JEE Main 24 June 2022(First Shift)

Total Time: 180

Total Marks: 300.0

Solution 1

$\therefore B=\frac{\Delta P}{\left(-\frac{\Delta V}{V}\right)}$

$\Rightarrow \Delta P=3\times 10^{10}\times(0.02)$

$\quad\quad = 6\times 10^{8}\ \mathrm{N/m^2}$

Hence, the correct answer is option C.

Solution 2

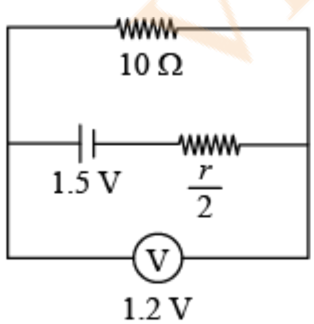
Magnetic force $\vec{F}\perp\vec{v}$

⇒ $W_b = 0$

⇒ ΔKE = 0 and speed remains constant.

Hence, the correct answer is option A.

Solution 3



$\frac{1.5\times 10}{10+\frac{r}{2}}=1.2$

$\Rightarrow r=5\ \Omega$

Hence, the correct answer is option C.

Solution 4

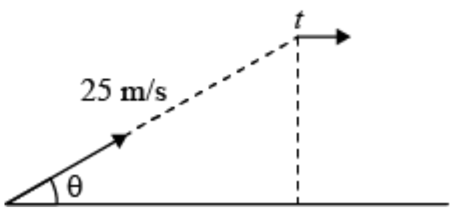
$$[S] = \frac{[C]}{[m] \times [\Delta T]}$$

$$\text{and,} \ \ [L] = \frac{[Q]}{[m]}$$

⇒ They have different dimensions

Hence, the correct answer is option D.

Solution 5



$$t = \frac{25 \ \sin \ \theta}{g}$$

$$\text{and,} \ R = \frac{(25)^2(2\sin\theta\cos\theta)}{g}$$

$$\Rightarrow R = \frac{25 \times 25 \times 2}{g} \times \frac{gt}{25} \times \cos\theta$$

$$\Rightarrow R = 50t \ \cos\theta$$

$$\therefore \tan\theta = \frac{gt}{25} \times \frac{50t}{R}$$

$$= \frac{20t^2}{R}$$

$$\Rightarrow \theta = \cot^{-1}\left(\frac{R}{20t^2}\right)$$

Hence, the correct answer is option D.

Solution 6

$$S = \frac{u^2}{2a} = \frac{u^2}{2(\mu g)}$$

$$= \frac{(9.8)^2}{2 \times 0.5 \times (9.8)}$$

$$= \frac{9.8}{1}$$

$$= 9.8 \text{ m}$$

Hence, the correct answer is option B.

Solution 7

$$T = m\omega^2 r$$
$$\Rightarrow 80 = 0.1 \times \left(2\pi \times \frac{K}{\pi} \times \frac{1}{60}\right)^2 \times 2$$
$$\Rightarrow \frac{800}{2} = \frac{K^2}{900}$$
$$\Rightarrow K = 30 \times 20 = 600$$

Hence, the correct answer is option C.

Solution 8

Since the droplet is at rest

⇒ Net force = 0

⇒ $mg = qE$

$$\Rightarrow q = \frac{mg}{E} = 2 \times 10^{-9}\ \mathrm{C}$$

Hence, the correct answer is option B.

Solution 9

$$W = \int \overrightarrow{F} \cdot d\overrightarrow{r}$$
$$= \int_1^2 4x dx + \int_2^3 3y^2 dy$$
$$= \left[2x^2\right]_1^2 + \left[y^3\right]_2^3$$
$$= 2 \times 3 + (27 - 8)$$
$$= 25\ \mathrm{J}$$

Hence, the correct answer is option C.

Solution 10

According to the given information

$$\frac{GM}{(R+h)^2} = \frac{1}{3} \times \frac{GM}{R^2}$$
$$\Rightarrow R + h = \sqrt{3}R$$
$$\Rightarrow h = \left(\sqrt{3} - 1\right)R \simeq 4685\ \mathrm{km}$$

Hence, the correct answer is option B.

Solution 11

$I = I_0\cos(\omega t)$ say

$\Rightarrow$ At maximum $\omega t_1 = 0$ or $t_1 = 0$

Then at rms value $I = \frac{I_0}{\sqrt{2}}$

$\Rightarrow \omega t_2 = \frac{\pi}{4}$

$\Rightarrow \omega\left(t_2 - t_1\right) = \frac{\pi}{4}$

$\Delta\, t = \frac{\pi}{4\omega} = \frac{\pi T}{4\times 2\pi}$

$= \frac{1}{400}\ \text{s or } 2.5\ \text{ms}$

Hence, the correct answer is option A.

Solution 12

$y_1 = 5\sin(2\pi x - 2\pi vt)$

$y_2 = 3\sin(2\pi x - 2\pi vt + 3\pi)$

$\Rightarrow$ Phase difference = 3π

$\Rightarrow A_{\text{net}} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(3\pi)}$

$\Rightarrow A_{\text{net}} = 2\ \text{cm}$

Hence, the correct answer is option A.

Solution 13

$H = 4.5 \times 10^{-2}$

So $B = \mu_0\mu H$

Thus $E = \frac{c}{n}B$ (where $n \Rightarrow$ refractive index)

So $E = \frac{3\times10^{8}\times4\pi\times10^{-7}\times1.61\times4.5\times10^{-2}}{\sqrt{1.61\times6.44}}$

$E = 8.48$

Hence, the correct answer is option C.

Solution 14

An atom based on classical theory of Rutherford's model should collapse as the electrons in continuous circular motion that is a continuously accelerated charge should emit EM waves and so should lose energy. These electrons losing energy should soon fall into heavy nucleus collapsing the whole atom.

Hence, the correct answer is option C.

Solution 15

$$^{220}A \rightarrow {}^{105}B + {}^{115}C$$
$$\Rightarrow Q = [105 \times 6.4 + 115 \times 6.4] - [220 \times 5.6] \text{ MeV}$$
$$\Rightarrow Q = 176 \text{ MeV}$$

Hence, the correct answer is option D.

Solution 16

$$\nu_c = 3.5 \times 10^9 \text{ Hz}$$
$$\therefore \lambda = \frac{c}{\nu_c} = \frac{3\times10^8}{3.5\times10^9}$$

$$\therefore \text{ Size of antenna } = \frac{\lambda}{4}$$
$$= \frac{8.57\times10^{-2}}{4}$$
$$= 21.4 \text{ mm}$$

Hence, the correct answer is option C.

Solution 17

$$\text{Initially } : \frac{1}{4} = 1 - \frac{300}{T_H}$$
$$\Rightarrow T_H = 400 \text{ K}$$
$$\text{Finally } : \text{ Efficiency becomes } \frac{1}{2}$$
$$\Rightarrow \frac{1}{2} = 1 - \frac{300}{T'_H}$$
$$\Rightarrow T'_H = 600 \text{ K}$$

⇒ Temperature of the source increases by 200°C.

Hence, the correct answer is option B.

Solution 18

Field inside the dielectric $= \frac{\sigma}{k\varepsilon_o}$

According to the given information,

$$\frac{\sigma}{k\varepsilon_o} = 3.6 \times 10^7$$
$$\Rightarrow \frac{\frac{Q}{A}}{k\varepsilon_o} = 3.6 \times 10^7$$
$$\Rightarrow k = 2.33$$

Hence, the correct answer is option D.

Solution 19

$$B=\frac{\mu_o I}{2r}$$

$$B_a=\frac{\mu_o I r^2}{2\left(r^2+\frac{r^2}{4}\right)}$$

$$\Rightarrow \frac{B_a}{B}=\left(\frac{2}{\sqrt{5}}\right)^3$$

$$\Rightarrow B_a=\left(\frac{2}{\sqrt{5}}\right)^3 B$$

Hence, the correct answer is option C.

Solution 20

Thermal current is same so

$$\frac{dQ}{dt}=\frac{\Delta T_1}{\frac{l_1}{K_1 A}}=\frac{\Delta T_2}{\frac{l_2}{K_2 A}}$$

$$\text{or } \frac{20}{16}\times K' =\frac{80}{8}\times K$$

$$\Rightarrow K'=8\text{ K}$$

Hence, the correct answer is option B.

Solution 21

Because the vessel is closed, it will be an isochoric process.

To double the speed, temperature must be 4 times $\left(v\alpha\sqrt{T}\right)$

So T_f = 1600 K, T_i = 400 K

Number of moles are $\frac{56}{28}=2$

$$\text{so } Q=\text{nCv } \Delta T=2\times\frac{5}{2}\times 2\times 1200$$

$$=12000 \text{ cal } =12\,k \text{ cal}$$

Solution 22

$$\frac{1}{f_l}=\left(\frac{\mu_e}{\mu_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here $|R_1|=|R_2|=R$

$$\Rightarrow \frac{1}{f_{l_1}}=(1.5-1)\left(\frac{2}{R}\right)=\frac{1}{15}$$

$$\Rightarrow \frac{1}{R}=\frac{1}{15} \text{ or } R=15 \text{ cm}$$

For the concave lens made up of liquid

$$\frac{1}{f_{l_2}}=(1.25-1)\left(-\frac{2}{R}\right)=-\frac{1}{30} \text{ cm}$$

Now for equivalent lens

$$\frac{1}{f_e}=\frac{2}{f_{l_1}}+\frac{1}{f_{l_2}}$$

$$=\frac{2}{15}-\frac{1}{30}=\frac{3}{30}=\frac{1}{10}$$

or $f_e=10$ cm

Solution 23

$$R_B=\frac{10\times10^{-3}}{10\times10^{-6}}$$

$$=10^3\ \Omega$$

$$\therefore A_v=\left(\frac{\Delta I_C}{\Delta I_B}\right)\times\left(\frac{R_C}{R_B}\right)$$

$$=\frac{1.5\times10^{-3}}{10\times10^{-6}}\times\frac{5\times10^3}{1\times10^3}$$

$$=\frac{1.5\times5}{10}\times(1000)$$

$$=750$$

Solution 24

$$I_{\text{rms}}=\frac{V_{\text{rms}}}{z}$$

$$z=X_2=\omega_2$$

$$=2\pi\times50\times\frac{200}{1000}$$

$$=20\pi$$

$$\therefore I_{\text{rms}}=\frac{220}{20\pi}=\frac{11}{\pi}$$

$\therefore I_{\text{peak}} = \sqrt{2} \times \frac{11}{\pi}$

$= \frac{\sqrt{2 \times 121}}{\pi}$

$= \frac{\sqrt{242}}{\pi}$

Solution 25

Position of 1st maxima is $\frac{3\lambda D}{2a}$

⇒ According to given values, required separation

$= \frac{3}{2} \times (655 \text{ nm} - 650 \text{ nm}) \times \frac{2\text{m}}{0.5 \text{ mm}}$

⇒ Required separation = 3×10^{-5} m.

Solution 26

Let us say the work function is ϕ

$\Rightarrow 2\phi = \phi + \frac{1}{2}mv_1^2 \quad \ldots\ldots(1)$

and $5\phi = \phi + \frac{1}{2}mv_2^2 \quad \ldots\ldots(2)$

From (1) and (2)

$\frac{v_2^2}{v_1^2} = \frac{4}{1}$ or $\frac{v_2}{v_1} = 2$

Solution 27

Based on the situation

$h = -ut_1 + \frac{1}{2}gt_1^2 \rightarrow$ throwing up $\ldots\ldots$(i)

$h = ut_2 + \frac{1}{2}gt_2^2 \rightarrow$ throwing down $\ldots\ldots$(ii)

$h = \frac{1}{2}gt^2 \rightarrow$ dropping $\ldots\ldots$(iii)

and $0 = u(t_1 - t_2) - \frac{1}{2}g(t_1 - t_2)^2 \quad \ldots\ldots$(iv)

Solving above equations $t = \sqrt{t_1 t_2}$

$\Rightarrow t = \sqrt{6 \times 1.5} = 3 \text{ s}$

Solution 28

$mg\left(h + \frac{h}{2}\right) = \frac{1}{2}k\left(\frac{h}{2}\right)^2$

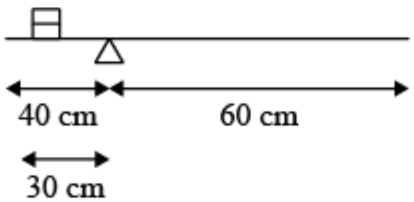
$\Rightarrow 0.1 \times 10 \times (0.15) = \frac{1}{2}k(0.05)^2$

$\Rightarrow k = 120 \text{ N/m}$

Solution 29

At balancing point, we know that emf is proportional to the balancing length.
i.e., emf $\propto$ balancing length
Now, let the emf's be 3ε and 2ε.
$\Rightarrow 3\varepsilon = k(75)$ .....(1)
and $2\varepsilon = k(l)$ .....(2)
$\Rightarrow l = 50$ cm
$\Rightarrow$ Difference is (75 – 50) cm = 25 cm.

Solution 30



If λ is the mass per unit length of the scale then
$0.02 \times (30) \times 10 + \lambda 40 \times 20 \times 10 = \lambda 60 \times 30 \times 10$
$0.006 = \lambda 10$
Or $100\lambda = 0.06$ kg
$= 6 \times 10^{-2}$ kg
$\Rightarrow x = 6$

Solution 31

$$C_{15}H_{30} + \frac{45}{2}O_2 \rightarrow 15\,CO_2 + 15H_2O$$

One litre of fuel has a mass (0.756) × 1000 g.

$\therefore$ Moles of $C_{15}H_{30}$ = $\frac{756}{210}$

Moles of O_2 required = $\frac{45}{2} \times \frac{756}{210}$

Mass of O_2 required = $\frac{45}{2} \times \frac{756}{210} \times 32\text{ g} = 2592\text{ g}$

Mass of CO_2 formed = $15 \times \frac{756}{210} \times 44 = 2376\text{ g}$

Hence, the correct answer is option C.

Solution 32

For degenerate orbitals, only the value of *m* must be different. The value of '*n*' and '*l*' must be the same. Hence, the pair of electrons with quantum numbers given in (B) are degenerate.

Hence, the correct answer is option B.

Solution 33

Complex/compound Hybridisation of central atoms

List-I	List-II
(A) $[PtCl_4]^{2-}$	(III) dsp^2
(B) BrF_5	(IV) sp^3d^2
(C) PCl_5	(I) sp^3d
(D) $[Co(NH_3)_6]^{3+}$	(II) d^2sp^3

Hence, the most appropriate answer is given in option (B)

Hence, the correct answer is option B.

Solution 34

$$\begin{array}{lcccc} & A(g) & \rightleftharpoons & B(g) & + & \frac{1}{2}C(g) \\ t=0 & p_i & & - & & - \\ \underset{\text{(eq. m conditions)}}{t=t} & p_i - p_i\alpha & & \mathrm{p_i}\alpha & & \frac{p_i\alpha}{2} \end{array}$$

$$\therefore \mathrm{P}\,(\text{equilibrium pressure}) = p_i - p_i\alpha + p_i\alpha + \frac{p_i\alpha}{2}$$
$$= p_i\left(1+\frac{\alpha}{2}\right)$$

$$\therefore p_i = \frac{P}{\left(1+\frac{\alpha}{2}\right)}$$

$$K_P = \frac{\left(p_i\frac{\alpha}{2}\right)^{\frac{1}{2}} \times p_i\alpha}{p_i(1-\alpha)} = \frac{p^{\frac{1}{2}}\alpha^{\frac{3}{2}}}{\left(1+\frac{\alpha}{2}\right)^{\frac{1}{2}}(1-\alpha)} \times \frac{1}{2^{\frac{1}{2}}}$$

$$= \frac{p^{\frac{1}{2}}\alpha^{\frac{3}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$$

Hence, the correct answer is option B.

Solution 35

Oil in water emulsions can sometimes separate into two layers on standing. The most relevant example for the above case is milk, which can separate into two layers on standing for a longer time. Therefore, statement (I) is correct. On adding an excess of electrolyte, coagulation occurs and emulsion is further destabilised. Therefore, statement (II) is incorrect.

Hence, the correct answer is option C.

Solution 36

Oxides
Na_2O → Basic
As_2O_3 → Amphoteric
N_2O → Neutral
NO → Neutral
Cl_2O_7 → Acidic
Hence, only one amphoteric oxide is present.

Hence, the correct answer is option B.

Solution 37

Ores	Formula
(A) Sphalerite	(IV) ZnS
(B) Calamine	(III) $ZnCO_3$
(C) Galena	(II) PbS
(D) Siderite	(I) $FeCO_3$

Hence, the correct answer is option A.

Solution 38

Hydrogen combines with nitrogen to produce
Ammonia in Haber's process.

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

In this process, iron oxide is used with small amounts of K_2O and Al_2O_3 to increase the rate of attainment of equilibrium.
Optimum conditions for the production of ammonia are a pressure of 200 atm and a temperature of 700 K.
Earlier, iron was used as a catalyst with molybdenum as promoter in this reaction.

Hence, the correct answer is option B.

Solution 39

(A) Both LiCl and $MgCl_2$ are covalent in nature due to high polarizing power of Li^+ and Mg^{+2} ions.
Hence, they are soluble in ethanol.

(B) Oxides of Li_2O and MgO do not form super oxide.

(C) LiF is least soluble among all other alkali metal fluorides due to high lattice energy of LiF.

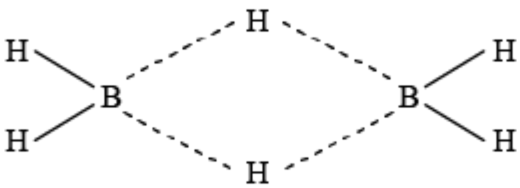
(D) Li_2O is least soluble among all other alkali metal oxides.

Hence, Statements (A) and (C) are correct.

Hence, the correct answers are options A.

Solution 40

Structure of B_2H_6



It has two 3-centre-2-electron bonds and four 2-centre-2-electron bonds.
Hence, all B – H bonds are not equivalent.
It is an electron deficient compound as the octet of boron is incomplete.
Hence, it can behave as a Lewis acid.
It can be synthesized from both BF_3 and $NaBH_4$

$$2\,BF_3 \;+\; 6\,NaH \xrightarrow{450\text{ K}} B_6H_6 \;+\; 6\,NaF$$

$$2\,NaBH_4 \;+\; I_2 \longrightarrow B_6H_6 \;+\; 2NaI + H_2$$

It is a non-planar molecule.
Hence, only Statements (C) and (D) are correct.

Hence, the correct answer is option C.

Solution 41

The most stable trihalide is NF_3
Order of stability: $NF_3 > NCl_3 > NBr_3 > NI_3$
NCl_3 is explosive is nature.
NBr_3 and NI_3 are known only as ammoniates. The stability of trihalides decreases down the group due to weakening of N – X bond and inability of N to accommodate large sized halogen atoms (Cl, Br, I) around it.

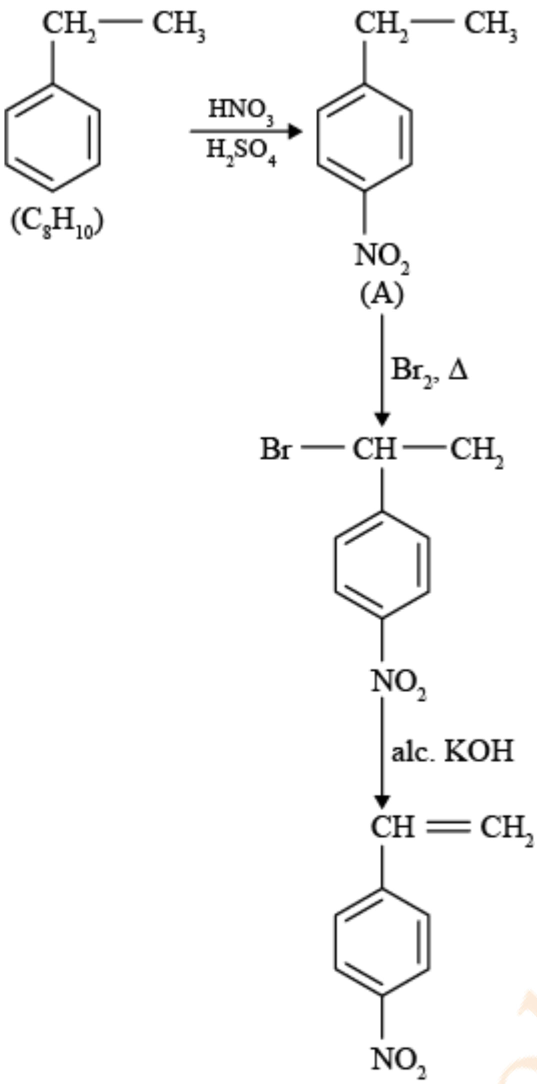
Hence, the correct answer is option A.

Solution 42

P^{+3} is not present in the enamel of teeth. The compound present is [$3Ca_3(PO_4)_2 \cdot CaF_2$] which contains Ca^{+2}, P^{+5} & F^-.

Hence, the correct answer is option B.

Solution 43



Hence, the correct answer is option B.

Solution 44

Statement (I) is correct as monocarboxylic acids with even number of carbon atoms show better packing efficiency in solid state, **statement (II)** is also correct as the solubility of carboxylic acids decreases with increase in molar mass due to increase in the hydrophobic portion with increase in the number of carbon atoms.

Hence, the correct answer is option A.

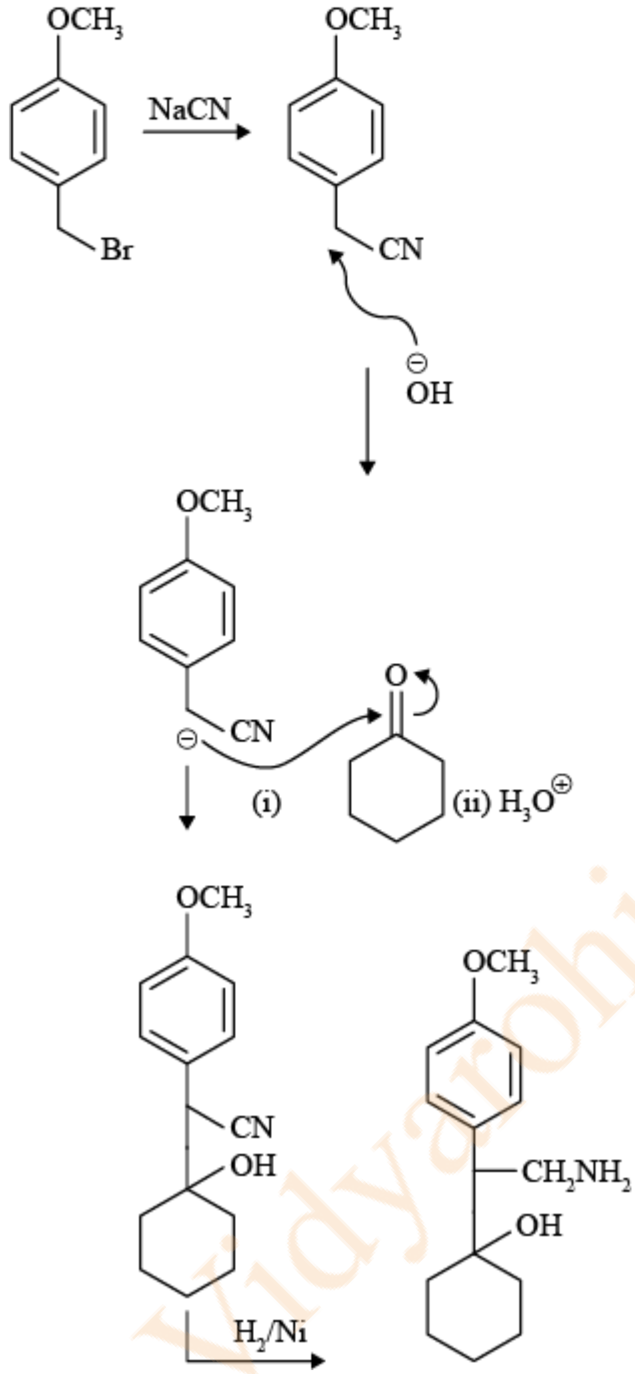
Solution 45

 is a conjugated diketone.

In rest of the diketones given in the question, the two (C = O) groups are not in conjugation with each other.

Hence, the correct answer is option C.

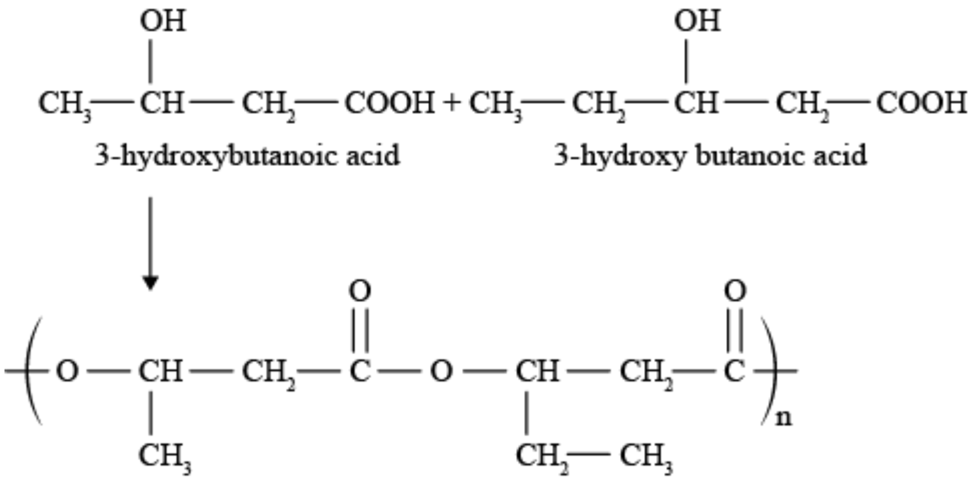
Solution 46



Hence, the correct answer is option D.

Solution 47

Polyesters are formed by condensation reaction between alcohols and carboxylic acid.
Poly-β-hydroxybutyrate-co-β-hydroxy valerate (PHBV) is a polymer obtained by condensation reaction of 3-hydroxybutanoic acid with 3-hydroxypentanoic acid.



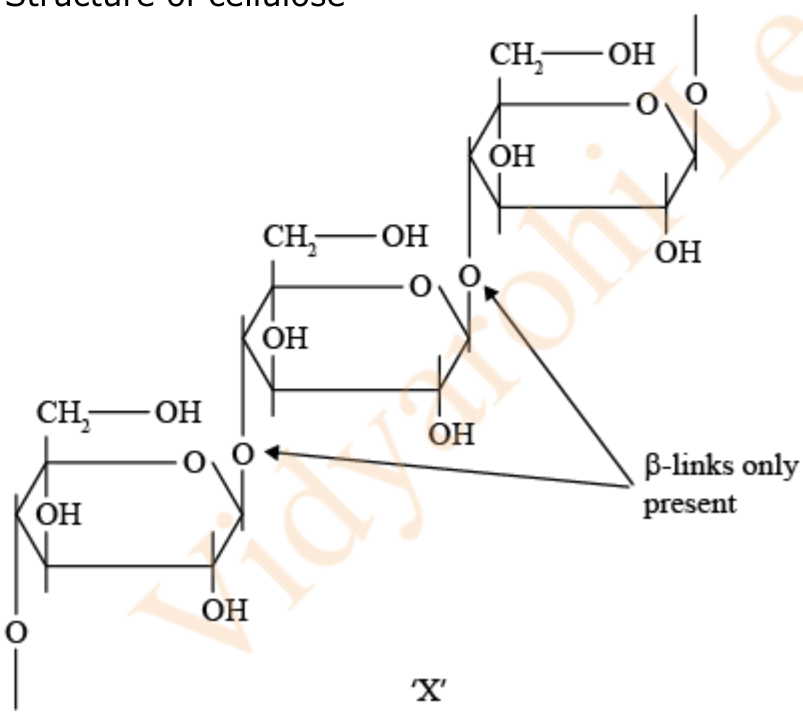
PHBV

Hence, PHBV is a polyester.

Hence, the correct answer is option D.

Solution 48

Cellulose contains β-glycosidic linkages only.
Structure of cellulose



'X'

On boiling with dil. H_2SO_4 at 393 K under 2-3 atm, 'X' forms glucose, which given gluconic acid on treatment with bromine water.

Hence, the correct answer is option B.

Solution 49

Penicillin G is a narrow spectrum antibiotic.

Hence, the correct answer is option D.

Solution 50

On addition of dimethylglyoxime to alkaline solution of Ni^{+2}, a bright red ppt. is obtained.
$Ni^{+2} + 2dmg \rightarrow [Ni(dmg)_2]^{+2}$ (Bright red ppt)

Hence, the correct answer is option A.

Solution 51

Since X occupies hcp lattice,
Number of particles of type X in a unit cell = 6
Number of particles of type $\mathrm{Y} = \frac{2}{3} \times 12 = 8$

$$\begin{aligned} \therefore \text{ Percentage of element X} &= \frac{6}{14} \times 100 \\ &= \frac{300}{7} \\ &= 42.85 \\ &\simeq 43\% \end{aligned}$$

Solution 52

$$\underset{1-x}{2O_3(g)} \rightleftharpoons \underset{\frac{3x}{2}}{3O_2(g)}$$

Given, x = 0.5

$$\therefore k_p = \frac{[3(0.5)]^3 \times 1}{[2]^3 \times (0.5)^2 \times 1.25}$$

$$\therefore k_p = \frac{27}{8} \times \frac{0.5}{1.25} = 1.35$$

$$\begin{aligned} \Delta G^\circ &= -2.303 \text{ RT } \log k_p \\ &= -2.303 \times 8.3 \times 300 \log 1.35 \\ &= -8.3 \times 300 \ln(1.35) \\ &= -747 \text{ J mol}^{-1} \end{aligned}$$

Solution 53

7.47 = C × 0.083 × 300

(π = CRT)

(Where C represents the concentration of glucose solution and π represents osmotic pressure)

$$C = \frac{7.47}{0.083 \times 300} \left(\text{mol L}^{-1}\right)$$

$$\begin{aligned} \text{which in gm/L} &= \frac{7.47}{0.083 \times 300} \times 180 \\ &= 54 \text{ gm/l} \end{aligned}$$

Solution 54

$$E_{cell} = E^0_{cell} - \frac{0.06}{2} \log \frac{[H^{\oplus}]^2}{[Cu^{+2}]}$$

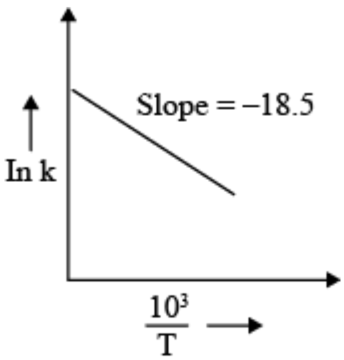
$$0.576 = 0.34 - 0.03 \log \frac{[H^{\oplus}]^2}{[0.01]}$$

$$0.576 - 0.34 = -0.03 \log [H^{\oplus}]^2 + 0.03 \log (0.01)$$

$$= 0.06\,pH - 0.06$$

$$pH \simeq 4.93 = 5$$

Solution 55



$$\ln k = \ln A - \frac{E_a}{RT}$$

$$\therefore \text{ Slope of the graph} = -\frac{E_a}{R \times 10^3} = -18.5$$

$$\therefore E_a = 18.5 \times 8.31 \times 1000 \simeq 154 \text{ kJ mol}^{-1}$$

Solution 56

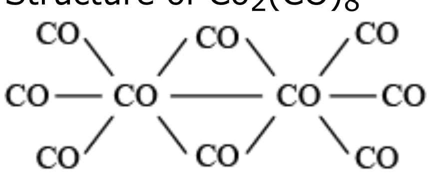
Chromate ion → CrO_4^{2-}, oxidation state of Cr = +6

Dichromate ion → CrO_4^{2-}, oxidation state of Cr = +6

∴ Difference in oxidation state = zero

Solution 57

Structure of $Co_2(CO)_8$



Number of Co – Co bonds = 1 = X
Number of terminal CO ligands = 6 = Y
∴ X + Y = 1 + 6 = 7

Solution 58

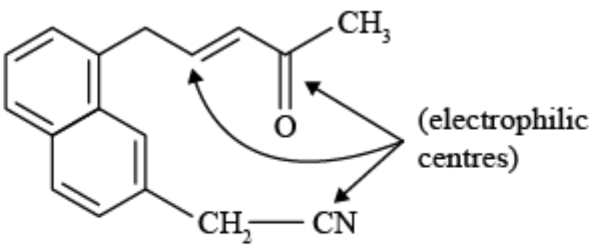
Millimoles of used acid $= \frac{30 \times 0.25}{2}$

Millimoles of NH_3 = 30 × 0.25 = 7.5

Mass% of nitrogen $= \frac{7.5}{0.166} \times 10^{-3} \times 14 \times 100 \simeq 63\%$

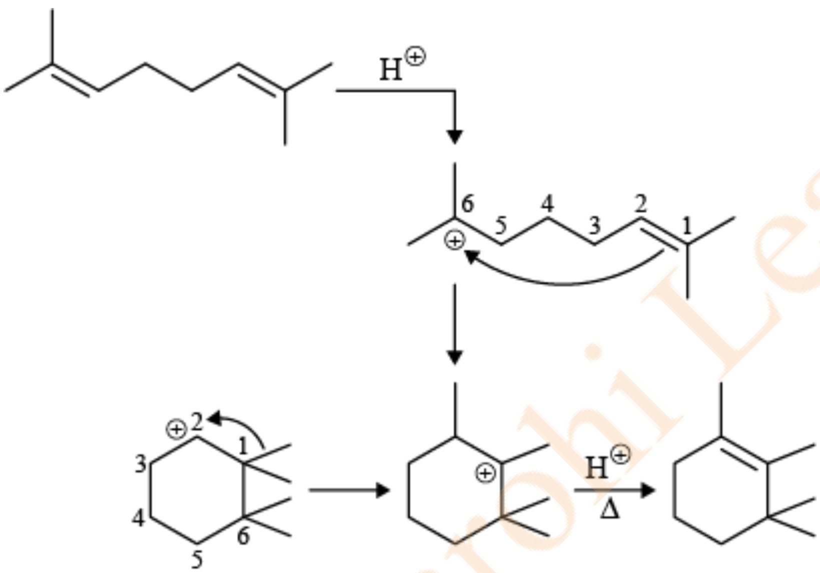
Solution 59

Given compounds :



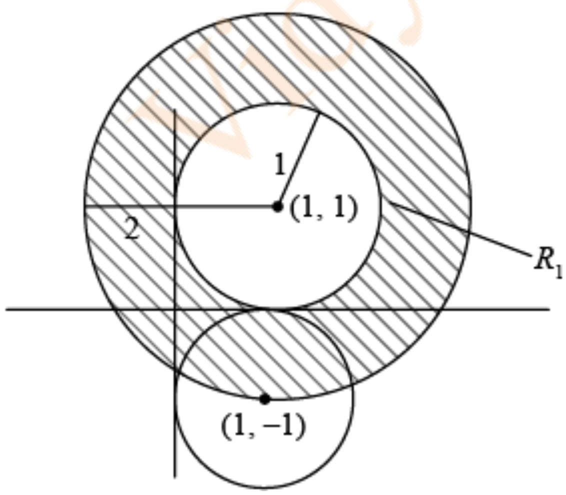
Number of electrophilic centres = 3

Solution 60



Number of sp^2 hybridised carbon atoms = 2

Solution 61



Set A represents region 1, i.e., R_1 and clearly set B has infinite points in it.

Hence, the correct answer is option D.

Solution 62

$$3^{2022}=(10-1)^{1011}$$
$$={}^{1011}C_0(10)^{1011}(-1)^0+{}^{1011}C_1(10)^{1010}(-1)^1+\ldots\ldots+{}^{1011}C_{1010}(10)^1(-1)^{1010}$$
$$=5k-1,\ \text{where } k\in I$$

So when divided by 5, it leaves remainder 4.

Hence, the correct answer is option D.

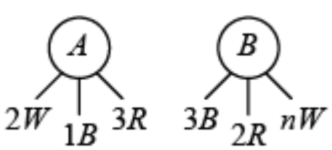
Solution 63

$$S=4\pi r^2$$
$$\frac{dS}{dt}=8\pi r\frac{dr}{dt}$$
$$\frac{dS}{dt}=\text{constant}$$
$$\text{so }\Rightarrow r\frac{dr}{dt}=k\ (\text{Let})$$
$$r\,dr=k\cdot dt$$
$$\Rightarrow\frac{r^2}{2}=kt+C$$

$$\text{at } t=0,\ r=3$$
$$\frac{9}{2}=C$$
$$\text{at } t=5,\ r=7$$
$$\frac{49}{2}=k\cdot 5+\frac{9}{2}$$
$$\Rightarrow k=4$$
$$\text{At } t=9,$$
$$\frac{r^2}{2}=\frac{81}{2}$$
$$\text{So, } r=9$$

Hence, the correct answer is option A.

Solution 64



$$P(1R \text{ and } 1B) = P(A) \cdot P\left(\frac{1R\,1B}{A}\right) + P(B) \cdot P\left(\frac{1R\,1B}{B}\right)$$

$$= \frac{1}{2} \cdot \frac{{}^3C_1 \cdot {}^1C_1}{{}^6C_2} + \frac{1}{2} \cdot \frac{{}^2C_1 \cdot {}^3C_1}{{}^{n+5}C_2}$$

$$P\left(\frac{1R\,1B}{A}\right) = \frac{\frac{1}{2} \cdot \frac{3}{15}}{\frac{1}{2} \cdot \frac{3}{15} + \frac{1}{2} \cdot \frac{6.2}{(n+5)(n+4)}} = \frac{6}{11}$$

$$\Rightarrow \frac{\frac{1}{10}}{\frac{1}{10} + \frac{6}{(n+5)(n+4)}} = \frac{6}{11}$$

$$\Rightarrow \frac{11}{10} = \frac{6}{10} + \frac{36}{(n+5)(n+4)}$$

$$\Rightarrow \frac{5}{10 \times 36} = \frac{1}{(n+5)(n+4)}$$

$$\Rightarrow n^2 + 9n - 52 = 0$$

$$\Rightarrow n = 4 \text{ is only possible value}$$

Hence, the correct answer is option C.

Solution 65

For tangent to parabola $y = x^2$ at (2, 4)

$$\left.\frac{dy}{dx}\right|_{(2,\,4)} = 4$$

Equation of tangent is $y - 4 = 4(x - 2)$

$\Rightarrow 4x - y - 4 = 0$

Family of circle can be given by

$$(x-2)^2 + (y-4)^2 + \lambda(4x - y - 4) = 0$$

As it passes through (0, 6)

$$2^2 + 2^2 + \lambda(-10) = 0$$

$$\Rightarrow \lambda = \frac{4}{5}$$

Equation of circle is

$$(x-2)^2 + (y-4)^2 + \frac{4}{5}(4x - y - 4) = 0$$

$$\Rightarrow \left(x^2 + y^2 - 4x - 8y + 20\right) + \left(\frac{16}{5}x - \frac{4}{5}y - \frac{16}{5}\right) = 0$$

$$A = -4 + \frac{16}{5},\ C = 20 - \frac{16}{5}$$

$$\text{So, } A + C = 16$$

Hence, the correct answer is option A.

Solution 66

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix}$$

$$= 1\,(10\alpha\ -\ 9\alpha)\ -\ 1\,(5\alpha\ -\ 3)\ +\ 1\left(3\alpha^2\ -\ 2\alpha\right)$$

$$=\ \alpha\ -\ 5\alpha\ +\ 3\ +\ 3\alpha^2\ -\ 2\alpha$$

$$= 3\alpha^2 - 6\alpha\ +\ 3$$

For inconsistency $\Delta = 0$, i.e., $\alpha = 1$
Now check for $\alpha = 1$
$x + y + z = 1$ .....(i)
$x + 2y + 3z = -1$ .....(ii)
$x + 3y + 5z = 4$ .....(iii)
By (ii) × 2 − (i) × 1
$x + 3y + 5z = -3$
So, equations are inconsistent for $\alpha = 1$

Hence, the correct answer is option B.

Solution 67

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$$

$$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = 15$$

$$\Rightarrow \frac{\frac{\lambda^2}{9} + \frac{2}{3}}{\frac{1}{9}} = 15$$

$$\Rightarrow \frac{\lambda^2}{9} = 1$$

$$\Rightarrow \lambda^2 = 9$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)\ \left(\alpha^2 + \beta^2 - \alpha\beta\right)$$

$$= \left(\frac{-\lambda}{3}\right)\left(\frac{\lambda^2}{9} - 3\left(\frac{-1}{3}\right)\right) = \left(\frac{-\lambda}{3}\right)\left(\frac{\lambda^2}{9} + 1\right) = \frac{-2\lambda}{3}$$

$$6\left(\alpha^3 + \beta^3\right)^2 = 6 \cdot \frac{4\lambda^2}{9} = 24$$

Hence, the correct answer is option B.

Solution 68

Let $\tan^{-1}x = t \in \left(\frac{-\pi}{2},\ \frac{\pi}{2}\right)$

$\cot^{-1}x = \frac{\pi}{2} - t$

$f(t) = t^3 + \left(\frac{\pi}{2} - t\right)^3$

$\Rightarrow f'(t) = 3t^2 - 3\left(\frac{\pi}{2} - t\right)^2$

$f'(t) = 0$ at $t = \frac{\pi}{4}$

$f(t)\Big|_{\min} = \frac{\pi^3}{64} + \frac{\pi^3}{64} = \frac{\pi^3}{32}$

Max will occur around $t = -\frac{\pi}{2}$

Range of $f(t) = \left[\frac{\pi^3}{32},\ \frac{7\pi^3}{8}\right)$

$k \in \left[\frac{1}{32},\ \frac{7}{8}\right)$

Hence, the correct answer is option A.

Solution 69

$|A| = a^2 + 1$

$|\text{adj } A| = \left(a^2 + 1\right)^2$

$S = \left\{1,\ \sqrt{3},\ \sqrt{5},\ \sqrt{7},\ \ldots,\ \sqrt{49}\right\}$

$$\sum_{a\in S}\det\ (\text{adj } A) = \left(1^2+1\right)^2 + (3+1)^2 + (5+1)^2 + \ldots + (49+1)^2$$

$$= 2^2\left(1^2 + 2^2 + 3^2 + \ldots + 25^2\right)$$

$$= 4 \cdot \frac{25\cdot 26\cdot 51}{6}$$

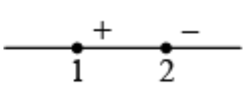
$$= 100 \cdot 221$$

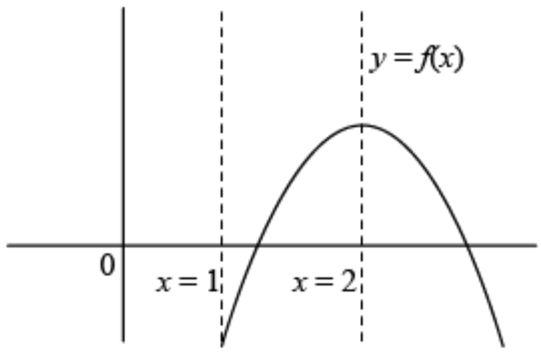
$\Rightarrow \lambda = 221$

Hence, the correct answer is option B.

Solution 70

$f'(x) = \frac{4}{x-1} - 4x + 4 = \frac{4\left(2x - x^2\right)}{x-1}$





So maxima occurs at x = 2

$f(2) = 4\cdot0 - 2\cdot2^2 + 4\cdot2 + 5 = 5$

So clearly $f(x) = -1$ has exactly 2 solutions

$$f''(x) = \frac{4(2-2x)(x-1)}{(x-1)^2} - (2x - x^2)$$

So $f'(e) - f''(2) > 0$

So option c is not correct

Hence, the correct answer is option C.

Solution 71

$m_{op} - m_{Tangent}$

$\frac{y_1}{x_1} = 3x_1^2 + 6x_1$

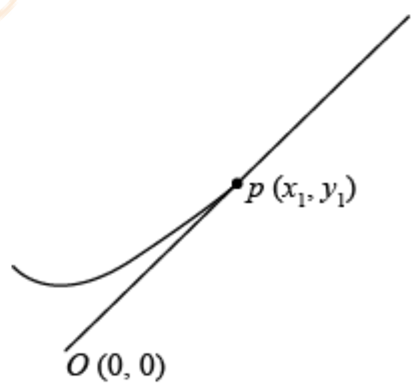
$\Rightarrow \frac{x_1^3+3x_1^2+5}{x_1} = 3x_1^2 + 6x_1$

$\Rightarrow x_1^3 + 3x_1^2 + 5 = 3x_1^3 + 6x_1^2$

$\Rightarrow 2x_1^3 + 3x_1^2 - 5 = 0$

$\Rightarrow (x_1 - 1)\left(2x_1^2 + 5x_1 + 5\right) = 0$

So, $(x_1,\ y_1) = (1,\ 9)$



Hence, the correct answer is option D.

Solution 72

$f(x) = |(2x-1)(x+2)| + \frac{\sin 2x}{2}$

$0 \le x < \frac{1}{2}\ \ f(x) = (1-2x)(x+2) + \frac{\sin 2x}{2}$

$f'(x) = -4x - 3 + \cos 2x < 0$

For $x \ge \frac{1}{2}:\ f'(x) = 4x + 3 + \cos 2x > 0$

So, minima occurs at $x=\frac{1}{2}$

$$f(x)\Big|_{\min}=\left|2\left(\frac{1}{2}\right)^2+\frac{3}{2}-2\right|+\sin\left(\frac{1}{2}\right)\cdot\cos\left(\frac{1}{2}\right)$$

$$=\frac{1}{2}\sin 1$$

So, maxima is possible at x = 0 or x = 1

Now checking for x = 0 and x = 1, we can see it attains its maximum value at x = 1

$$f(x)\Big|_{\max}=|2+3-2|+\frac{\sin 2}{2}$$

$$=3+\frac{1}{2}\sin 2$$

Sum of absolute maximum and minimum value$=3+\frac{1}{2}(\sin 1+\sin 2)$

Hence, the correct answer is option B.

Solution 73

$$a_1+a_2+\ldots+a_n=192$$

$$\Rightarrow \frac{n}{2}(a_1+a_n)=192 \qquad \ldots..(1)$$

$$a_2+a_4+a_6+\ \ldots+a_n=120$$

$$\Rightarrow \frac{n}{4}(a_1+1+a_n)=120 \qquad \ldots..(2)$$

From (2) and (1)

$$\frac{480}{n}-\frac{384}{n}=1$$

$$\Rightarrow n=96$$

Hence, the correct answer is option B.

Solution 74

$$\frac{dx}{dy}-\frac{2x}{y}=y^2(y+1)e^y$$

$$\text{If}=e^{\int -\frac{2}{y}dy}=e^{-2\ln y}=\frac{1}{y^2}$$

Solution is given by

$$x\cdot\frac{1}{y^2}=\int y^2(y+1)e^y\cdot\frac{1}{y^2}dy$$

$$\Rightarrow \frac{x}{y^2}=\int(y+1)e^y dy$$

$$\Rightarrow \frac{x}{y^2}=ye^y+c$$

$\Rightarrow x = y^2\left(ye^y + c\right)$
at, $y = 1,\ x = 0$
$\Rightarrow 0 = 1\left(1.\, e^1 + c\right)$
$\Rightarrow c = -e$
at $y = e,\ x = e^2\left(e \cdot e^e - e\right)$

Hence, the correct answer is option A.

Solution 75

$\frac{x^2}{\left(\frac{b^2}{a^2}\right)} - \frac{y^2}{b^2} = 1$

Tangent in slope form

$\Rightarrow y = mx \pm \sqrt{\frac{b^2}{a^2}m^2 - b^2}$
i.e., same as $y = \frac{\lambda x}{2} - \frac{\mu}{2}$

Comparing coefficients,
$m = \frac{\lambda}{2},\ \frac{b^2}{a^2}m^2 - b^2 = \frac{\mu^2}{4}$
Eliminating $m,\ \frac{b^2}{a^2} \cdot \frac{\lambda^2}{4} - b^2 = \frac{\mu^2}{4}$
$\Rightarrow \frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 4$

Hence, the correct answer is option D.

Solution 76

$\because \hat{b} = \vec{c} + 2\left(\vec{c} \times \hat{a}\right)$

$\Rightarrow \hat{b} \cdot \vec{c} = \left|\vec{c}\right|^2 \qquad \ldots\ldots \text{(i)}$

$\therefore \hat{b} - \vec{c} = 2\left(\vec{c} \times \vec{a}\right)$

$\Rightarrow \left|\hat{b}\right|^2 + \left|\vec{c}\right|^2 - 2\hat{b} \cdot \vec{c} = 4\left|\vec{c}\right|^2\left|\vec{a}\right|^2 \sin^2 \frac{\pi}{12}$

$\Rightarrow 1 + \left|\vec{c}\right|^2 - 2\left|\vec{c}\right|^2 = 4\left|\vec{c}\right|^2\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$

$\Rightarrow 1 = \left|\vec{c}\right|^2\left(3 - \sqrt{3}\right)$

$\Rightarrow 36\left|\vec{c}\right|^2 = \frac{36}{3-\sqrt{3}} = 6\left(3 + \sqrt{3}\right)$

Hence, the correct answer is option C.

Solution 77

$3P(X = 0) = P(X = 1)$

$3 \cdot {}^{n}C_0 P^0 (1-P)^n = {}^{n}C_1 P^1 (1-P)^{n-1}$

$\frac{3}{n} = \frac{P}{1-P}$

$\Rightarrow \frac{1}{11} = \frac{P}{1-P}$

$\Rightarrow 1 - P = 11P$

$\Rightarrow P = \frac{1}{12}$

$\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$

$\Rightarrow \frac{{}^{33}C_{15} P^{15} (1-P)^{18}}{{}^{33}C_{18} P^{18} (1-P)^{15}} - \frac{{}^{33}C_{16} P^{16} (1-P)^{17}}{{}^{33}C_{17} P^{17} (1-P)^{16}}$

$\Rightarrow \left(\frac{1-P}{P}\right)^3 - \left(\frac{1-P}{P}\right)$

$\Rightarrow 11^3 - 11 = 1320$

Hence, the correct answer is option A.

Solution 78

$-1 \leq \frac{x^2-5x+6}{x^2-9} \leq 1$ and $x^2-3x+2>0, \ \neq 1$

$\frac{(x-3)(2x+1)}{x^2-9} \geq 0 \Big| \frac{5(x-3)}{x^2-9} \geq 0$

Solution to this inequality is $x \in \left[\frac{-1}{2}, \infty\right) - \{3\}$ for $x^2 - 3x + 2 > 0$ and ≠ 1

$x \in (-\infty, 1) \cup (2, \infty) - \left\{\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right\}$

Combining the two solution sets (taking intersection)

$x \in \left[-\frac{1}{2}, 1\right) \cup \left(2, \infty\right) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

Hence, the correct answer is option D.

Solution 79

tanθ (sinθ + 1) – sin2θ = 0

tanθ (sinθ + 1 – 2cos^{2}θ) = 0

⇒ tanθ = 0 or 2sin^{2}θ + sinθ – 1 = 0

⇒ (2sinθ + 1)(sinθ – 1) = 0

$\Rightarrow \sin\theta = \frac{-1}{2}$ or 1

But, sinθ = 1 not possible

$\theta = 0, \pi, -\pi, -\frac{\pi}{6}, \frac{-5\pi}{6}$

$n(S) = 5$

$$T = \sum \cos 2\theta = \cos 0^\circ + \cos 2\pi + \cos(-2\pi) + \cos\left(-\frac{5\pi}{3}\right) + \cos\left(-\frac{\pi}{3}\right)$$

$$= 4$$

Hence, the correct answer is option B.

Solution 80

Let x : $(p \Delta q) \Rightarrow (p \Delta \sim q) \vee (\sim p \Delta q)$

Case-I

When Δ is same as ∨

Then $(p \Delta \sim q) \vee (\sim p \Delta q)$ becomes $(p \vee \sim q) \vee (\sim p \vee q)$ which is always true, so x becomes a tautology.

Case-II

When Δ is same as ∧

Then $(p \wedge q) \Rightarrow (p \wedge \sim q) \vee (\sim p \wedge q)$

If $p \wedge q$ is T, then $(p \wedge \sim q) \vee (\sim p \wedge q)$ is F

So x cannot be a tautology.

Case-III

When Δ is same as ⇒

Then $(p \Rightarrow \sim q) \vee (\sim p \Rightarrow q)$ is same at $(\sim p \vee \sim q) \vee (p \vee q)$, which is always true, so x becomes a tautology.

Case-IV
When Δ is same as ⇔
Then $(p \Leftrightarrow q) \Rightarrow (p \Leftrightarrow \sim q) \vee (\sim p \Leftrightarrow q)$
$p \Leftrightarrow q$ is true when p and q have same truth values, then $p \Leftrightarrow \sim q$ and $\sim p \Leftrightarrow q$ both are false. Hence x
cannot be a tautology.
So finally x can be ∨ or ⇒.

Hence, the correct answer is option B.

Solution 81

$\because 3f(c) + 2f(a) + f(d) = f(b)$

Value of *f*(*c*)	**Value of *f*(*a*)**	**Number of functions**
0	1	7
	2	5
	3	3
	4	2
1	0	6
	2	2
	3	1
2	0	3
	1	1
3	0	1
	Total Number of functions =	**31**

Solution 82

Let student marks x correct answers and y incorrect. So $3x - 2y = 5$ and $x + y \leq 5$ where $x, y \in W$
Only possible solution is $(x, y) = (3, 2)$
Student can mark correct answer by only one choice but for incorrect answer, there are two choices. So total number of ways of scoring 5 marks =
$^{5}C_{3}(1)^{3} \cdot (2)^{2} = 40$

Solution 83

Clearly B is $\left(-\frac{3}{\sqrt{a}},\ +\sqrt{a}\right)$ and C is $\left(-\frac{3}{\sqrt{a}},\ -\sqrt{a}\right)$

$$\text{Area of } \Delta \text{ ACD} = \frac{1}{2}\begin{vmatrix} \frac{3}{\sqrt{a}} & \sqrt{a} & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \left|3\sqrt{a}\sin\theta + 3\sqrt{a}\cos\theta\right| = 3\sqrt{a}\left|\sin\theta + \cos\theta\right|$$

$$\Rightarrow \Delta_{\max} = 3\sqrt{a}\cdot\sqrt{2} = 12$$

$$\Rightarrow a = \left(2\sqrt{2}\right)^2 = 8$$

Solution 84

Let $A(3\lambda + 7, -\lambda + 1, \lambda - 2)$ and $B(2\mu, 3\mu + 7, \mu)$

So, DR's of $AB \propto 3\lambda - 2\mu + 7, -(\lambda + 3\mu + 6), \lambda - \mu - 2$

$$\text{Clearly } \frac{3\lambda-2\mu+7}{1} = \frac{\lambda+3\mu+6}{4} = \frac{\lambda-\mu-2}{2}$$

$$\Rightarrow 5\lambda - 3\mu = -16 \qquad \ldots\ldots(\text{i})$$

$$\text{And } \lambda - 5\mu = 10 \qquad \ldots\ldots(\text{ii})$$

From (i) and (ii) we get $\lambda = -5$, $\mu = -3$

So, A is $(-8, 6, -7)$ and B is $(-6, -2, -3)$

$$AB = \sqrt{4 + 64 + 16}$$

$$\Rightarrow (AB)^2 = 84$$

Solution 85

$\because f(-1) = 2$ and $f(1) = 3$
For $x \in (-1, 1)$, $(4x^2 - 1) \in [-1, 3)$ hence $f(x)$ will be discontinuous at $x = 1$ and also
whenever $4x^2 - 1 = 0, 1$ or 2

$$\Rightarrow x = \pm\frac{1}{2}, \pm\frac{1}{\sqrt{2}} \text{ and } \pm\frac{\sqrt{3}}{2}$$

So there are total 7 points of discontinuity.

Solution 86

$$f(\theta) = \sin\theta \left(1 + \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(t)dt\right) + \cos\theta \left(\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} t f(t)dt\right)$$

Clearly $f(\theta) = a\sin\theta + b\cos\theta$

Where $a = 1 + \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (a\sin t + b\cos t)dt$

$\Rightarrow a = 1 + 2b \qquad \ldots\ldots(1)$

and $b = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (at\ \sin t + bt\ \cos t)dt$

$\Rightarrow b = 2a \qquad \ldots\ldots(2)$

From (1) and (2) we get

$a = -\frac{1}{3}$ and $b = -\frac{2}{3}$

So, $f(\theta) = -\frac{1}{3}(\sin\theta + 2\cos\theta)$

$\Rightarrow \left|\int_0^{\frac{\pi}{2}} f(\theta)d\theta\right| = \frac{1}{3}(1 + 2 \times 1) = 1$

Solution 87

Let $f(x) = \frac{x^2-9}{x-5}$

$\Rightarrow f'(x) = \frac{(x-1)(x-9)}{(x-5)^2}$

So, $\alpha = f(1) = 2$ and $\beta = \min(f(0), f(2)) = \frac{5}{3}$

$$\text{Now,} \int_{-1}^{3} \max\left\{\frac{x^2-9}{x-5},\ x\right\}dx = \int_{-1}^{\frac{9}{5}} \frac{x^2-9}{x-5}dx + \int_{\frac{9}{5}}^{3} x dx$$

$$= \int_{-1}^{\frac{9}{5}} \left(x + 5 + \frac{16}{x-5}\right)dx + \frac{x^2}{2}\Bigg|_{\frac{9}{5}}^{3}$$

$$= \frac{28}{25} + 14 + 16\ \ln\left(\frac{8}{15}\right) + \frac{72}{25} = 18 + 16\ \ln\left(\frac{8}{15}\right)$$

Clearly, $\alpha_1 = 18$ and $\alpha_2 = 16$, so $\alpha_1 + \alpha_2 = 34$.

Solution 88

$\because$ (α, β) lies on the given ellipse, $25\alpha^2 + 4\beta^2 = 1 \qquad \ldots..(1)$

Tangent to the parabola, $y = mx + \frac{1}{m}$ passes through (α, β). So, $\alpha m^2 - \beta m + 1 = 0$ has roots m_1 and $4m_1$,

$m_1 + 4m_1 = \frac{\beta}{\alpha}$ and $m_1 \cdot 4m_1 = \frac{1}{\alpha}$

Gives that $4\beta^2 = 25\alpha$ .....(2)

From (1) and (2)

$25(\alpha^2 + \alpha) = 1$ .....(3)

Now, $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$

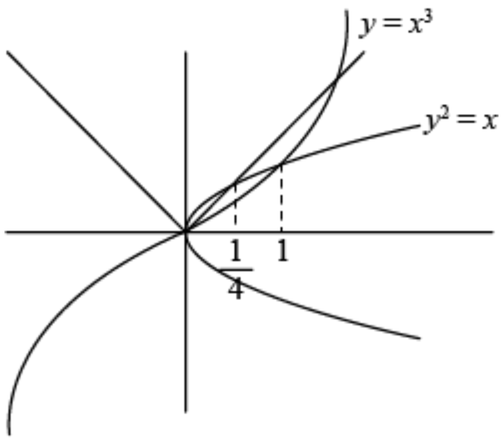
$= 25(2\alpha + 1)^2 + 2500\,(2\alpha + 1)^2$

$= 2525\,(4\alpha^2 + 4\alpha + 1)$ from equation (3)

$= 2525\left(\frac{4}{25} + 1\right)$

$= 2929$

Solution 89



$C_1 : y = x^3$

$C_2 : y^2 = x$

and $C_3 = y = 2|x|$

C_1 and C_2 intersect at (1, 1)

C_2 and C_3 intersect at $\left(\frac{1}{4}, \frac{1}{2}\right)$

Clearly $R_1 = \int_0^{\frac{1}{4}} \left(\sqrt{x} - 2x\right) dx = \frac{2}{3}\left(\frac{1}{8}\right) - \frac{1}{16} = \frac{1}{48}$

and $R_1 + R_2 = \int_0^1 \left(\sqrt{x} - x^3\right) dx = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

So, $\frac{R_1 + R_2}{R_1} = \frac{\frac{5}{12}}{\frac{1}{48}}$

$\Rightarrow 1 + \frac{R_2}{R_1} = 20$

$\Rightarrow \frac{R_2}{R_1} = 19$

Solution 90

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix} = -a\hat{i} - \hat{j} + (a-1)\hat{k}$$

$$\vec{a}_1 - \vec{a}_2 = -\hat{i} + \hat{j} + \hat{k}$$

$$\text{Shortest distance} = \left| \frac{\left(\vec{a}_1 - \vec{a}_2\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|} \right|$$

$$\Rightarrow \sqrt{\frac{2}{3}} = \frac{2(a-1)}{\sqrt{a^2+1+(a-1)^2}}$$

$$\Rightarrow 6\left(a^2 - 2a + 1\right) = 2a^2 - 2a + 2$$

$$\Rightarrow (a-2)(2a-1) = 0$$

$$\Rightarrow a = 2 \text{ because } a \in z$$