rightarrow \left|\overrightarrow{a} \times \overrightarrow{b}\right| = 15\sqrt{2}$ sq. units

Solution 25

The equation of the given line is

$5x - 25 = 14 - 7y = 35z$

$\Rightarrow \frac{x-5}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z}{\frac{1}{35}}$

$\Rightarrow \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1} \qquad \ldots\ldots(1)$

Thus, the vector equation of parallel line is given by

$\overrightarrow{b} = 7\hat{i} - 5\hat{j} + \hat{k}$

The vector equation of the line passing through the point $A(1,\ 2,\ -1)$ i

$\overrightarrow{a} = \hat{i} + 2\hat{j} - \hat{k}$

Therefore the required vector equation of the line is

$\overrightarrow{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda\left(7\hat{i} - 5\hat{j} + \hat{k}\right)$

Also, the required cartesian equation of the line is

$\Rightarrow \frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$

Section C

Solution 26

$$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^3 = A^2 . A = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$A^3 - 23A = 40I = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23 \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

= 0

Hence proved.

Solution 27

$\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

Let $x = \sin\ \theta\ \ .....\left(1\right)$

$\sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$

$\sec^{-1}\left(\frac{1}{\cos\theta}\right)$

$\sec^{-1}(\sec\theta)$

$\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Let $x = \sin\theta\ \ .....2$

$\sin^{-1}\left(2\sin\theta.\sqrt{\cos^2\theta}\right)$

$\sin^{-1}(2\sin\theta\cos\theta)$

$\sin^{-1}(\sin 2\theta)$

From $\left(1\right)$, $\theta = \sin^{-1}x$

Now, $\sin^{-1}x$ differentiate w.r.tx

$\frac{1}{\sqrt{1-\mathrm{x}^2}}.....\left(\mathrm{a}\right)$

Divide $\left(\mathrm{a}\right)$ by $\left(\mathrm{b}\right)$

From $\left(2\right)$, $\theta = \sin^{-1}x$

Now, $2\sin^{-1}\mathrm{x}$ differentiate w.r.tx

$2\frac{1}{\sqrt{1-\mathrm{x}^2}}.....\left(\mathrm{b}\right)$

$$\frac{\frac{1}{\sqrt{1-\mathrm{x}^2}}}{\frac{2}{\sqrt{1-\mathrm{x}^2}}} = \frac{1}{2}$$

OR

$y=\tan x+\sec x$

Differentiate both sides w.r.t x

$\frac{dy}{dx}=\sec^2 x+\sec x\ \tan x$

$=\frac{1}{\cos^2 x}+\frac{\sin x}{\cos^2 x}$

$=\frac{1+\sin x}{\cos^2 x}$

$=\frac{1+\sin x}{1-\sin^2 x}=\frac{1+\sin x}{(1+\sin x)(1-\sin x)}=\frac{1}{1-\sin x}$

Differentiate $\frac{dy}{dx}$ both sides w.r.t x

$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1}{1-\sin x}\right)$

$=\frac{-1}{(1-\sin x)^2}\times\frac{d}{dx}(1-\sin x)$

$=\frac{-1}{(1-\sin x)^2}\times(0-\cos x)$

$=\frac{\cos x}{(1-\sin x)^2}$

Hence Proved.

Solution 28

$\mathrm{I}=\int_0^{2\pi}\frac{1}{1+\mathrm{e}^{\sin x}}\mathrm{d}x \qquad \ldots(1)$

applying property of Integration

$\int_a^b f\left(x\right)dx=\int_a^b f\left(a+b-x\right)dx$

$I=\int_0^{2\pi}\frac{1}{1+e^{\sin(2\pi-x)}}dx \qquad \sin\left(2\pi-\theta\right)=-\sin\theta$

$I=\int_0^{2\pi}\frac{1}{1+e^{-\sin x}}dx$

$I=\int_0^{2\pi}\frac{1}{1+\frac{1}{e^{\sin x}}}dx$

$\mathrm{I}=\int_0^{2\pi}\frac{e^{\sin x}}{\mathrm{e}^{\sin x}+1}\mathrm{d}x \qquad \ldots(2)$

Adding equations (1) & (2)

$$2I = \int_0^{2\pi} \left(\frac{e^{\sin x}}{1+e^{\sin x}} + \frac{1}{1+e^{\sin x}}\right) dx$$

$$2I = \int_0^{2\pi} \left(\frac{e^{\sin x}+1}{1+e^{\sin x}}\right) dx$$

$$2I = \int_0^{2\pi} dx$$

$$2I = [x]_0^{2\pi}$$

$2I$ = (2π – 0)
$2I$ = 2π
$\therefore$ $\boxed{\mathrm{I} = \pi}$

OR

$$I = \int \frac{x^4}{(x-1)(x^2+1)} dx$$

Since the degree of numerator is greater than the degree of denominator, we must divide first.
$(x - 1)(x^2 + 1) = x^3 - x^2 + x - 1$

$$\begin{array}{r|l} & x+1 \\ x^3-x^2+x-1 & x^4 \\ & x^4-x^3+x^2-x \\ & - \quad + \quad - \quad - \\ \hline & x^3-x^2+x \\ & x^3-x^2+x-1 \\ & - \quad + \quad - \quad + \\ \hline & 1 \end{array}$$

Using Dividend = (Divisor × Quotient + Remainder)
$x^4 = (x + 1)(x^3 - x^2 + x - 1) + 1$

$$I = \int \frac{x^4}{(x-1)(x^2+1)} dx$$

$$I = \int \frac{(x+1)(x-1)(x^2+1)+1}{(x-1)(x^2+1)} dx$$

$$I = \int \frac{(x+1)(x-1)(x^2+1)}{(x-1)(x^2+1)} dx + \int \frac{dx}{(x-1)(x^2+1)}$$

$$I = \int \left(x+1\right) dx + \int \frac{dx}{(x-1)(x^2+1)}$$

$I = I_1 + I_2$
$I_1 = \int (x + 1)dx$

$$\boxed{\mathrm{I}_1 = \frac{x^2}{2} + x + \mathrm{C}}$$

$$I_2 = \int \frac{1}{(x-1)(x^2+1)} dx$$

Using partial fraction, we get:

$\frac{1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)}$

1 = A (x^2 + 1) + [B(x) + C] (x – 1)

When x = 1,
1 = A(1 + 1)
1 = 2A
$\therefore \boxed{A = \frac{1}{2}}$

When x = 0
1 = A – C
∴ C = A – 1
$C = \frac{1}{2} - 1$

$\boxed{C = \frac{-1}{2}}$

When x = 2
1 = 5A + 2B + C
$1 = \frac{5}{2} + 2B - \frac{1}{2}$
$\therefore \boxed{B = \frac{-1}{2}}$

$$I_2 = \int \left(\frac{1}{2}\left(\frac{1}{x-1}\right) - \frac{1}{2}\left(\frac{x+1}{x^2+1}\right)\right)dx$$

$$I_2 = \frac{1}{2}\int \frac{1}{x-1}dx - \frac{1}{2}\left[\int \frac{x}{x^2+1}dx + \int \frac{1}{x^2+1}dx\right]$$

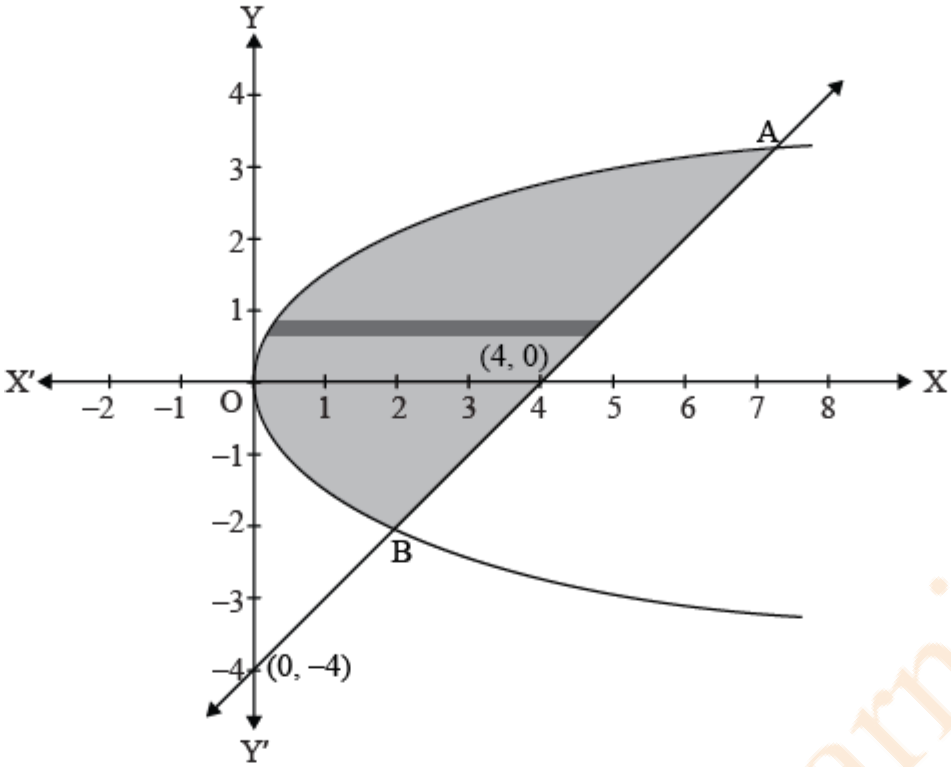
$$I_2 = \frac{1}{2}\log\left(x-1\right) - \frac{1}{2}\left[\frac{1}{2}\log\left(x^2+1\right) + \tan^{-1}x\right]$$

$$I_2 = \frac{1}{2}\log\left(x-1\right) - \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\tan^{-1}x$$

Now,
$I = I_1 + I_2$
$= \frac{x^2}{2} + x + \frac{1}{2}\log\left(x-1\right) - \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\tan^{-1}x + C$

Solution 29

{(x, y) : $y^2 \le 2x$ and $y \ge x - 4$}



For line
$y = x - 4$

x	y
0	–4
4	0

Required region is ABOA
Take horizonal strip
For A and B,
Solve $y^2 = 2x$ .....(1)
And $y = x - 4$ .....(2)
$\because\ x = \frac{y^2}{2}$
Putting in (2), we get

$$y = \frac{y^2}{2} - 4$$

$$2y = y^2 - 8$$

$$y^2 - 2y - 8 = 0$$

$$\left(y - 4\right)\left(y + 2\right) = 0$$

$$y = -2,\ 4$$

$$y = -2 \text{ or } y = 4$$

$$\Rightarrow x = 2 \text{ or } x = 8$$

$$\therefore \text{ Area} = \int_{-2}^{4}\left(y+4\right)-\left(\frac{y^2}{2}\right)dy$$

$$= \left[\frac{y^2}{2}+4y-\frac{y^3}{6}\right]_{-2}^{4}$$

$$= \left(\frac{4^2}{2}+4\left(4\right)-\frac{4^3}{6}\right)-\left(\frac{(-2)^2}{2}+4\left(-2\right)-\frac{(-2)^3}{6}\right)$$

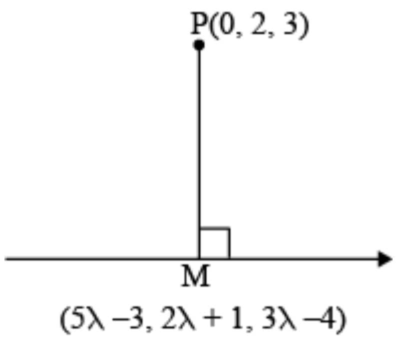
$$= \left[\left(8+16-\frac{64}{6}\right)-\left(2-8+\frac{8}{6}\right)\right]$$

$$= 24-\frac{64}{6}-2+8-\frac{8}{6}$$

$$= 18 \text{ sq. unit}$$

Solution 30

Equation of given line is $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda\,(\text{say})$



Any point on this line can be expressed as $5\lambda-3,\ 2\lambda+1,\ 3\lambda-4$

For some value of λ, let these be the coordinates of point M (i.e. Foot of $\perp r$ from P to the line)

$\therefore$ d·r·s of line PM are

$5\lambda - 3 - 0,\ 2\lambda + 1 - 2,\ 3\lambda - 4 - 3$

i.e. $5\lambda - 3,\ 2\lambda - 1,\ 3\lambda - 7$

Since PM $\perp r$ to the given line

$\therefore 5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$(\because a_1a_2 + b_1b_2 + c_1c_2 = 0)$

$\Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$

$38\lambda = 38$

$\lambda = 1$

Hence coordinates of point M are

$5\lambda - 3,\ 2\lambda + 1,\ 3\lambda - 4$

Put $\lambda = 1$ $5(1) - 3,\ 2(1) + 1,\ 3(1) - 4$

$(2, 3, -1)$

OR

Given $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

Taking modulus of both side, we get

$$\left|\vec{a}+\vec{b}+\vec{c}\right|=0$$

squaring both sides, we get

$$\left|\vec{a}+\vec{b}+\vec{c}\right|^2=0$$

$$\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2+2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=0$$

$$|3|^2+|4|^2+|2|^2+2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=0$$

$$9+16+4+2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=0$$

$$2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=-29$$

$$\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=\frac{-29}{2}$$

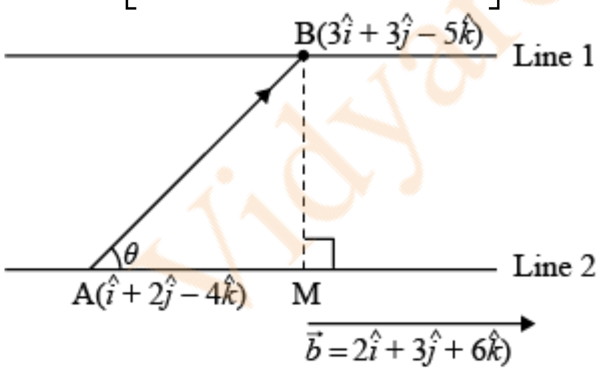
Solution 31

Equation of given lines are

$$\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right) \quad \ldots.(1)$$

and $\vec{r}=\left(3\hat{i}+3\hat{j}-5\hat{k}\right)+2\mu\left(2\hat{i}+3\hat{j}+6\hat{k}\right) \quad \ldots..(2)$

We observe that the two lines are parallel as they are parallel to the same vector $\left[\text{ie } \vec{b}=2\hat{i}+3\hat{j}+6\hat{k}\right]$



Our aim is to find length BM (see figure)

Now $\mathrm{BM}=\left|\overrightarrow{\mathrm{AB}}\right| \sin\theta$

$=\frac{\left|\overrightarrow{\mathrm{AB}}\right|\left|\vec{b}\right| \sin\theta}{\left|\vec{b}\right|}$ $\left(\text{ie multiplied and divided by } \left|\vec{b}\right|\right)$

$=\frac{\left|\overrightarrow{\mathrm{AB}}\times\vec{b}\right|}{\left|\vec{b}\right|}$

Now $\overrightarrow{\mathrm{AB}}=P.V.$ of point $B-P.V.$ of point A

$=\left(3\hat{i}+3\hat{j}-5\hat{k}\right)-\left(\hat{i}+2\hat{j}-4\hat{k}\right)$

$=2\hat{i}+\hat{j}-\hat{k}$

$$\overrightarrow{\mathrm{AB}}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 2 & 3 & 6\end{vmatrix}$$

$=(6+3)\hat{i}-(12+2)\hat{j}+(6-2)\hat{k}$

$=9\hat{i}-14\hat{j}+4\hat{k}$

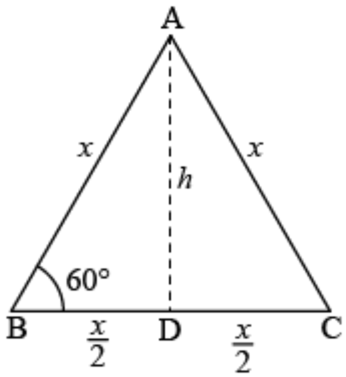
$\therefore \left|\overrightarrow{\mathrm{AB}}\times\vec{b}\right|=\sqrt{9^2+(-14)^2+4^2}=\sqrt{81+196+16}=293$

also $\left|\vec{b}\right|=\sqrt{2^2+3^2+6^2}=\sqrt{4+9+36}=\sqrt{49}=7$

$\therefore$ Required distance $=\frac{\left|\overrightarrow{\mathrm{AB}}\times\vec{b}\right|}{\left|\vec{b}\right|}=\frac{\sqrt{293}}{7}$

Section D

Solution 32



Consider an equilateral ΔABC of side '*x*'
AD is the median of ΔABC
∠A = ∠B = ∠C = 60°
AB = AC = BC = *x*
Let *h* be the length of side AD
In ΔABC

$$\sin B = \sin 60 = \frac{AD}{AB}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{x}$$

$$\sin B = \sin 60 = \frac{AD}{AB}$$

$$\therefore h = \frac{\sqrt{3}}{2}x \qquad \ldots\ldots(1)$$

Now it is given that

$$\frac{dh}{dt} = +2\sqrt{3}\ \text{cm}/\text{sec}$$

$$\frac{dx}{dt} \quad \text{to be figured out}$$

Differentiating eq (1) w.r.t

$$h = \frac{\sqrt{3}}{2}x$$

$$\frac{dh}{dt} = \frac{\sqrt{3}}{2}\frac{dx}{dt}$$

$$+2\sqrt{3} = \frac{\sqrt{3}}{2}\frac{dx}{dt}$$

$$\frac{dx}{dt} = +4\ \text{cm}/\text{sec}$$

Hence, the side of the equilateral triangle is increasing at the rate of 4 cm/sec.

OR

Given: Sum of two number is 5
Sum of cubes of these numbers is least.
To find: Sum of squares of these numbers.

Let the two numbers be x and y
$x + y = 5$
$\Rightarrow y = 5 - x$ .....(1)
$f(x) = x^3 + y^3$ is least
Using equation (1), we have:
$f(x) = x^3 + (5 - x)^3$ .....(2)
Differentiating w.r.t 'x', we get:

$$f'(x) = 3x^2 - 3(5-x)^2$$
$$= 3x^2 - 3\left(25 + x^2 - 10x\right)$$
$$= 3x^2 - 75 - 3x^2 + 30x$$
$$= 30x - 75$$

$$f'(x) = 0$$
$$\Rightarrow 30x - 75 = 0$$
$$\Rightarrow x = \frac{75}{30} = \frac{5}{2}$$

Differentiating $f(x)$ w.r.t 'x', we get:

$$f''\left(x\right) = \frac{d}{dx}(30x - 75)$$
$$= 30$$
$$f''\left(\frac{5}{2}\right) = 30 > 0$$

Hence, $f(x)$ will attain minimum value at $x = \frac{5}{2}$

Now,

$$x = \frac{5}{2}$$
$$y = 5 - \frac{5}{2} = \frac{5}{2}$$

Sum of squares = $x^2 + y^2$

$$= \left(\frac{5}{2}\right)^2 \times \left(\frac{5}{2}\right)^2 = \frac{50}{4} = 12.5$$

Solution 33

$$I=\int_{0}^{\frac{\pi}{2}} \sin\ 2x\ \tan^{-1}(\sin x)dx$$

$$I=\int_{0}^{\frac{\pi}{2}} 2\sin x\ \cos x\ \tan^{-1}(\sin x)dx$$

$$\text{Let}\quad \sin x \longrightarrow t \qquad 0 \longrightarrow \frac{\pi}{2}$$

$$\cos x dx \longrightarrow dt \qquad 0 \longrightarrow 1$$

$$I=2\int_{0}^{1} t\tan^{-1}\left(t\right)dt=2I_1$$

$$I_1=\int \underset{\text{II}}{t}\underset{\text{I}}{\tan^{-1}}\left(t\right)dt$$

using integration by parts

$$I_1=\tan^{-1}t\int tdt-\int\left(\frac{d}{dt}\left(\tan^{-1}(t)\right)\int tdt\right)dt$$

$$I_1=\frac{t^2}{2}\tan^{-1}\left(t\right)-\frac{1}{2}\int\frac{t^2}{1+t^2}dt$$

$$I_1=\frac{t^2}{2}\tan^{-1}\left(t\right)-\frac{1}{2}\int\frac{1+t^2-1}{1+t^2}dt$$

$$I_1=\frac{t^2}{2}\tan^{-1}\left(t\right)-\frac{1}{2}\int dt+\frac{1}{2}\int\frac{dt}{1+t^2}$$

$$I_1=\left[\frac{t^2}{2}\tan^{-1}\left(t\right)-\frac{1}{2}t+\frac{1}{2}\tan^{-1}\left(t\right)\right]_0^1$$

Now putting limits

$$I_1=\left[\left(\frac{1}{2}\tan^{-1}(1)-\frac{1}{2}+\frac{1}{2}\tan^{-1}(1)\right)-\left(0-\left(\frac{0}{2}\right)+\left(\frac{1}{2}\tan^{-1}(0)\right)\right)\right]$$

$$I_1=\left(\tan^{-1}(1)-\frac{1}{2}\right)$$

$$I_1=\left(\frac{\pi}{4}-\frac{1}{2}\right)$$

$$\text{Now}\ \ I=2I_1=2\left(\frac{\pi}{4}-\frac{1}{2}\right)$$

$$\boxed{I=\frac{\pi}{2}-1}$$

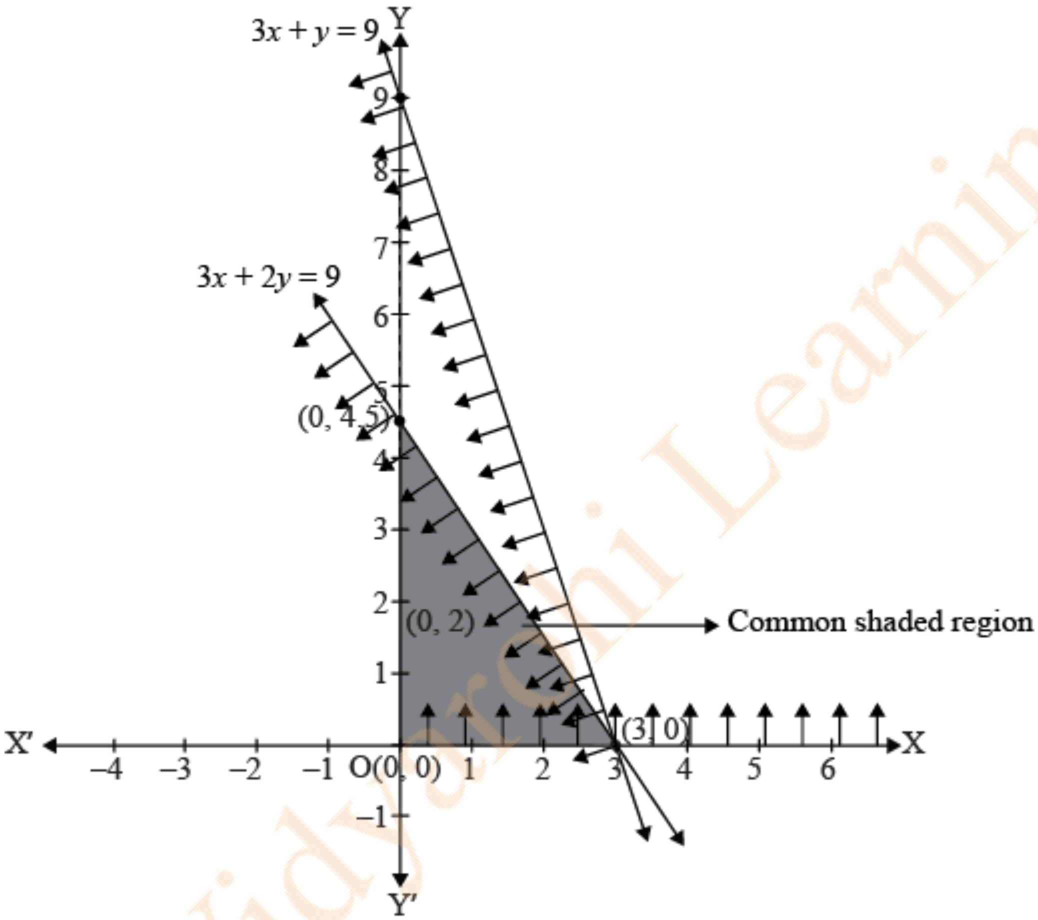
Solution 34

$3x + 2y = 9$

x	0	3
y	4.5	0

$3x + 2y = 9$

x	0	3
y	9	0



Corner Points → (0, 0), (3, 0) and (0, 4.5)

Points	Z = 70x + 40y
(0, 0)	0
(3, 0)	210
(0, 4.5)	180

Z is maximized at (3, 0) and value of z is 210.

Solution 35

Let event
A : Student knows the answer
B: Student guesses the answer

$\mathrm{P}(\mathrm{A})=\frac{3}{5},\ \mathrm{P}(\mathrm{B})=\frac{2}{5} \qquad [\text{Given}]$

E: Answer is correct

$\mathrm{P}\left(\frac{\mathrm{E}}{\mathrm{B}}\right)=\frac{1}{3}\ (\text{Given}),\ \ \mathrm{P}\left(\frac{\mathrm{E}}{\mathrm{A}}\right)=1$

$\mathrm{P}\left(\frac{\mathrm{A}}{\mathrm{E}}\right)=?$

By Bayes Theorem

$$\mathrm{P}\left(\frac{\mathrm{A}}{\mathrm{E}}\right)=\frac{\mathrm{P(A)}\cdot\mathrm{P}\left(\frac{\mathrm{E}}{\mathrm{A}}\right)}{\mathrm{P(A)}\,\mathrm{P}\left(\frac{\mathrm{E}}{\mathrm{A}}\right)+\mathrm{P(B)}\,\mathrm{P}\left(\frac{\mathrm{E}}{\mathrm{B}}\right)}$$

$$=\frac{\frac{3}{5}\times 1}{\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{3}}=\frac{\frac{3}{5}}{\frac{3}{5}+\frac{2}{15}}=\frac{\frac{3}{5}}{\frac{9+2}{15}}$$

$$=\frac{3}{5}\times\frac{15}{11}=\frac{9}{11}$$

OR

Let *x* : Prize of ticket

x values can be 2, 4 or 8.

$\text{For } x=2,\ \mathrm{P}(x)=\frac{3}{10}$

$\text{For } x=4,\ \mathrm{P}(x)=\frac{5}{10}$

$\text{For } x=8,\ \mathrm{P}(x)=\frac{2}{10}$

Prob. distribution table

x	2	4	8
P(*x*)	$\frac{3}{10}$	$\frac{5}{10}$	$\frac{2}{10}$

$$\text{Mean Value }=2\times\frac{3}{10}+4\times\frac{5}{10}+8\times\frac{2}{10}$$

$$=\frac{6}{10}+\frac{20}{10}+\frac{16}{10}$$

$$=\frac{42}{10}$$

$$=4.2$$

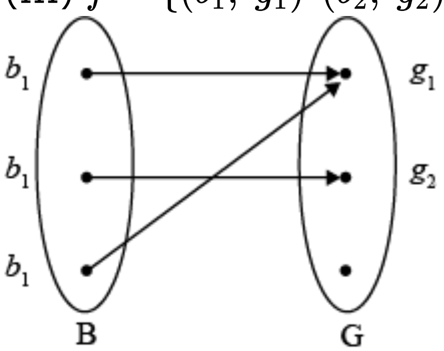
Solution 36

(I) Total number of relations from B to G $= 2^{3\times 2}$
$= 2^6$
$= 64$
(II) Total number of functions from B to G $= 2^3$
$= 8$
(III) $R = \{(x, y)\ ;\ x$ and y are students of same$\}$, R: B → B
For Reflexive, $(x, x) \in R \to x$ and x are students of same sex which is true
So, it is reflective relation
For Symmetric, If $(x, y) \in R \to x$ and y are of same sex then $(y, x) \in R \to y$ and x are of same sex which is true.
So, it is symmetric relation.

For transitive, if $(x, y) \in R \to x$ and y are of same sex
$(y, z) \in R \to y$ and z are of same sex then $(x, z) \in R \to x$ and z are of same sex which is true
So, it is transitive relation.
∴ It is an equivalence Relation.

OR

(III) $f = \{(b_1,\ g_1)\ (b_2,\ g_2)\ (b_3,\ g_1)\}$



It is many-one and onto function.
So, it is a surjective function.

Solution 37

Let cost of 1 pen $= x$
1 bag $= y$
1 instrument box $= z$
For Gautam total price = 160
$5x + 3y + z = 160$
For Vikram, total price = 190
$\therefore 2x + y + 3z = 190$
For Ankur, total price = 250
$\therefore x + 2y + 4z = 250$

(I) In the matrix form
AX = B

$$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$$

(II) |A|

Expanding along row 1.

$$= 5\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 5(-2) - 3(5) + 1(3)$$

$$= -10 - 15 + 3$$

$$= -22$$

(III) $A^{-1} = \frac{\text{adj } A}{|A|}$

Minors	**Co-factors**
$M_{11} = -2$	$C_{11} = -2$
$M_{12} = 5$	$C_{12} = -5$
$M_{13} = 3$	$C_{13} = 3$
$M_{21} = 10$	$C_{21} = -10$
$M_{22} = 19$	$C_{22} = 19$
$M_{23} = 7$	$M_{23} = -7$
$M_{31} = 8$	$C_{31} = 8$
$M_{32} = 13$	$C_{32} = -13$
$M_{33} = -1$	$M_{33} = -1$

Co-factor matrix

$$C = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}$$

$$\text{adj } A = C^T = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} \frac{1}{11} & \frac{5}{11} & \frac{-4}{11} \\ \frac{5}{22} & \frac{-19}{22} & \frac{13}{22} \\ \frac{-3}{22} & \frac{7}{22} & \frac{1}{22} \end{bmatrix}$$

OR

(III) $P = A^2 - 5A$

$$A^2 = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix}$$

$$5A = \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

Now

$$P = A^2 - 5A$$

$$= \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$$

Solution 38

(I) $\left(x^2 - y^2\right)dx + 2xy\, dy = 0$

$2xy\, dy = -\left(x^2 - y^2\right)dx$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

$$\frac{dy}{dx} = \frac{y^2}{2xy} - \frac{x^2}{2xy}$$

$$\frac{dy}{dx} = \frac{y}{2x} - \frac{x}{2y}$$

$$F(x,\ y) = \frac{y}{2x} - \frac{x}{2y}$$

$$F(\lambda x,\ \lambda y) = \frac{\lambda y}{2\lambda x} - \frac{\lambda x}{2\lambda y}$$

$$= \frac{y}{2x} - \frac{x}{2y}$$

$$= \lambda^{\circ} F(x,\ y)$$

So, $F(x,\ y)$ is $g\left(\frac{y}{x}\right)$ type homogeneous function of 0 degree.

(II) $\frac{dy}{dx} = \frac{y}{2x} - \frac{x}{2y}$

Let $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + \frac{xdv}{dx} = \frac{vx}{2x} - \frac{x}{2vx}$

$v + \frac{xdv}{dx} = \frac{v}{2} - \frac{1}{2v}$

$\frac{xdv}{dx} = \frac{v^2-1}{2v} - v$

$\frac{xdv}{dx} = \frac{-v^2-1}{2v}$

$\frac{2vdv}{v^2+1} = -\frac{dx}{x}$

Integrate both sides

$\int \frac{2v}{v^2+1}dv = -\int \frac{dx}{x}$

Let $v^2 + 1 = t$

Differentiate w.r.t 'v'

$2v = \frac{dt}{dv}$

$2vdv = dt$

$\int \frac{dt}{t} = -\int \frac{dx}{x}$

$\log_e t = -\log_e x + \log_e C$

$\log_e t = \log\left(\frac{C}{x}\right)$

$t = \frac{C}{x}$

Substitute $t = v^2 + 1$

$v^2 + 1 = \frac{C}{x}$

Substitute $v = \frac{y}{x}$

$\frac{y^2}{x^2} + 1 = \frac{C}{x}$

$\frac{y^2}{x^2} = \frac{C}{x} - 1$

$y^2 = x^2\left(\frac{C-x}{x}\right)$

$y^2 = x\left(C - x\right)$

$y^2 = Cx - x^2$

Thus, the required general solution is $y^2 = Cx - x^2$.