NDA I 2016_Mathematics

Total Time: 150

Total Marks: 300.0

Q.No.1: Suppose ω is a cube root of unity with ω ≠ 1. Suppose *P* and *Q* are the points on the complex plane defined by ω and ω^2. If *O* is the origin, then what is the angle between *OP* and *OQ*?

A. 60°
B. 90°
C. 120°
D. 150°

Marks:[2.50]

Q.No.2: Suppose there is a relation * between the positive numbers *x* and *y* given by *x* * *y* if and only if $x \leq y^2$. Then which one of the following is correct?

A. * is reflexive but not transitive and symmetric
B. * is transitive but not reflexive and symmetric
C. * is symmetric and reflexive but not transitive
D. * is symmetric but not reflexive and transitive

Marks:[2.50]

Q.No.3: If $x^2 - px + 4 > 0$ for all real values of *x*, then which one of the following is correct?

A. $|p| < 4$
B. $|p| \leq 4$
C. $|p| > 4$
D. $|p| \geq 4$

Marks:[2.50]

Q.No.4: If $z = x + iy = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{-25}$, where $i = \sqrt{-1}$, then what is the fundamental amplitude of $\frac{z-\sqrt{2}}{z-i\sqrt{2}}$?

A. ⊓
B. $\frac{\pi}{2}$
C. $\frac{\pi}{3}$
D. $\frac{\pi}{4}$

Marks:[2.50]

Q.No.5: If $f\left(x_1\right) - f\left(x_2\right) = f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)$ for x_1, $x_2 \in (-1, 1)$, then what is $f(x)$ equal to?

A. $\ln\left(\frac{1-x}{1+x}\right)$
B. $\ln\left(\frac{2+x}{1-x}\right)$
C. $\tan^{-1}\left(\frac{1-x}{1+x}\right)$
D. $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

Marks:[2.50]

Q.No.6: What is the range of the function $y = \frac{x^2}{1+x^2}$ where, $x \in \mathbf{R}$?

A. [0, 1)
B. [0, 1]
C. (0, 1)
D. (0, 1]

Marks:[2.50]

Q.No.7: A straight line intersects *x* and *y* axes at *P* and *Q*, respectively. If (3, 5) is the middle point of *PQ*, then what is the area of the triangle *OPQ*?

A. 12 square units
B. 15 square units
C. 20 square units
D. 30 square units

Marks:[2.50]

Q.No.8: If a circle of radius *b* units with centre at (0, *b*) touches the line $y = x - \sqrt{2}$, then what is the value of *b*?

A. $2 + \sqrt{2}$

B. $2 - \sqrt{2}$

C. $2\sqrt{2}$

D. $\sqrt{2}$

Marks:[2.50]

Q.No.9: Consider the function $f(\theta) = 4(\sin^2\theta + \cos^4\theta)$

What is the maximum value of the function *f*(θ)?

A. 1

B. 2

C. 3

D. 4

Marks:[2.50]

Q.No.10: Consider the function $f(\theta) = 4(\sin^2\theta + \cos^4\theta)$

What is the minimum value of the function *f*(θ)?

A. 0

B. 1

C. 2

D. 3

Marks:[2.50]

Q.No.11: Consider the function $f(\theta) = 4(\sin^2\theta + \cos^4\theta)$

Consider the following statements:

1. *f*(θ) = 2 has no solution.
2. $f(\theta) = \frac{7}{2}$ has no solution.

Which of the above statements is/are correct?

A. 1 only

B. 2 only

C. Both 1 and 2

D. Neither 1 nor 2

Marks:[2.50]

Q.No.12: Consider the curves

$f\left(x\right) = x\left|x\right| - 1$ and $g\left(x\right) = \begin{cases} \frac{3x}{2}, & x > 0 \\ 2x, & x \leq 0 \end{cases}$

Where do the curves intersect?

A. At (2, 3) only

B. At (−1, −2) only

C. At (2, 3) and (−1, −2)

D. Neither at (2, 3) nor at (−1, −2)

Marks:[2.50]

Q.No.13: Consider the curves

$f\left(x\right) = x\left|x\right| - 1$ and $g\left(x\right) = \begin{cases} \frac{3x}{2}, & x > 0 \\ 2x, & x \leq 0 \end{cases}$

What is the area bounded by the curves?

A. $\frac{17}{6}$ square units

B. $\frac{8}{3}$ square units

C. 2 square units

D. $\frac{1}{3}$ square units

Marks:[2.50]

Q.No.14: Consider the curves

$f\left(x\right) = \frac{27\left(x^{2/3} - x\right)}{4}$

How many solutions does the function $f(x) = 1$ have?

A. One

B. Two

C. Three

D. Four

Marks:[2.50]

Q.No.15: Consider the curves

$f\left(x\right) = \frac{27\left(x^{2/3} - x\right)}{4}$

How many solutions does the function $f(x) = -1$ have?

A. One

B. Two

C. Three

D. Four

Marks:[2.50]

Q.No.16: Consider the functions
$f(x) = xg(x)$ and $g(x) = \left[\frac{1}{x}\right]$ where, [.] is the greatest integer function.
What is $\int_{\frac{1}{3}}^{\frac{1}{2}} g(x)\,dx$ equal to?

A. $\frac{1}{6}$
B. $\frac{1}{3}$
C. $\frac{5}{18}$
D. $\frac{5}{36}$

Marks:[2.50]

Q.No.17: Consider the functions
$f(x) = xg(x)$ and $g(x) = \left[\frac{1}{x}\right]$ where, [.] is the greatest integer function.
What is $\int_{\frac{1}{3}}^{1} f(x)dx$ equal to?

A. $\frac{37}{72}$
B. $\frac{2}{3}$
C. $\frac{17}{72}$
D. $\frac{37}{144}$

Marks:[2.50]

Q.No.18: Consider the function
$f(x) = |x - 1| + x^2$
where, $x \in \mathbf{R}$.
Which one of the following statements is correct?

A. $f(x)$ is continuous but not differentiable at $x = 0$
B. $f(x)$ is continuous but not differentiable at $x = 1$
C. $f(x)$ is differentiable at $x = 1$
D. $f(x)$ is not differentiable at $x = 0$ and $x = 1$

Marks:[2.50]

Q.No.19: Consider the function
$f(x) = |x - 1| + x^2$
where, $x \in \mathbf{R}$.
Which one of the following statements is correct?

A. $f(x)$ is increasing in $\left(-\infty, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, \infty\right)$

B. $f(x)$ is decreasing in $\left(-\infty, \frac{1}{2}\right)$ and increasing in $\left(\frac{1}{2}, \infty\right)$

C. $f(x)$ is increasing in $(-\infty, 1)$ and decreasing in $(1, \infty)$

D. $f(x)$ is decreasing in $(-\infty, 1)$ and increasing in $(1, \infty)$

Marks:[2.50]

Q.No.20: Consider the function
$f(x) = |x - 1| + x^2$
where, $x \in \mathbf{R}$. Which one of the following statements is correct?

A. $f(x)$ has local minima at more than one point in $(-\infty, \infty)$

B. $f(x)$ has local maxima at more than one point in $(-\infty, \infty)$

C. $f(x)$ has local minimum at one point only in $(-\infty, \infty)$

D. $f(x)$ has neither maxima nor minima in $(-\infty, \infty)$

Marks:[2.50]

Q.No.21: Consider the function
$f(x) = |x - 1| + x^2$
where, $x \in \mathbf{R}$. What is the area of the region bounded by x-axis, the curve $y = f(x)$ and the two ordinates $x = \frac{1}{2}$ and $x = 1$?

A. $\frac{5}{12}$ square unit

B. $\frac{5}{6}$ square unit

C. $\frac{7}{6}$ square units

D. 2 square units

Marks:[2.50]

Q.No.22: Consider the function
$f(x) = |x - 1| + x^2$
where, $x \in \mathbf{R}$. What is the area of the region bounded by x-axis, the curve $y = f(x)$ and the two ordinates $x = 1$ and $x = \frac{3}{2}$?

A. $\frac{5}{12}$ square unit

B. $\frac{7}{12}$ square unit

C. $\frac{2}{3}$ square unit

D. $\frac{11}{12}$ square unit

Marks:[2.50]

Q.No.23: Given that $a_n = \int_0^{\pi} \frac{\sin^2\{(n+1)x\}}{\sin 2x} dx$

Consider the following statements:
1. The sequence $\{a_{2n}\}$ is in AP with common difference zero.
2. The sequence $\{a_{2n+1}\}$ is in AP with common difference zero.
Which of the above statements is/are correct?

A. 1 only
B. 2 only
C. Both 1 and 2
D. Neither 1 nor 2

Marks:[2.50]

Q.No.24: Given that $a_n = \int_0^{\pi} \frac{\sin^2\{(n+1)x\}}{\sin 2x} dx$. What is $a_{n-1} - a_{n-4}$ equal to?

A. –1
B. 0
C. 1
D. 2

Marks:[2.50]

Q.No.25: Consider the equation $x + |y| = 2y$.
Which of the following statements are ***not*** correct?
1. y as a function of x is not defined for all real x
2. y as a function of x is not continuous at $x = 0$.
3. y as a function of x is differentiable for all x.
Select the correct answer using the code given below.

A. 1 and 2 only
B. 2 and 3 only
C. 1 and 3 only
D. 1, 2 and 3

Marks:[2.50]

Q.No.26: Consider the equation $x + |y| = 2y$.
What is the derivative of y as a function of x with respect to x for $x < 0$?

A. 2
B. 1
C. $\frac{1}{2}$
D. $\frac{1}{3}$

Marks:[2.50]

Q.No.27: Consider the lines $y = 3x$, $y = 6x$ and $y = 9$. What is the area of the

triangle formed by these lines?

A. $\frac{27}{4}$ square units

B. $\frac{27}{2}$ square units

C. $\frac{19}{4}$ square units

D. $\frac{19}{2}$ square units

Marks:[2.50]

Q.No.28: Consider the lines $y = 3x$, $y = 6x$ and $y = 9$. The centroid of the triangle is at which one of the following points?

A. (3, 6)

B. $\left(\frac{3}{2}, 6\right)$

C. (3, 3)

D. $\left(\frac{3}{2}, 9\right)$

Marks:[2.50]

Q.No.29: Consider the function $f(x) = (x - 1)^2 (x + 1) (x - 2)^3$
What is the number of points of local minima of the function $f(x)$?

A. None

B. One

C. Two

D. Three

Marks:[2.50]

Q.No.30: Consider the function $f(x) = (x - 1)^2 (x + 1) (x - 2)^3$
What is the number of points of local maxima of the function $f(x)$?

A. None

B. One

C. Two

D. Three

Marks:[2.50]

Q.No.31: Let $f(x)$ and $g(x)$ be twice differentiable functions on [0, 2] satisfying $f''(x) = g''(x)$, $f'(1) = 4$, $g'(1) = 6$, $f(2) = 3$ and $g(2) = 9$. Then what is $f(x) - g(x)$ at $x = 4$ equal to?

A. –10

B. –6

C. –4

D. 2

Marks:[2.50]

Q.No.32: Consider the curves $y = |x - 1|$ and $|x| = 2$. What is/are the points(s) of intersection of the curves?

A. (−2, 3) only

B. (2, 1) only

C. (−2, 3) and (2, 1)

D. Neither (−2, 3) nor (2, 1)

Marks:[2.50]

Q.No.33: Consider the curves $y = |x - 1|$ and $|x| = 2$
What is the area of the region bounded by the curves and x-axis?

A. 3 square units

B. 4 square units

C. 5 square units

D. 6 square units

Marks:[2.50]

Q.No.34: Consider the function $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where, p is a constant. What is the value of $f'(0)$?

A. p^3

B. $3p^3$

C. $6p^3$

D. $-6p^3$

Marks:[2.50]

Q.No.35: Consider the function $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where, p is a constant
What is the value of p for which $f''(0) = 0$?

A. $-\frac{1}{6}$ or 0

B. −1 or 0

C. $-\frac{1}{6}$ or 1

D. –1 or 1

Marks:[2.50]

Q.No.36: Consider triangle ABC in which $\cos A + \cos B + \cos C = \sqrt{3} \sin \frac{\pi}{3}$. What is the value of $\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$?

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{16}$

Marks:[2.50]

Q.No.37: Consider triangle ABC in which $\cos A + \cos B + \cos C = \sqrt{3} \sin \frac{\pi}{3}$. What is the value of $\cos\left(\frac{A+B}{2}\right) \cos\left(\frac{B+C}{2}\right) \cos\left(\frac{C+A}{2}\right)$?

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{1}{16}$

D. None of the above

Marks:[2.50]

Q.No.38: Given that tan α and tan β are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$. What is $\tan(\alpha + \beta)$ equal to?

A. $b(c - 1)$

B. $c(b - 1)$

C. $c(b - 1)^{-1}$

D. $b(c - 1)^{-1}$

Marks:[2.50]

Q.No.39: Given that tan α and tan β are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$. What is $\sin(\alpha + \beta) \sec \alpha \sec \beta$ equal to?

A. b

B. $-b$

C. c

D. $-c$

Marks:[2.50]

Q.No.40: Consider the two circles
$(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$. What is the distance between the centres of the two circles?

A. 5 units

B. 6 units

C. 8 units

D. 10 units

Marks:[2.50]

Q.No.41: Consider the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$. If the circles intersect at two distinct points, then which one of the following is correct?

A. $r = 1$

B. $1 < r < 2$

C. $r = 2$

D. $2 < r < 8$

Marks:[2.50]

Q.No.42: Consider the two lines
$x + y + 1 = 0$ and $3x + 2y + 1 = 0$. What is the equation of the line passing through the point of intersection of the given lines and parallel to x-axis?

A. $y + 1 = 0$

B. $y - 1 = 0$

C. $y - 2 = 0$

D. $y + 2 = 0$

Marks:[2.50]

Q.No.43: Consider the two lines
$x + y + 1 = 0$ and $3x + 2y + 1 = 0$. What is the equation of the line passing through the point of intersection of the given lines and parallel to y-axis?

A. $x + 1 = 0$

B. $x - 1 = 0$

C. $x - 2 = 0$

D. $x + 2 = 0$

Marks:[2.50]

Q.No.44: Consider the equation $k \sin x + \cos 2x = 2k - 7$. If the equation possesses solutions, then what is the minimum value of k?

A. 1

B. 2
C. 4
D. 6

Marks:[2.50]

Q.No.45: Consider the equation $k \sin x + \cos 2x = 2k - 7$. If the equation possesses solution, then what is the maximum value of k?

A. 1
B. 2
C. 4
D. 6

Marks:[2.50]

Q.No.46: Consider the function $f(x) = \frac{a^{[x]+x}-1}{[x]+x}$. Where [.] denotes the greatest integer function. What is $\lim_{x \to 0^+} f(x)$ equal to?

A. 1
B. ln a
C. $1 - a^{-1}$
D. Limit does not exit

Marks:[2.50]

Q.No.47: Consider the function $f(x) = \frac{a^{[x]+x}-1}{[x]+x}$. What is $\lim_{x \to 0^-} f(x)$ equal to?

A. 0
B. ln a
C. $1 - a^{-1}$
D. Limit does not exist

Marks:[2.50]

Q.No.48: Let z_1, z_2 and z_3 be non-zero complex numbers satisfying $z^2 = i\bar{z}$, where, $i = \sqrt{-1}$.
What is $z_1 + z_2 + z_3$ eqaul to?

A. i
B. $-i$
C. 0
D. 1

Marks:[2.50]

Q.No.49: Let z_1, z_2 and z_3 be non-zero complex numbers satisfying $z^2 = i\bar{z}$, where, $i = \sqrt{-1}$.
Consider the following statements:
1. $z_1z_2z_3$ is purely imaginary.
2. $z_1z_2 + z_2z_3 + z_3z_1$ is purely real.
Which of the above statements is/are correct?

A. 1 only
B. 2 only
C. Both 1 and 2
D. Neither 1 nor 2

Marks:[2.50]

Q.No.50: Given that $\log_x y$, $\log_z x$, $\log_y z$ are in GP, $xyz = 64$ and x^3, y^3, z^3 are in AP.
Which one of the following is correct?
x, y and z are

A. in AP only
B. in GP only
C. in both AP and GP
D. neither in AP nor in GP

Marks:[2.50]

Q.No.51: Given that $\log_x y$, $\log_z x$, $\log_y z$ are in GP, $xyz = 64$ and x^3, y^3, z^3 are in AP.
Which one of the following is correct?
xy, yz and zx are

A. in AP only
B. in GP only
C. in both AP and GP
D. neither in AP nor in GP

Marks:[2.50]

Q.No.52: Let *z* be a complex number satisfying $\left|\frac{z-4}{z-8}\right| = 1$ and $\left|\frac{z}{z-2}\right| = \frac{3}{2}$
What is |*z*| equal to?

A. 6
B. 12
C. 18
D. 36

Marks:[2.50]

Q.No.53: Let *z* be a complex number satisfying $\left|\frac{z-4}{z-8}\right| = 1$ and $\left|\frac{z}{z-2}\right| = \frac{3}{2}$
What is $\left|\frac{z-6}{z+6}\right|$ equal to?

A. 3
B. 2
C. 1
D. 0

Marks:[2.50]

Q.No.54: A function *f*(*x*) is defined as follows:

$$f(x) = \begin{cases} x+\pi & \text{for} \quad x \in [-\pi, 0) \\ \pi \cos x & \text{for} \quad x \in \left[0, \frac{\pi}{2}\right] \\ \left(x - \frac{\pi}{2}\right)^2 & \text{for} \quad x \in \left(\frac{\pi}{2}, \pi\right) \end{cases}$$

Consider the following statements:
1. The function *f*(*x*) is continuous at *x* = 0.
2. The function *f*(*x*) is continuous at $x = \frac{\pi}{2}$.

Which of the above statements is/are correct?

A. 1 only
B. 2 only
C. Both 1 and 2
D. Neither 1 nor 2

Marks:[2.50]

Q.No.55: A function *f*(*x*) is defined as follows:

$$f(x) = \begin{cases} x+\pi & \text{for} \quad x \in [-\pi, 0) \\ \pi \cos x & \text{for} \quad x \in \left[0, \frac{\pi}{2}\right] \\ \left(x - \frac{\pi}{2}\right)^2 & \text{for} \quad x \in \left(\frac{\pi}{2}, \pi\right) \end{cases}$$

Consider the following statements:
1. The function *f*(*x*) is differentiable at *x* = 0.

2. The function $f(x)$ is differentiable at $x = \frac{\pi}{2}$.

Which of the above statements is/are correct?

A. 1 only

B. 2 only

C. Both 1 and 2

D. Neither 1 nor 2

Marks:[2.50]

Q.No.56: Let α and $\beta\,(\alpha < \beta)$ be the roots of the equation $x^2 + bx + c = 0$, where, $b > 0$ and $c < 0$.

Consider the following:

1. $\beta < -\alpha\alpha$
2. $\beta < |\alpha|$

Which of the above statements is/are correct?

A. 1 only

B. 2 only

C. Both 1 and 2

D. Neither 1 nor 2

Marks:[2.50]

Q.No.57: Let α and $\beta\,(\alpha < \beta)$ be the roots of the equation $x^2 + bx + c = 0$, where, $b > 0$ and $c < 0$.

Consider the following:

1. $\alpha + \beta + \alpha\beta > 0$
2. $\alpha^2\beta + \beta^2\alpha > 0|$

Which of the above statements is/are correct?

A. 1 only

B. 2 only

C. Both 1 and 2

D. Neither 1 nor 2

Marks:[2.50]

Q.No.58: Consider a parallelogram whose vertices are $A(1, 2)$, $B(4, y)$, $C(x, 6)$ and $D(3, 5)$ taken in order.

What is the value of $AC^2 - BD^2$?

A. 25

B. 30

C. 36

D. 40

Marks:[2.50]

Q.No.59: Consider a parallelogram whose vertices are *A*(1, 2), *B*(4, *y*), *C*(*x*, 6) and *D*(3, 5) taken in order.
What is the point of intersection of the diagonals?

A. $\left(\frac{7}{2}, 4\right)$

B. (3, 4)

C. $\left(\frac{7}{2}, 5\right)$

D. (3, 5)

Marks:[2.50]

Q.No.60: Consider a parallelogram whose vertices are *A*(1, 2), *B*(4, *y*), *C*(*x*, 6) and *D*(3, 5) taken in order.
What is the area of the parallelogram?

A. $\frac{7}{2}$square units

B. 4 square units

C. $\frac{11}{2}$ square units

D. 7 square units

Marks:[2.50]

Q.No.61:
Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that
$f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$ for $x \in \mathbb{R}$. What is *f*(1) equal to?

A. –2

B. –1

C. 0

D. 4

Marks:[2.50]

Q.No.62: Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that
$f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$ for $x \in \mathbb{R}$.
What is $f'(1)$ equal to?

A. –6

B. –5

C. 1

D. 0

Marks:[2.50]

Q.No.63: Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = x^3 + x^2\ f'(1) + xf''(2) + f'''(3)$ for $x \in \mathbb{R}$.
What is $f'''(10)$ equal to?

A. 1
B. 5
C. 6
D. 8

Marks:[2.50]

Q.No.64: Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = x^3 + x^2\ f'(1) + xf''(2) + f'''(3)$ for $x \in \mathbb{R}$.
Consider the following:
1. $f(2) = f(1) - f(0)$
2. $f''(2) - 2f'(1) = 12$
Which of the above is/are correct?

A. 1 only
B. 2 only
C. Both 1 and 2
D. Neither 1 nor 2

Marks:[2.50]

Q.No.65: A plane P passes through the line of intersection of the planes $2x - y + 3z = 2$, $x + y - z = 1$ and the point (1, 0, 1). What are the direction ratios of the line of intersection of the given planes?

A. $(2, -5, -3)$
B. $(1, -5, -3)$
C. $(2,\ 5,\ 3)$
D. $(1,\ 3,\ 5)$

Marks:[2.50]

Q.No.66: A plane P passes through the line of intersection of the planes $2x - y + 3z = 2$, $x + y - z = 1$ and the point (1, 0, 1). What is the equation of the plane P?

A. $2x + 5y - 2 = 0$
B. $5x + 2y - 5 = 0$
C. $x + z - 2 = 0$
D. $2x - y - 2z = 0$

Marks:[2.50]

Q.No.67: A plane P passes through the line of intersection of the planes $2x - y + 3z = 2$, $x + y - z = 1$ and the point (1, 0, 1). If the plane *P* touches the sphere $x^2 + y^2 + z^2 = r^2$, then what is *r* equal to?

A. $\frac{2}{\sqrt{29}}$

B. $\frac{4}{\sqrt{29}}$

C. $\frac{5}{\sqrt{29}}$

D. 1

Marks:[2.50]

Q.No.68: Consider the function $f(x) = \left|x^2 - 5x + 6\right|$. What is *f* ′(4) equal to?

A. −4

B. −3

C. 3

D. 2

Marks:[2.50]

Q.No.69: Consider the function $f(x) = \left|x^2 - 5x + 6\right|$. What is $f''(2.5)$ equal to?

A. −3

B. −2

C. 0

D. 2

Marks:[2.50]

Q.No.70: Let, *f*(*x*) be the greatest integer function and *g*(*x*) be the modulus function.

What is $(g \circ f)\left(-\frac{5}{3}\right) - (f \circ g)\left(-\frac{5}{3}\right) \lim_{x \to \infty}$ equal to?

A. −1

B. 0

C. 1

D. 2

Marks:[2.50]

Q.No.71: Let, *f*(*x*) be the greatest integer function and *g*(*x*) be the modulus function.

What is $(f \circ f)\left(-\frac{9}{5}\right) + (g \circ g)\ (-2)$ equal to?

A. –1
B. 0
C. 1
D. 2

Marks:[2.50]

Q.No.72: Consider a circle passing through the origin and the points (*a, b*) and (*–b*, *–a*).
On which line, does the centre of the circle lie?

A. $x + y = 0$
B. $x - y = 0$
C. $x + y = a + b$
D. $x - y = a^2 - b^2$

Marks:[2.50]

Q.No.73: Consider a circle passing through the origin and the points (*a, b*) and (*–b*, *–a*).
What is the sum of the squares of the intercepts cut off by the circle on the axes?

A. $\left(\frac{a^2+b^2}{a^2-b^2}\right)^2$
B. $2\left(\frac{a^2+b^2}{a-b}\right)^2$
C. $4\left(\frac{a^2+b^2}{a-b}\right)^2$
D. None of the above

Marks:[2.50]

Q.No.74: Let, $\widehat{a},\ \hat{b}$ be two unit vectors and θ be the angle between them. What is $\cos\left(\frac{\theta}{2}\right)$ equal to?

A. $\frac{\left|\widehat{a}-\hat{b}\right|}{2}$

B. $\frac{\left|\widehat{a}+\hat{b}\right|}{2}$

C. $\frac{\left|\widehat{a}-\hat{b}\right|}{4}$

D. $\frac{\left|\widehat{a}+\hat{b}\right|}{4}$

Marks:[2.50]

Q.No.75: Let, $\widehat{a},\ \hat{b}$ be two unit vectors and θ be the angle between them. What is $\sin\left(\frac{\theta}{2}\right)$ equal to?

A. $\frac{\left|\widehat{a}-\hat{b}\right|}{2}$

B. $\frac{|\widehat{a}+b|}{2}$

C. $\frac{\left|\widehat{a}-\hat{b}\right|}{4}$

D. $\frac{\left|\widehat{a}+\hat{b}\right|}{4}$

Marks:[2.50]

Q.No.76: Consider the following statements:

1. There exists $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which $\tan^{-1}(\tan\ \theta) \neq \theta$.

2. $\sin^{-1}\left(\frac{1}{3}\right) - \sin^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\left(\frac{2\sqrt{2}\left(\sqrt{3}-1\right)}{15}\right)$

Which of the above statements is/are correct?

A. 1 only

B. 2 only

C. Both 1 and 2

D. Neither 1 nor 2

Marks:[2.50]

Q.No.77: Consider the following statements:

1. $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \pi$

2. There exists
$x, y \in [-1, 1],\ \text{where},\ x \neq y\ \text{such that}\ \sin^{-1} x + \cos^{-1} y = \frac{\pi}{2}$.
Which of the above statements is/are correct?

A. 1 only
B. 2 only
C. Both 1 and 2
D. Neither 1 nor 2

Marks:[2.50]

Q.No.78: What are the order and degree, respectively, of the differential equation whose solution is $y = cx + c^2 - 3c^{3/2} + 2$, where *c* is a parameter?

A. 1, 2
B. 2, 2
C. 1, 3
D. 1, 4

Marks:[2.50]

Q.No.79: What is $\int_{-2}^{2} x dx - \int_{-2}^{2} [x]\, dx$ equal to, where [.] is greatest integer function?

A. 0
B. 1
C. 2
D. 4

Marks:[2.50]

Q.No.80: If $\int_{-2}^{5} f(x)\, dx = 4\ \text{and}\ \int_{0}^{5} \{1 + f(x)\}\, dx = 7$ then what is $\int_{-2}^{0} f(x)\, dx$ equal to?

A. –3
B. 2
C. 3
D. 5

Marks:[2.50]

Q.No.81: If $\lim_{x \to 0} \phi(x) = a^2$, where *a* ≠ 0, then what is $\lim_{x \to 0} \phi\left(\frac{x}{a}\right)$ equal to?

A. a^2
B. a^{-2}
C. $-a^2$

D. $-a$

Marks:[2.50]

Q.No.82: What is $\lim_{x \to 0} e^{-\frac{1}{x^2}}$ equal to?

A. 0
B. 1
C. –1
D. Limit does not exist

Marks:[2.50]

Q.No.83: If A is a square matrix, then what is adj(A^{-1}) – (adj A)$^{-1}$ equal to?

A. 2 $|A|$
B. Null matrix
C. Unit matrix
D. None of the above

Marks:[2.50]

Q.No.84: What is the binary equivalent of the decimal number 0.3125?

A. 0.0111
B. 0.1010
C. 0.0101
D. 0.1101

Marks:[2.50]

Q.No.85: Let R be a relation on the Set N of natural numbers defined by '$nRm \Leftrightarrow n$ is a factor of m'. Then which one of the following is correct?

A. R is reflexive, symmetric but not transitive
B. R is transitive, symmetric but not reflexive
C. R is reflexive, transitive but not symmetric
D. R is an equivalence relation

Marks:[2.50]

Q.No.86: What is $\int_0^{4\pi} |\cos\ x|\, dx$ equal to?

A. 0
B. 2
C. 4
D. 8

Marks:[2.50]

Q.No.87: What is the number of natural numbers less than or equal to 1000 which are neither divisible by 10 nor 15 nor 25?

A. 860
B. 854
C. 840
D. 824

Marks:[2.50]

Q.No.88: (*a*, 2*b*) is the mid-point of the line segment joining the points (10, −6) and (*k*, 4). If *a* − 2*b* = 7, then what is the value of *k*?

A. 2
B. 3
C. 4
D. 5

Marks:[2.50]

Q.No.89: Consider the following statements:
1. If *ABC* is an equilateral triangle, then 3 tan(*A* + *B*) tan*C* = 1
2. If *ABC* is a triangle in which *A* = 78º, *B* = 66º, then $\tan\left(\frac{A}{2}+C\right) < \tan\ A$
3. If *ABC* is any triangle, then $\tan\left(\frac{A+B}{2}\right)\ \sin\ \left(\frac{C}{2}\right) < \cos\left(\frac{C}{2}\right)$

Which of the above statements is/are correct?

A. 1 only
B. 2 only
C. 1 and 2
D. 2 and 3

Marks:[2.50]

Q.No.90: If *A* = (cos 12º − cos 36º) (sin 96º + sin 24º) and *B* = (sin 60º − sin 12º) (cos 48º − cos 72º), then what is $\frac{A}{B}$ equal to?

A. –1

B. 0

C. 1

D. 2

Marks:[2.50]

Q.No.91: What is the mean deviation from the mean of the numbers 10, 9, 21, 16, 24?

A. 5.2

B. 5.0

C. 4.5

D. 4.0

Marks:[2.50]

Q.No.92: Three dices are thrown simultaneously. What is the probability that the sum of the three faces is at least 5?

A. $\frac{17}{18}$

B. $\frac{53}{54}$

C. $\frac{103}{108}$

D. $\frac{215}{216}$

Marks:[2.50]

Q.No.93: Two independent events *A* and *B* have $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$. What is the probability that exactly one of the two events *A* or *B* occurs?

A. $\frac{1}{4}$

B. $\frac{5}{6}$

C. $\frac{5}{12}$

D. $\frac{7}{12}$

Marks:[2.50]

Q.No.94: A coin is tossed three times. What is the probability of getting head and tail alternately?

A. $\frac{1}{8}$

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Marks:[2.50]

Q.No.95: If the total number of observations is 20, $\Sigma x_i = 1000 \text{ and } \Sigma x_i^2 = 84000$, then what is the variance of the distribution?

A. 1500

B. 1600

C. 1700

D. 1800

Marks:[2.50]

Q.No.96: A card is drawn from a well-shuffled deck of 52 cards. What is the probability that it is queen of spade?

A. $\frac{1}{52}$

B. $\frac{1}{13}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Marks:[2.50]

Q.No.97: If two dice are thrown, then what is the probability that the sum on the two faces is greater than or equal to 4?

A. $\frac{13}{18}$

B. $\frac{5}{6}$

C. $\frac{11}{12}$

D. $\frac{35}{36}$

Marks:[2.50]

Q.No.98: A certain type of missile hits the target with probability p = 0.3. What is the least number of missiles should be fired so that there is at least an

80% probability that the target is hit?

A. 5

B. 6

C. 7

D. None of the above

Marks:[2.50]

Q.No.99: For two mutually exclusive events *A* and *B*, *P*(*A*) = 0.2 and $P\left(\overline{A} \cap B\right) = 0.3$. What is equal to $P\left(A|\left(A \cup B\right)\right)$ equal to?

A. $\frac{1}{2}$

B. $\frac{2}{5}$

C. $\frac{2}{7}$

D. $\frac{2}{3}$

Marks:[2.50]

Q.No.100: What is the probability of 5 Sundays in the month of December?

A. $\frac{1}{7}$

B. $\frac{2}{7}$

C. $\frac{3}{7}$

D. None of the above

Marks:[2.50]

Q.No.101: If *m* is the geometric mean of $\left(\frac{y}{z}\right)^{\log(yz)}$, $\left(\frac{z}{x}\right)^{\log(zx)}$ and $\left(\frac{x}{y}\right)^{\log(xy)}$ then what is the value of *m*?

A. 1

B. 3

C. 6

D. 9

Marks:[2.50]

Q.No.102: A point is chosen at random inside a rectangle measuring 6 inches by 5 inches. What is the probability that the randomly selected point is at least one inch from the edge of the rectangle?

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{2}{5}$

Marks:[2.50]

Q.No.103: The mean of the series x_1, x_2,...,x_n is $\overline{X}$. If x_2 is replaced by λ, then what is the new mean?

A. $\overline{X} - x_2 + \lambda$

B. $\frac{\overline{X} - x_2 - \lambda}{n}$

C. $\frac{\overline{X} - x_2 + \lambda}{n}$

D. $\frac{n\overline{X} - x_2 + \lambda}{n}$

Marks:[2.50]

Q.No.104: For the data 3, 5, 1, 6, 5, 9, 5, 2, 8, 6 the mean, median and mode are *x*, *y* and *z*, respectively. Which one of the following is correct?

A. $x = y \neq z$

B. $x \neq y = z$

C. $x \neq y \neq z$

D. $x = y = z$

Marks:[2.50]

Q.No.105: Consider the following statements in respect to a histogram:
1. The total area of the rectangles in a histogram is equal to the total area bounded by the corresponding frequency polygon and the *x*-axis.
2. When class intervals are unequal in a frequency distribution, the area of the rectangle is proportional to the frequency.
Which of the above statements is/are correct?

A. 1 only

B. 2 only

C. Both 1 and 2

D. Neither 1 nor 2

Marks:[2.50]

Q.No.106: A fair coin is tossed 100 times. What is the probability of getting tails an odd number of times?

A. $\frac{1}{2}$

B. $\frac{3}{8}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Marks:[2.50]

Q.No.107: What is the number of ways in which 3 holidays travel tickets are to be given to 10 employees of an organization if each employee is eligible for any one or more of the tickets?

A. 60

B. 120

C. 500

D. 1000

Marks:[2.50]

Q.No.108: If one root of the equation $(l - m)\ x^2 + lx + 1 = 0$ is double the other and l is real, then what is the greatest value of m?

A. $-\frac{9}{8}$

B. $\frac{9}{8}$

C. $-\frac{8}{9}$

D. $\frac{8}{9}$

Marks:[2.50]

Q.No.109: What is the number of four-digit decimal numbers (< 1) in which no digit is repeated?

A. 3024

B. 4536

C. 5040

D. None of the above

Marks:[2.50]

Q.No.110: What is a vector of unit length orthogonal to both the vectors and $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+3\hat{i}-\hat{k}$?

A. $\frac{-4\hat{i}+3\hat{j}-\hat{k}}{\sqrt{26}}$

B. $\frac{-4\hat{i}+3\hat{j}+\hat{k}}{\sqrt{26}}$

C. $\frac{-3\hat{i}+2\hat{j}-\hat{k}}{\sqrt{14}}$

D. $\frac{-3\hat{i}+2\hat{j}+\hat{k}}{\sqrt{14}}$

Marks:[2.50]

Q.No.111: If $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then which one of the following is correct?

A. $\overrightarrow{a}+\overrightarrow{b}+\overrightarrow{c}=\overrightarrow{0}$

B. $\overrightarrow{a}+\overrightarrow{b}+\overrightarrow{c}=\text{unit vector}$

C. $\overrightarrow{a}+\overrightarrow{b}=\overrightarrow{c}$

D. $\overrightarrow{a}=\overrightarrow{b}+\overrightarrow{c}$

Marks:[2.50]

Q.No.112: What is the area of the parallelogram having diagonals $3\hat{i}+\overrightarrow{j}-\overrightarrow{2k}$ and $\hat{i}-3\hat{j}+4\overrightarrow{k}$?

A. $5\sqrt{5}$ square units

B. $4\sqrt{5}$ square units

C. $5\sqrt{3}$ square units

D. $15\sqrt{2}$ square units

Marks:[2.50]

Q.No.113: Consider the following in respect to the matrix $A=\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$:

1. $A^2 = -A$
2. $A^3 = 4A$

Which of the above statements is/are correct?

A. 1 only

B. 2 only
C. Both 1 and 2
D. Neither 1 nor 2

Marks:[2.50]

Q.No.114: Which of the following determinants have value 'zero'?

1. $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$
2. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$
3. $\begin{vmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix}$

Select the correct answer using the code given below.

A. 1 and 2 only
B. 2 and 3 only
C. 1 and 3 only
D. 1, 2 and 3

Marks:[2.50]

Q.No.115: What is the acute angle between the lines represented by the equations $y - \sqrt{3}x - 5 = 0 \text{ and } \sqrt{3}y - x + 6 = 0$?

A. 30º
B. 45º
C. 60º
D. 75º

Marks:[2.50]

Q.No.116: The system of linear equations $kx + y + z = 1$, $x + ky + z = 1$ and $x + y + kz = 1$ has a unique solution under which one of the following conditions?

A. $k \neq 1 \text{ and } k \neq -2$
B. $k \neq 1 \text{ and } k \neq 2$
C. $k \neq -1 \text{ and } k \neq -2$
D. $k \neq -1 \text{ and } k \neq 2$

Marks:[2.50]

Q.No.117: What is the number of different messages that can be represented by three 0's and two 1's?

A. 10

B. 9

C. 8

D. 7

Marks:[2.50]

Q.No.118: If $\log_a(ab) = x$, then what is $\log_b(ab)$ equal to?

A. $\frac{1}{x}$

B. $\frac{x}{x+1}$

C. $\frac{x}{1-x}$

D. $\frac{x}{x-1}$

Marks:[2.50]

Q.No.119: If $y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$ then what is $\left(\frac{dy}{dx}\right)_{x=10}$ equal to?

A. 10

B. 2

C. 1

D. 0

Marks:[2.50]

Q.No.120: Suppose ω_1 and ω_2 are two distinct cube roots of unity different from 1. Then what is $(\omega_1 - \omega_2)^2$ equal to?

A. 3

B. 1

C. –1

D. –3

Marks:[2.50]