NDA I 2017_Mathematic

Total Time: 150

Total Marks: 300.0

Solution 1

Given that S is the set and the relation R = {(*x, y*) where *x* and *y* are born on the same day}
Let's consider three persons *a, b* and *c*.
Reflexive:
a ∈S ⇒(*a, a*)∈R means if *a* belongs to Delhi then *a* and *b* are born on the same day so the relation is reflexive.

Symmetric:
(*a, b*)∈R then (*b, a*)∈R means if *a* & *b* are born on the same day then *b* & *a* are also born on the same day so the relation is symmetric.

Transitive:
(*a, b*)∈R and (*b, c*)∈R ⇒(*a, c*)∈R means if *a* & *b* are born on the same day and *b* & *c* are born on the same then *a* & *c* are also born on the same day so the relation is transitive

Since the relation is relfexive, symmetric and transitive so the relation is equivalent.
Hence, the correct answer is option A.

Solution 2

Given, *A* = {1,2,3,4,5,6,7,8,9,10}
Number of subsets containing 2 elements = number of ways of selecting 2 elements out of 10 = ${}^{10}C_2$ =45
Number of subsets containig 3 elements = number of ways of selecting 3 elements out of 10 = ${}^{10}C_3$ =120
Therefore, the number of subsets containig 2 or 3 elements = 45 + 120 = 165.
Hence, the correct answer is option C.

Solution 3

Given expression: $i^{2n}+i^{2n+1}+i^{2n+2}+i^{2n+3}$
$=i^{2n}+i^{2n}\times i+i^{2n}\times i^2+i^{2n}\times i^2\times i$
$=i^{2n}\left(1+i+i^2+i^2\times i\right)$
$=i^{2n}\left(1+i-1-i\right)$
$=i^{2n}\times 0$
$=0$

Hence, the correct answer is option A.

Solution 4

Given equation is $x^2+kx+1=0$
Let's suppose α and β are the roots of the given equation.
Therefore,
$\alpha+\beta=\frac{-b}{a}=\frac{-k}{1}=-k$

and $\alpha\beta = \frac{c}{a} = \frac{1}{1} = 1$
Difference between the roots is strictly lesser than $\sqrt{5}$.
$\Rightarrow |\alpha - \beta| < \sqrt{5}$

$\Rightarrow \left(\alpha - \beta\right)^2 < 5$

$\Rightarrow \left(\alpha + \beta\right)^2 - 4\alpha\beta < 5$

$\Rightarrow \left(-k\right)^2 - 4 \times 1 < 5$

$\Rightarrow k^2 - 4 < 5$
$\Rightarrow k^2 < 9$
$\Rightarrow k^2 - 3^2 < 0$
$\Rightarrow -3 < k < 3 \ldots \left(i\right)$

Also,

$|k| \geq 2$
$\Rightarrow k \leq -2 \; and \; k \geq 2 \ldots (ii)$

From (i) and (ii), we have (−3, −2] ∪ [2,3)
Hence, the correct answer is option A.

Solution 5

Given that the roots of the equations $x^2 + px + q = 0$ and $x^2 + lx + m = 0$ are in the same ratio.
Let the ratio of roots is *a : b*.
Therefore, let the roots of the equation $x^2 + px + q = 0$ are *ac* and *bc*.
Sum of roots = $\frac{-p}{1} = -p$
∴ *c*(*a* + *b*) = −*p*
$\Rightarrow c = \frac{-p}{a+b}$
and
Product of the roots = $\frac{q}{1} = q$
$\Rightarrow ac \times bc = q$
⇒ $abc^2 = q$
$\Rightarrow ab\left(\frac{-p}{a+b}\right)^2 = q$

$\Rightarrow ab = \frac{q\left(a+b\right)^2}{p^2} \qquad \ldots\ldots \left(i\right)$

Similarly, let the roots of the equation $x^2 + lx + m = 0$ be *ad* and *bd*.
Sum of roots = $\frac{-l}{1} = -l$
∴ *d*(*a* + *b*) = −*l*
$\Rightarrow d = \frac{-l}{a+b} \qquad \ldots\ldots \left(ii\right)$
and
Product of roots = $\frac{m}{1} = m$
$abd^2 = m$

$\Rightarrow ab\left(\frac{-l}{a+b}\right)^2 = m$

$\Rightarrow \left(\frac{q(a+b)^2}{p^2}\right)\left(\frac{l^2}{(a+b)^2}\right) = m$

$\Rightarrow \frac{ql^2}{p^2} = m$

$\Rightarrow l^2q = p^2m$

Hence, the correct answer is option A.

Solution 6

Given expression is $\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$

The two brackets represent the complex cube root of unity.

Let $\omega = \left(\frac{-1+i\sqrt{3}}{2}\right)$ $then\ \omega^2 = \left(\frac{-1-i\sqrt{3}}{2}\right)$

So, the expression is $\omega^n + \left(\omega^2\right)^n$

Since any number can be written in the form of 3*k*, 3*k*+1 or 3*k*+2 but *n* is not a multiple of 3 so *n* can be 3*k*+1 or 3*k*+2.

When *n* = 3*k*+1 then expression is

$\omega^{3k+1} + \left(\omega^2\right)^{3k+1}$

$= \left(\omega^3\right)^k \times \omega + \left(\omega^3\right)^{2k} \times \omega^2$

$= \omega + \omega^2 \quad \left(\omega^3 = 1\right)$

$= -1$

When *n* = 3*k*+2 then expression is $\omega^{3k+2} + \left(\omega^2\right)^{3k+2}$

$= \left(\omega^3\right)^k \times \omega^2 + \left(\omega^3\right)^{2k} \times \left(\omega^4\right)$

$= \omega^2 + \omega^4 \qquad \left(\omega^3 = 1\right)$

$= \omega^2 + \omega \quad \left(\omega^3 = 1\right)$

$= -1$

Hence, the correct answer is option B.

Solution 7

Since repetition is not allowed so number of numbers formed from the digits 1, 2 and 3 is 1 × 2 × 3 = 6

Each digit will occur twice in every ones, tens and hundreds places.

Sum of digits at ones = 2(1+2+3)=12 so 2 will remain at ones and 1 will be carried to tens.

Sum of digits at tens = 2(1+2+3) + 1 = 13 so 3 will remain at tens and 1 will be carried to hundreds.

Sum of digits at hundreds = 2(1+2+3)+1=13

Therefore, the sum of all the 3-digit numbers formed from the digits 1, 2 and 3 is 1332.

Hence, the correct answer is the option D.

Solution 8

Series is 0.3 + 0.33 + 0.333 + ….. to *n* terms

= 3(0.1 + 0.11 + 0.111 + …. to *n* terms)

$= \frac{3}{9}\left(0.9 + 0.99 + 0.999 + \ldots \text{ to } n \text{ terms}\right)$

$= \frac{1}{3}\left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots \text{ to } n \text{ terms}\right)$

$= \frac{1}{3}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots \text{ to } n \text{ terms}\right]$

$= \frac{1}{3}\left[\left(1+1+1+\ldots \text{to } n \text{ terms}\right) - \left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\ldots \text{ to } n \text{ terms}\right)\right]$

$= \frac{1}{3}\left[n - \frac{\left(\frac{1}{10}\right)\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$

$= \frac{1}{3}\left[n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

Hence, the correct answer is option A.

Solution 9

$$\left(1+\omega\right)\left(1+\omega^2\right)\left(1+\omega^3\right)\left(1+\omega+\omega^2\right)$$
$$= \left(1+\omega\right)\left(1+\omega^2\right)\left(1+\omega^3\right)\times 0 \qquad \left(1+\omega+\omega^2=0\right)$$
$$= 0.$$

Hence, the correct answer is option C.

Solution 10

Given, $S_m = n$

$\Rightarrow \frac{m}{2}\left\{2a+\left(m-1\right)d\right\} = n$

$\Rightarrow 2a+\left(m-1\right)d = \frac{2n}{m} \qquad .....(i)$

Also, $S_n = m$

$\Rightarrow \frac{n}{2}\left\{2a+\left(n-1\right)d\right\} = m$

$\Rightarrow 2a+\left(n-1\right)d = \frac{2m}{n} \qquad .....(ii)$

Solving (i) and (ii) using elimination method, we have

$d = \frac{-2(m+n)}{mn}$

Substituting *d* in equation (i), we have

$a = \frac{m^2+n^2+mn-m-n}{mn}$

$S_{m+n} = \frac{m+n}{2}\left\{2a+(m+n-1)d\right\}$

$= \frac{m+n}{2}\left\{\frac{2\left(m^2+n^2+mn-m-n\right)}{mn} - \frac{2(m+n-1)(m+n)}{mn}\right\}$

$= \left(m+n\right)\left\{\frac{m^2+n^2+mn-m-n-m^2-n^2-2mn+m+n}{mn}\right\}$

$= \left(m+n\right)\left(-1\right)$

$= -\left(m+n\right).$

Hence, the correct answer is option D.

Solution 11

$\frac{1+2i}{1-(1-i)^2}$

$= \frac{1+2i}{1-(1+i^2-2i)}$

$= \frac{1+2i}{1+2i}$

$= 1$

$= 1 + 0i \ = x + iy \quad \left(\text{say}\right)$

Modulus = $\sqrt{x^2+y^2} = \sqrt{1^2+0^2} = 1$
Principal argument = $\tan^{-1}\frac{y}{x} = \ \tan^{-1}\frac{0}{1} = \ \tan^{-1}(\tan\ 0) = 0$
(modulus, principal argument) = (1, 0)
Hence, the correct answer is option A.

Solution 12

Real zeroes or roots of a polynomial is determined by number of times the graph of polynomial intersects the x-axis.
Since the graph is completely above the x-axis that means there are no real zeroes or roots so both the zeroes or roots are complex.
Hence, the correct answer is option C.

Solution 13

$|z+4| \leq 3$

$\Rightarrow -3 \leq (z+4) \leq 3$

$\Rightarrow -7 \leq z \leq -1$

The two extreme values of z are −7 and −1.
When z = −7 then $|z+1| = |-7+1| = |-6| = 6$
When z = −1 then $|z+1| = |-1+1| = |0| = 0$
Thus, the maximum value of z is 6.
Hence, the correct answer is option C.

Solution 14

Given equation is $z^2 = 2\overline{z}$
Let $z = x + iy$ then $\bar{z} = x - iy$
Therefore, $(x+iy)^2 = 2(x-iy)$

$x^2 + i^2y^2 + i2xy = 2x - i2y$

$\Rightarrow (x^2-y^2) + i2xy = 2x - i2y$

$\Rightarrow x^2 - y^2 = 2x \qquad \ldots\ldots (i)$

and

$2xy = -2y \qquad \ldots\ldots (ii)$

From (ii),
$y(x + 1) = 0 \Rightarrow y = 0$ or $x = -1$
Substituting y = 0 in (i),
$x^2 - 0^2 = 2x \ \Rightarrow x^2 - 2x = 0 \ \Rightarrow x(x-2) = 0 \ \Rightarrow x = 0,\ 2$
Thus, the solutions are (0,0), (2,0).
Substituting x = −1 in (i),
$(-1)^2 - y^2 = 2(-1) \ \Rightarrow 1 - y^2 = -2 \ \Rightarrow y^2 = 3 \ \Rightarrow y = \pm\sqrt{3}$
Thus, the solutions are $\left(-1, \sqrt{3}\right),\ \left(-1, -\sqrt{3}\right)$
Since all these solutions are satisfying the original equation so there are 4 solutions possible.
Hence, the correct answer is option C.

Solution 15

Given equation is $x^2+bx+c=0$
Since $\cot\alpha$ and $\cot\beta$ are the roots of the equation so,
$\cot\alpha+\cot\beta=-b$ and $\cot\alpha\cdot\cot\beta=c$
Therefore,
$\cot(\alpha+\beta)=\frac{\cot\alpha\cdot\cot\beta-1}{\cot\alpha+\cot\beta}=\frac{c-1}{-b}=\frac{1-c}{b}$
Hence, the correct answer is option B.

Solution 16

Given equation is $x^2+bx+c=0$
Let's suppose α and β are the roots of the equation.
$\alpha+\beta=-b$ and $\alpha\beta=c$
According to question,
$$\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$$
$$\Rightarrow -b=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$$
$$\Rightarrow -b=\frac{(\alpha+\beta)^2-2\alpha\beta}{c^2}$$
$$\Rightarrow -b=\frac{b^2-2c}{c^2}$$
$$\Rightarrow -bc^2=b^2-2c$$
$$\Rightarrow 2c=b^2+bc^2$$

Now, let's check with the given options
If $\frac{1}{c}$, b and $\frac{c}{b}$ are in AP then $2b=\frac{1}{c}+\frac{c}{b}\ \Rightarrow 2b^2c=b+c^2$
If they are in GP, then $b^2=\frac{1}{c}\times\frac{c}{b}\Rightarrow b^3=1$
If they are in HP, then c, $\frac{1}{b}$ *and* $\frac{b}{c}$ are in AP so, $\frac{2}{b}=c+\frac{b}{c}\ \Rightarrow \frac{2}{b}=\frac{c^2+b}{c}\ \Rightarrow 2c=b^2+bc^2$
Hence, the correct answer is option C.

Solution 17

Given equation is $ax^2+x+c=0$
Let's suppose the roots are α *and* β.
$\alpha+\beta=\frac{-1}{a}$ *and* $\alpha\beta=\frac{c}{a}$
According to question,
$$\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$$
$$\Rightarrow \frac{-1}{a}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$$
$$\Rightarrow \frac{-1}{a}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$
$$\Rightarrow \frac{-1}{a}=\frac{\frac{1}{a^2}-\frac{2c}{a}}{\frac{c^2}{a^2}}$$
$$\Rightarrow \frac{-1}{a}=\frac{1-2ac}{c^2}$$
$$\Rightarrow -c^2=a-2a^2c$$
$$\Rightarrow 2a^2c=a+c^2$$

If a, ca^2 *and* c^2 are in AP then
$ca^2-a=c^2-ca^2$
$\Rightarrow 2ca^2=a+c^2$

Hence, the correct answer is option A.

Solution 18

Using the formula, ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

$$\left({}^7C_0 + {}^7C_1\right) + \left({}^7C_1 + {}^7C_2\right) + \left({}^7C_2 + {}^7C_3\right) + \ldots + \left({}^7C_5 + {}^7C_6\right) + \left({}^7C_6 + {}^7C_7\right)$$

$$= \left({}^7C_0 + {}^7C_1 + {}^7C_3 + \ldots + {}^7C_6\right) + \left({}^7C_1 + {}^7C_2 + {}^7C_3 + \ldots + {}^7C_7\right)$$

$$= \left(2^7 - {}^7C_7\right) + \left(2^7 - {}^7C_0\right)$$

$$= (128 - 1) + (128 - 1)$$

$$= 256 - 2$$

$$= 254.$$

Hence, the correct answer is option A.

Solution 19

There are three consonents (Q,T,N) and 5 vowels in the word 'EQUATION'.
The word should start and end with a consonent.
Selection of two consonents out of 3 can be made in ${}^3C_2$ ways.
These two consonents can be arranged in 2! ways further.
Rest 6 places can be filled with remaining 6 letters in 6! ways.
So, the number of words = ${}^3C_2$ × 2! × 6! = 3 × 2 × 720 = 4320.
Hence, the correct answer is option B.

Solution 20

$a_5 = S_5 - S_4$

$$= \left(5^2 - 2 \times 5\right) - \left(4^2 - 2 \times 4\right) \ = \ 15 - 8 \ = \ 7.$$

Hence, the correct answer is option B.

Solution 21

Sum of all 2-digit odd numbers = 11 + 13 + 15 + ... + 99
These numbers form an AP where *a* = 11, *d* = 2, $a_n = 99$
To find '*n*', let's use formula $a_n = a + (n-1)d$

$$99 = 11 + (n-1)(2)$$

$$88 = (n-1)\ (2)$$

$$44 = n - 1$$

$$n = 45$$

Therefore, the sum of all the two digit odd numbers will be

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{45}{2}\{2 \times 11 + (45-1)(2)\}$$

$$= 45\,(11 + 44)$$

$$= 45 \times 55 \ = \ 2475$$

Hence, the correct answer is option A.

Solution 22

Given series:

$\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots.$ to n terms

$=\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)+\ldots$ to n terms

$=(1+1+1+1+\ldots$ to n terms$)-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots \text{ to } n \text{ terms}\right)$

$=n-\frac{\left(\frac{1}{2}\right)\left\{1-\frac{1}{2^n}\right\}}{1-\frac{1}{2}}$

$=n-1+\frac{1}{2^n}=2^{-n}+n-1$

Hence, the correct answer is option C.

Solution 23

Let's check the statements one by one-
(i) let $x \in (A-B) \cup B \Rightarrow x \in (A-B)$ or $x \in B \Rightarrow (x \in A$ and $x \notin B)$ or $x \in B \Rightarrow (x \in A$ or $x \in B)$ and $(x \notin B$ or $x \in B) \Rightarrow x \in (A \cup B)$
So, the statement is false.

(ii) let $x \in (A-B) \cup A \Rightarrow x \in (A-B)$ or $x \in A \Rightarrow (x \in A$ and $x \notin B)$ or $x \in A \Rightarrow (x \in A$ or $x \in A)$ and $(x \notin B$ or $x \in A) \Rightarrow x \in A$
So, the statement is correct.

(iii) let $x \in (A-B) \cap B \Rightarrow x \in (A-B)$ and $x \in B \Rightarrow (x \in A$ and $x \notin B)$ and $x \in B \Rightarrow (x \in A$ and $x \in B)$ and $(x \notin B$ and $x \in B)$
$\Rightarrow x \in (A \cap B)$ and $x \in \phi \Rightarrow x \in \phi$
So, the statement is correct.

(iv) Given $A \subseteq B$ that means all the elements of A are also the elements of B.
So, $(A \cup B)$ will have same elements as B. Therefore, $(A \cup B) = B$
So, the statement is correct.
Thus, statements (ii), (iii) and (iv) are corrrect.
Hence, the correct answer is option B.

Solution 24

```
   1  p  1  0  1
 + 0  1  0  q  1
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  1 0 0  r   0  0
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In this column method of addition,
1st column, $1+1=(2)_{10}=(10)_2$ so, 1 is carried to the next column.
2nd column, 1(carry) + 0 + q = 1 + q. Here, 0 is written in the last row that means $q = 1$
3rd column, 1(carry) + 1 + 0 = $(2)_{10}=(10)_2$ so, $r = 0$ and 1 carried to the next row.
4th row, 1(carry) + p + 1 = $(2)_{10} + p = 0 + p$ but 0 is written in the last row that means $p = 0$
Thus, $p = 0$, $q = 1$ and $r = 0$
Hence, the correct answer is option A.

Solution 25

$S=\left\{x: x^2+1=0,\ x \text{ is real}\right\}$
Solving the equation $x^2+1=0$, we get two unreal solutions. That means there is no real solution of x.
So, S is an empty set.
Hence, the correct answer is option D.

Solution 26

the (r+1)th term of a binomial $(x+y)^n$ = ${}^nC_r . x^{n-r} . y^r$

5th term of $\left(x-y\right)^n={}^nC_4 . x^{n-4} . \left(-y\right)^4=\frac{n(n-1)(n-2)(n-3)}{24} . x^{n-4} . y^4$

6th term of $\left(x-y\right)^{n} = {}^{n}C_5 . x^{n-5} . \left(-y\right)^{5} = -\frac{n(n-1)(n-2)(n-3)(n-4)}{120} . x^{n-5} . y^{5}$

The sum of 5th and 6th terms =

$$x^{n-4}y^{4} - \frac{n-4}{5}x^{n-5}y^{5} = 0$$

$$\Rightarrow x^{n-5}y^{4}\left\{x - \frac{n-4}{5}y\right\} = 0$$

$$\Rightarrow x^{n-5} = 0 \; or \; y^{4} = 0 \; or \; x - \frac{n-4}{5}y = 0 \; \Rightarrow \frac{x}{y} = \frac{n-4}{5}$$

Hence, the correct answer is option (b).

Solution 27

$$\text{Given : } \det\left(A^{3}\right) = 125$$

$$\Rightarrow (|A|)^{3} = 5^{3}$$
$$\Rightarrow |A| = 5$$
$$\Rightarrow \alpha^{2} - 4 = 5$$
$$\Rightarrow \alpha^{2} = 9$$
$$\Rightarrow \alpha = \pm 3$$

Hence, the correct answer is option (c).

Solution 28

B is a non-singular matrix and A is a square matrix.

$\left|B^{-1}AB\right| = \left|B^{-1}\right| . |A| . |B| = \frac{1}{|B|} . |A| . |B| = |A|$

Hence, the correct answer is option (b).

Solution 29

Given, $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$

Going through the choices one by one,
when x = 0, the determinant becomes

$\begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

Which is true.
Hence, the correct answer is option (d).

Solution 30

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \; and \; A^{T} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A . A^{T} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \times \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^{2}\alpha + \sin^{2}\alpha & \cos\alpha . \sin\alpha - \sin\alpha . \cos\alpha \\ \sin\alpha . \cos\alpha - \cos\alpha . \sin\alpha & \sin^{2}\alpha + \cos^{2}\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Which is an identity matrix.
Hence, the correct answer is option (b).

Solution 31

Given system of equations is
$x + 2y + 3z = 1$
$2x + y + 3z = 2$
$5x + 5y + 9z = 4$
det. of coefficient matrix-

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix} = 1\left(9 - 15\right) - 2\left(18 - 15\right) + 3\left(10 - 5\right) = -6 - 6 + 15 = 3 \neq 0$$

Since, the coefficient matrix is non-singular so the system of equations has unique solution.
Hence, the correct answer is option (a).

Solution 32

Given, $A = \begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $and\ C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$AB = C$

$$\begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 3x+3y-2y \\ 3x-2x+2y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 3x+y \\ x+2y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$3x + y = 4$

$x + 2y = -2$

Solving the above pair of equations, we have $x = 2$, $y = -2$

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & -8 \\ 8 & 12 \end{bmatrix}$$

Hence, the correct answer is option (d).

Solution 33

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+xyz & 1 \\ 1 & 1 & 1+xyz \end{vmatrix}$$

$By\ R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & xyz & 0 \\ 0 & 0 & xyz \end{vmatrix}$$

Expanding along R_3
$= x^2y^2z^2$
Hence, the correct answer is option (c).

Solution 34

$$\begin{vmatrix} x & y & 0 \\ 0 & x & y \\ y & 0 & x \end{vmatrix} = 0$$

Expanding along 1st row,

$x\left(x^2\right) - y\left(0 - y^2\right) = 0$

$x^3 + y^3 = 0$

$\left(x + y\right)\left(x^2 - xy + y^2\right) = 0$

$either\ x + y = 0\ or\ x^2 - xy + y^2 = 0$

When $x^2 - xy + y^2 = 0$

$\left(\frac{x}{y}\right)^2 - \frac{x}{y} + 1 = 0$

That means $\frac{x}{y}$ is a cube root of unity.

Hence, the correct answer is option (a).

Solution 35

Let's consider a matrix of order 3 × 3 as $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, the determinant of which is 1

Number of ways of different arrangements of given 3 rows = 3!
Number of ways of different arrangements of given 3 columns = 3!
Total number of matrices possible by interchanging the rows and columns = 3! + 3! = 12 (these matrices will have determinant value either 1 or −1)
We know that if any two rows or columns are interchanged together, the sign of determinant of the matrix changes.
Number of ways to interchange any two rows out of 3 = ${}^3C_2$ = 3
Number of ways to interchange any two columns out of 3 = ${}^3C_2$ = 3
Total number of ways to interchange any two rows or columns = 3+3 = 6
Clearly, there are 6 ways when we get the determinant negative (in this case −1) which is half of the total number of matrices possible.
Following the above conclusion, we can say that set B will have as many elements as set C.
Hence, the correct answer is option (b).

Solution 36

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \times \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta\cos\theta - \sin 2\theta\sin\theta & \cos 2\theta\sin\theta + \sin 2\theta\cos\theta \\ -\sin 2\theta\cos\theta - \cos 2\theta\sin\theta & -\sin 2\theta\sin\theta + \cos 2\theta\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

Hence, the correct answer is option (a).

Solution 37

Order of a matrix found on multiplying the two matrices of orders (mxn) and (nxp) is (mxp).

$$[x \quad y \quad z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Orders of the given matrices are (1x3), (3x3) and (3x1) respectively.
Order of matrix found on multiplying the matrices of orders (1x3) and (3x3) is (1x3).
then the order of matrix found on multiplying the matrices of orders (1x3) and (3x1) is (1x1).
Hence, the correct answer is option (b).

Solution 38

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^4 = A^2 \times A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, the correct answer is option (a).

Solution 39

$$Given: \ SinA = \frac{3}{5} \ and \ 450^\circ < A < 540^\circ \ \Rightarrow 225^\circ < \frac{A}{2} < 270^\circ$$

So, A is in 2nd quadrant and $\frac{A}{2}$ is in 3rd quadrant.

$$Cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

$$CosA = -\frac{4}{5} \left(A \ is \ in \ 2nd \ quadrant\right)$$

$$Cos^2 \frac{A}{2} = \frac{1+CosA}{2} = \frac{1-\frac{4}{5}}{2} = \frac{1}{10}$$

$$Cos \frac{A}{2} = \frac{-1}{\sqrt{10}} \left(\frac{A}{2} \ is \ in \ 3rd \ quadrant\right)$$

Hence, the correct answer is option (d).

Solution 40

$$\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$$

$$= \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ}$$

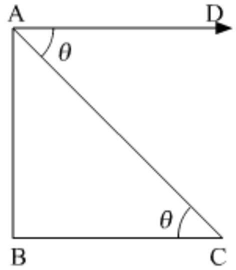
$$= \frac{2 \times 2\left(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)}{2\sin 10^\circ \cos 10^\circ}$$

$$= \frac{4(\cos 60^\circ \cos 10^\circ - \sin 60^\circ \sin 10^\circ)}{\sin 20^\circ}$$

$$= \frac{4\cos 70^\circ}{\sin 20^\circ} = \frac{4\cos(90^\circ - 20^\circ)}{\sin 20^\circ} = \frac{4\sin 20^\circ}{\sin 20^\circ} = 4$$

Hence, the correct answer is option (d).

Solution 41



In the figure, AB is the lighthouse 100 m high.

∠CAD = ∠ACB = $\theta = \tan^{-1}\left(\frac{5}{12}\right)$

We need to find BC.

In ΔABC,

$\tan\theta = \frac{AB}{BC}$

$\tan\left\{\tan^{-1}\frac{5}{12}\right\} = \frac{100}{BC}$

$\frac{5}{12} = \frac{100}{BC}$

$BC = \frac{1200}{5} = 240\ m.$

Hence, the correct answer is option (c).

Solution 42

$$\sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$$
$$=\sqrt{2}\left\{\sin\left(x+\frac{\pi}{6}\right)\frac{1}{\sqrt{2}}+\cos\left(x+\frac{\pi}{6}\right)\frac{1}{\sqrt{2}}\right\}$$
$$=\sqrt{2}\left\{\sin\left(x+\frac{\pi}{6}\right)\cos\frac{\pi}{4}+\cos\left(x+\frac{\pi}{6}\right)\sin\frac{\pi}{4}\right\}$$
$$=\sqrt{2}\sin\left(x+\frac{\pi}{6}+\frac{\pi}{4}\right)$$

For maximum value of the expression,

$$x+\frac{\pi}{6}+\frac{\pi}{4}=\frac{\pi}{2}$$
$$\Rightarrow x=\frac{\pi}{2}-\frac{\pi}{6}-\frac{\pi}{4}=\frac{\pi}{12}.$$

Hence, the correct answer is option (a).

Solution 43

$$K=\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$$
$$=\sin 10^\circ\sin 50^\circ\sin 70^\circ$$
$$=\frac{1}{2}\left(2\sin 50^\circ\sin 10^\circ\right)\sin 70^\circ$$
$$=\frac{1}{2}\left(\cos 40^\circ-\cos 60^\circ\right)\sin 70^\circ$$
$$=\frac{1}{2}\left[\frac{1}{2}\left(2\sin 70^\circ\cos 40^\circ\right)-\frac{1}{2}\sin 70^\circ\right]$$
$$=\frac{1}{4}\left[\sin 110^\circ+\sin 30^\circ-\sin 70^\circ\right]$$
$$=\frac{1}{4}\left[\sin 70^\circ+\sin 30^\circ-\sin 70^\circ\right]=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}.$$

Hence, the correct answer is option (c).

Solution 44

$\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}$

$= \frac{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}$

$= \tan\ \frac{\alpha+\beta}{2}$

Hence, the correct answer is option (a).

Solution 45

$\sin\theta = 3\sin(\theta+2\alpha)$

$\Rightarrow \frac{\sin(\theta+2\alpha)}{\sin\theta} = \frac{1}{3}$

$\Rightarrow \frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta} = \frac{1+3}{1-3}$

$\Rightarrow \frac{2\sin(\theta+\alpha)\cos\alpha}{2\cos(\theta+\alpha)\sin\alpha} = \frac{4}{-2}$

$\Rightarrow \frac{\tan(\theta+\alpha)}{\tan\alpha} = -2$

$\Rightarrow \tan\left(\theta+\alpha\right) = -2\tan\alpha$

$\Rightarrow \tan\left(\theta+\alpha\right) + 2\tan\alpha = 0$

Hence, the correct answer is option (b).

Solution 46

$Let\ A = 18^\circ$
$then\ 5A = 90^\circ$
$\Rightarrow 2A + 3A = 90^\circ$
$\Rightarrow 2A = 90^\circ - 3A$

$\Rightarrow \sin 2A = \sin\left(90^\circ - 3A\right)$

$\Rightarrow \sin 2A = \cos 3A$
$\Rightarrow 2\sin A\cos A = 4\cos^3 A - 3\cos A$
$\Rightarrow \cos A\left(4\cos^2 A - 2\sin A - 3\right) = 0$
$\Rightarrow \cos A\left(4 - 4\sin^2 A - 2\sin A - 3\right) = 0$
$\Rightarrow -\cos A\left(4\sin^2 A + 2\sin A - 1\right) = 0$

$either\ \cos A = 0\left(not\ possible\right)\ or\ 4\sin^2 A + 2\sin A - 1 = 0$

$\Rightarrow 4\sin^2 A + 2\sin A - 1 = 0$

$\sin A = \frac{-2\pm\sqrt{2^2-4\times4\times(-1)}}{2\times4}\ \left(A = 18^\circ\ that\ is\ an\ acute\ angle\ so\ \sin A\ is\ positive\right)$

$\therefore \sin A = \frac{-1+\sqrt{5}}{4}$

Now,

$\cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{-1+\sqrt{5}}{4}\right)^2 = \frac{10+2\sqrt{5}}{16}$

$\therefore \cos A = \frac{\sqrt{10+2\sqrt{5}}}{4}$

So,

$\tan A = \frac{\sin A}{\cos A} = \frac{-1+\sqrt{5}}{\sqrt{10+2\sqrt{5}}}$

Hence, the correct answer is option (a).

Solution 47

$x,\ y,\ z\ are\ in\ GP$

$\therefore y^2 = zx$

Now,

$\tan^{-1} x,\ \tan^{-1} y,\ \tan^{-1} z\ are\ in\ AP$

$\therefore 2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z$

$\Rightarrow \tan^{-1} \frac{2y}{1-y^2} = \tan^{-1} \frac{z+x}{1-zx}$

$\Rightarrow \frac{2y}{1-y^2} = \frac{z+x}{1-zx} = \frac{z+x}{1-y^2} \left(\because y^2 = zx\right)$

$\Rightarrow 2y = z + x$

$\Rightarrow x,\ y,\ z\ are\ in\ AP.$

$But\ x,\ y,\ z\ are\ given\ in\ GP$

$\Rightarrow x = y = z.$

Hence, the correct answer is option (a).

Solution 48

$\tan\left(2\alpha\right) = \tan\left[\left(\alpha+\beta\right) + \left(\alpha-\beta\right)\right] = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta).\tan(\alpha-\beta)} = \frac{2+1}{1-2\times 1} = -3$

Hence, the correct answer is option (a).

Solution 49

$A + B + C = 180^\circ$

$\Rightarrow B + C = 180^\circ - A$

$\Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2} \ldots . \left(\text{i}\right)$

$\text{Taking sin on both sides of } \left(\text{i}\right)$

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right) = \cos\frac{A}{2}$

$\text{Taking tan on both sides of } \left(\text{i}\right)$

$\Rightarrow \tan\left(\frac{B+C}{2}\right) = \tan\left(90^\circ - \frac{A}{2}\right) = cot\frac{A}{2}$

Statements 1 and 2 are correct.
Hence, the correct answer is option (b).

Solution 50

$(sec\theta - \cos ec\theta) = \frac{4}{3}$

$\Rightarrow \frac{1}{\cos\theta} - \frac{1}{\sin\theta} = \frac{4}{3}$

$\Rightarrow \frac{\sin\theta - \cos\theta}{\sin\theta\cos\theta} = \frac{4}{3}$

$\Rightarrow \frac{(\sin\theta - \cos\theta)^2}{(\sin\theta\cos\theta)^2} = \frac{16}{9}$

$\Rightarrow \frac{1 - 2\sin\theta\cos\theta}{\sin^2\theta\cos^2\theta} = \frac{16}{9}$

let $\sin\theta\cos\theta = x$

$\frac{1-2x}{x^2} = \frac{16}{9}$

$\Rightarrow 16x^2 + 18x - 9 = 0$

$\Rightarrow x = \frac{-18 \pm \sqrt{18^2 - 4\times16\times(-9)}}{2\times16} = \frac{3}{8}, \frac{-3}{2}$

$\Rightarrow \sin\theta\cos\theta = \frac{3}{8}, \frac{-3}{2} \quad \left(\frac{-3}{2}\ is\ invalid\right)$

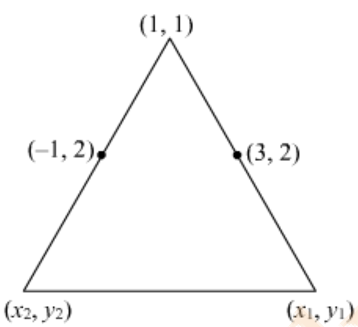
$\therefore \sin\theta\cos\theta = \frac{3}{8}$

$(\sin\theta - \cos\theta)^2 = \left(\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta\right) = 1 - 2\times\frac{3}{8} = \frac{1}{4}$

$\sin\theta - \cos\theta = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

Hence, the correct answer is option (b).

Solution 51



Let's assume the other two vertices are $(x_{1,}y_1)\ and\ (x_2, y_2)$.

$\therefore \frac{x_1+1}{2} = 3\ and\ \frac{y_1+1}{2} = 2$

$\Rightarrow x_1 = 5\ and\ y_1 = 3$

So, the vertex is (5,3)

Also,

$\frac{x_2+1}{2} = -1\ \ and\ \frac{y_2+1}{2} = 2$

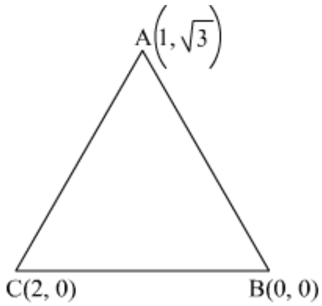
$\Rightarrow x_2 = -3\ \ and\ y_2 = 3$

So, the vertex is (-3,3)

Centroid= $\left(\frac{5+1-3}{3}, \frac{3+1+3}{3}\right) = \left(1, \frac{7}{3}\right)$

Hence, the correct answer is option (d).

Solution 52



Vertices of a triangle ABC are $A\left(1, \sqrt{3}\right),\ B\left(0, 0\right)\ and\ C\left(2, 0\right)$

$$AB = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2\ units$$

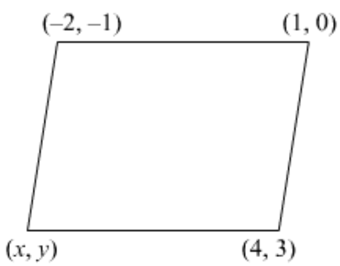
$$BC = \sqrt{2^2} = 2\ units$$

$$CA = \sqrt{(2-1)^2 + \left(0-\sqrt{3}\right)^2} = 2\ units$$

Thus, AB=BC=CA
So, ABC is an equilateral triangle.
Therefore, the incentre is same as the centroid, i.e. $\left(\frac{0+1+2}{3}, \frac{0+\sqrt{3}+0}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$

Hence, the correct answer is option (d).

Solution 53



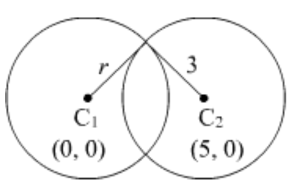
Coordinates of consecutive vertices of a parallelogram are given as (-2,-1), (1,0), (4,3).
Let's assume the fourth vertex is (x,y).
Property: Diagonals of a parallelogram bisect each other.
That means the mid-point of both the diagonals are same.

$$\frac{x+1}{2} = \frac{-2+4}{2}\ \ and\ \ \frac{y+0}{2} = \frac{-1+3}{2}$$

$$\Rightarrow x = 1\ \ \ and\ \ y = 2$$

Thus, the vertex of the fourth vertex is (1,2)
Hence, the correct answer is option (a).

Solution 54



For the circle $x^2 + y^2 = r^2$
Its centre is at (0,0) and the radius is r

For the circle

$x^2+y^2-10x+16=0$

$\left(x^2-10x\right)+(y-0)^2=-16$

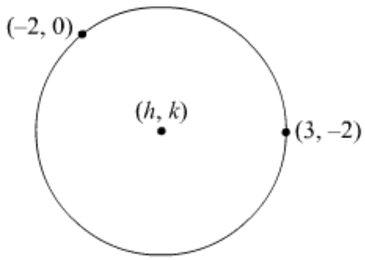
$\left(x^2-10x+5^2\right)+(y-0)^2=25-16$

$(x-5)^2+(y-0)^2=3^2$

Its centre is at (5,0) and the radius is 3
Clearly, the distance between the two centres is 5 units
So, (r-3)<5<(r+3) for the circles to intersect at two distinct points.
or 2 < r < 8

Hence, the correct answer is option (a).

Solution 55



Let the centre is O(x,y)
Let A(3,-2) and B(-2,0) are the points on the circle.
Therefore, OA=OB

$\sqrt{(h-3)^2+(k+2)^2}=\sqrt{(h+2)^2+(k-0)^2}$

$\Rightarrow h^2+9-6h+k^2+4+4k=h^2+4+4h+k^2$

$\Rightarrow 4k-6h+13=4h+4$

$\Rightarrow 4k-10h=-9$

Since the centre lies on the line 2x-y-3=0
So, 2h-k-3=0
Solving 4k-10h=-9 and 2h-k-3=0, we have $h=\frac{-3}{2}\ and\ k=-6$

So, the centre is $\left(-\frac{3}{2},-6\right)$ and radius is $\left(\frac{-3}{2}+2\right)^2+(-6)^2=\frac{1}{4}+36$

Equation of circle is

$\left(x+\frac{3}{2}\right)^2+(y+6)^2=\frac{1}{4}+36$

$\Rightarrow x^2+\frac{9}{4}+3x+y^2+36+12y=\frac{1}{4}+36$

$\Rightarrow x^2+y^2+3x+12y+2=0$

Hence, the correct answer is option (b).

Solution 56

Let the ratio be k:1 therefore,

$\left(\frac{2k-2}{k+1},\frac{-4k-2}{k+1}\right)=\left(\frac{-2}{7},\frac{-20}{7}\right)$

$\frac{2k-2}{k+1}=\frac{-2}{7}\ and\ \frac{-4k-2}{k+1}=\frac{-20}{7}$

$\therefore k=\frac{3}{4}$

$Ratio=3:4$

Hence, the correct answer is option (b).

Solution 57

$e=\frac{c}{a}\Rightarrow\frac{1}{4}=\frac{2}{a}\Rightarrow a=8$

$a^2=c^2+b^2$

$b^2=a^2-c^2=64-4=60$

$Ellipse:\ \frac{x^2}{64}+\frac{y^2}{60}=1$

Hence, the correct answer is option (a).

Solution 58

Since the line is parallel to 2x+3y+1=0 so let the equation to be 2x+3y+k=0
The line passes through (-1,2), therefore 2(-1)+3(2)+k=0
k=-4
So, the equation of the straight line is 2x+3y-4=0
Hence, the correct answer is option (a).

Solution 59

Slope m of the standrard linear equation $ax+by+c=0$ is $\frac{-a}{b}$

Here, $m_1=\frac{-\sqrt{2}}{\sqrt{3}}\ and\ m_2=\frac{-\sqrt{3}}{\sqrt{2}}$

Acute angle $=\tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\tan^{-1}\left|\frac{\frac{-\sqrt{2}}{\sqrt{3}}+\frac{\sqrt{3}}{\sqrt{2}}}{1+1}\right|=\tan^{-1}\left(\frac{1}{2\sqrt{6}}\right)$

Hence, the correct answer is option (a).

Solution 60

Coordinates of the vertices are (7,x), (y,-6) and (9,10). Centroid is (6,3).

$\therefore\ \frac{7+y+9}{3}=6\ \Rightarrow y=2$

$and\ \frac{x-6+10}{3}=3\Rightarrow x=5$

(x,y)=(5,2)
Hence, the correct answer is option (a).

Solution 61

Direction cosines are given (0,1,0)

$\therefore\cos\alpha=0\Rightarrow\cos\alpha=\cos 90^\circ\Rightarrow\alpha=90^\circ$

$\cos\beta=1\Rightarrow\cos\beta=\cos 0^\circ\Rightarrow\beta=0^\circ$

$\cos\gamma=0\Rightarrow\gamma=90^\circ$

Clearly, the line is parallel to y-axis.
Hence, the correct answer is option (b).

Solution 62

Let the point to be (x, y, z).

Distance between (x,y,z) and (0,0,0) is $\sqrt{x^2+y^2+z^2}$.....(i)

Distance between (x,y,z) and (a,0,0) is $\sqrt{(x-a)^2+y^2+z^2}$ ....(ii)

Distance between (x,y,z) and (0,b,0) is $\sqrt{x^2+(y-b)^2+z^2}$ ....(iii)

Distanve between (x,y,z) and (0,0,c) is $\sqrt{x^2+y^2+(z-c)^2}$ ...(iv)

Equating (i) and (ii), x=$\frac{a}{2}$

Equating (i) and (iii), y=$\frac{b}{2}$

Equating (i) and (iv), z=$\frac{c}{2}$

Thus, the point is $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

Hence, the correct answer is option (c).

Solution 63

Using vector concepts-

Vector $\overrightarrow{PQ} = \hat{i} + 3\hat{j} - 2\hat{k}$

$\overrightarrow{PR} = 2\hat{i} + 6\hat{j} - 4\hat{k} = 2\overrightarrow{PQ}$

$\overrightarrow{PS} = -\hat{i} - 3\hat{j} + 2\hat{k} = -1 \times \overrightarrow{PQ}$

Thus, vectors $\overrightarrow{PQ}, \overrightarrow{PR}\ and\ \overrightarrow{PS}$ are parallel with one point 'P' as common.
So, the points are collinear.
Hence, the correct answer is option (c).

Solution 64

The line passing through the points (1,2,-1) and (3,-1,2) is

$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+1}{3} = \lambda\left(Say\right)$

$x = 2\lambda + 1,\ y = -3\lambda + 2\ and\ z = 3\lambda - 1$

x must be zero at yz-plane so $0 = 2\lambda + 1\ \Rightarrow \lambda = \frac{-1}{2}$

$\therefore y = -3\left(\frac{-1}{2}\right) + 2 = \frac{3}{2} + 2 = \frac{7}{2}$ and $z = 3\left(\frac{-1}{2}\right) - 1 = \frac{-3}{2} - 1 = \frac{-5}{2}$

So, the point is $\left(0, \frac{7}{2}, \frac{-5}{2}\right)$

Hence, the correct answer is option (d).

Solution 65

Given lines are $\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}\ and\ \frac{x-f}{e} = \frac{y-0}{1} = \frac{z-h}{g}$

The lines will be perpendicular if $ae + cg + 1 = 0$

Hence, the correct answer is option (c).

Solution 66

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 2 \\ 1 & m & n \end{vmatrix} = 0$$

$3n - 2m + 2n - 2 + 2m - 3 = 0$

$n = 1$

Now,

$\left|\overrightarrow{C}\right| = \sqrt{6}$

$\sqrt{1 + m^2 + n^2} = \sqrt{6}$

$1 + m^2 + 1 = 6$

$m = \pm 2$

Hence, the correct answer is option (d).

Solution 67

Completing a parallelogram with $\overrightarrow{OA}\ and\ \overrightarrow{OC}$, $\overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OP}$

Similarly, $\overrightarrow{OB} + \overrightarrow{OD} = 2\overrightarrow{OP}$

$\therefore \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4\overrightarrow{OP}$

Hence, the correct answer is option (b).

Solution 68

$\overrightarrow{BA}+\overrightarrow{AD}=\overrightarrow{BD}....\left(i\right)$

$\overrightarrow{CD}+\overrightarrow{DA}=\overrightarrow{CA}....\left(ii\right)$

$Adding\ \left(i\right)\ and\ \left(ii\right),$

$\overrightarrow{BA}+\overrightarrow{CD}=\overrightarrow{BD}+\overrightarrow{CA}$

Hence, the correct answer is option (b).

Solution 69

$\vec{a}$ × $\vec{b}$ = $\vec{c}$ so $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.
$\vec{b}$ × $\vec{c}$ = $\vec{a}$ so $\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$.
Thus, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular to each other.
Also,

$\vec{a}\times\vec{b}=\vec{c}\Rightarrow\left|\vec{a}\right|\left|\vec{b}\right|\sin 90^{\circ}.1=\vec{c}\Rightarrow\left|\vec{a}\right|\left|\vec{b}\right|=\vec{c}$

$\vec{b}\times\vec{c}=\vec{a}\Rightarrow\left|\vec{b}\right|\left|\vec{c}\right|\sin 90^{\circ}.1=\vec{a}\Rightarrow\left|\vec{b}\right|\left|\vec{c}\right|=\vec{a}\Rightarrow\left|\vec{b}\right|\left|\vec{a}\right|\left|\vec{b}\right|=\vec{a}\Rightarrow\left|\vec{b}\right|=1$

and $\vec{a}=\vec{c}$

Hence, the correct answer is option (a).

Solution 70

$\vec{a}.\vec{b}=\left|\vec{a}\right|\left|\vec{b}\right|\cos 90^{\circ}$

$6+6-4\lambda=0$
$12=4\lambda$
$\lambda=3$

Hence, the correct answer is option (b).

Solution 71

$$\lim_{x\to 0}\frac{e^x-(1+x)}{x^2}=\lim_{x\to 0}\frac{\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....\right)-\left(1+x\right)}{x^2}=\lim_{x\to 0}\frac{\frac{x^2}{2!}+\frac{x^3}{3!}+...}{x^2}$$
$$=\lim_{x\to 0}\frac{1}{2!}+\frac{x}{3!}+...=\frac{1}{2}$$

Hence, the correct answer is option (b).

Solution 72

$$\int_0^{\frac{\pi}{2}}\frac{d\theta}{1+\cos\theta}=\int_0^{\frac{\pi}{2}}\frac{d\theta}{2\cos^2\frac{\theta}{2}}=\frac{1}{2}\int_0^{\frac{\pi}{2}}sec^2\frac{\theta}{2}d\theta=\frac{1}{2}\times 2\left[\tan\frac{\theta}{2}\right]_0^{\frac{\pi}{2}}=\frac{1}{2}\times 2\times 1=1$$

Hence, the correct answer is option (b).

Solution 73

$\int \frac{dx}{x(x^7+1)} = \frac{1}{7}\int \frac{7x^6}{x^7(x^7+1)}dx$

$Let\ x^7+1=t\ \Rightarrow 7x^6dx=dt$

$\therefore \frac{1}{7}\int \frac{7x^6}{x^7(x^7+1)}dx = \frac{1}{7}\int \frac{1}{t(t-1)}dt$

$\text{Using partial fraction,}$

$= \frac{1}{7}\int \frac{1}{t-1}dt - \frac{1}{7}\int \frac{1}{t}dt$

$= \frac{1}{7}\left(\ln|t-1| - \ln|t|\right) + c$

$= \frac{1}{7}\ln\left|\frac{t-1}{t}\right| + c \quad \left(c \text{ is any constant}\right)$

$= \frac{1}{7}\ln\left|\frac{x^7}{x^7+1}\right| + c$

Hence, the correct answer is option (d).

Solution 74

In the first quadrant, all the values of cosθ are positive and in second quadrant, all the values are negative. These values repeat in third and fourth quadrants. so for cosθ to be one-one and onto, X∊[0,π] and Y∊[-1,1].
Hence, the correct answer is option (a).

Solution 75

$f(x) = \frac{x}{x-1}$

$\therefore \frac{f(a)}{f(a+1)} = \frac{\frac{a}{a-1}}{\frac{a+1}{a}} = \frac{a^2}{a^2-1} = f\left(a^2\right)$

Hence, the correct answer is option (b).

Solution 76

$\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x}dx = \frac{1}{e}\int \frac{ex^{e-1}+e^x}{x^e+e^x}dx$

$Let\ x^e+e^x=t$

$\Rightarrow \left(ex^{e-1}+e^x\right)dx=dt$

$\therefore \frac{1}{e}\int \frac{ex^{e-1}+e^x}{x^e+e^x}dx = \frac{1}{e}\int \frac{1}{t}dt = \frac{1}{e}\ln|t| + c = \frac{1}{e}\ln|x^e+e^x| + c$

Hence, the correct answer is option (d).

Solution 77

$1.\ \text{L.H.S.} = (fofof)(-1) = (fof)(-2) = f(1) = -2$

$\text{R.H.S.} = (fofof)(1) = (fof)(-2) = f(1) = -2$

$\text{L.H.S.} = \text{R.H.S.}$

$\text{So, this statement is correct.}$

$2.\ \text{L.H.S.} = (fofof)(-1) - 4(fofof)(1) = (-2) - 4(-2) = -2 + 8 = 6$

$\text{R.H.S.} = (fof)(0) = f(-3) = 6$

$\text{L.H.S.} = \text{R.H.S.}$

So, this statement is correct.

Hence, the correct answer is option (c).

Solution 78

$f(x) = px + q,\ g(x) = mx + n$

$f(g(x)) = g(f(x))$

$p(mx + n) + q = m(px + q) + n$

$pmx + pn + q = pmx + mq + n$

$pn + q = mq + n$

$f(n) = g(q)$

Hence, the correct answer is option (c).

Solution 79

$$\lim_{x\to 1}\frac{F(x)-F(1)}{x-1} = \lim_{x\to 1}\frac{\sqrt{9-x^2}-\sqrt{8}}{x-1}$$

$$= \lim_{x\to 1}\frac{(9-x^2)^{1/2}-8^{1/2}}{(9-x^2)-8} \times \frac{1-x^2}{x-1}$$

$$= \lim_{x\to 1}\frac{(9-x^2)^{1/2}-8^{1/2}}{(9-x^2)-8} \times \frac{-(x^2-1^2)}{x-1} = \frac{1}{2}.8^{-1/2} \times -2.1^1 = \frac{-1}{2\sqrt{2}}$$

Alternate method:

$$\lim_{x\to 1}\frac{F(x)-F(1)}{x-1} = F'(1)$$

$$\text{Now, } F'(x) = \frac{-2x}{2\sqrt{9-x^2}}$$

$$\Rightarrow F'(1) = -\frac{1}{2\sqrt{2}}$$

Hence, the correct answer is option (c).

Solution 80

$$\frac{d^2x}{dy^2}$$

$$= \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right) \qquad \left(\text{using quotient rule}\right)$$

$$= \frac{-\frac{d}{dy}\left(\frac{dy}{dx}\right)}{\left(\frac{dy}{dx}\right)^2} = \frac{-\frac{dx}{dy}.\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\left(\frac{dy}{dx}\right)^2} = -\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3} = -\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^{-3}$$

Hence, the correct answer is option (c).

Solution 81

For any rational value of x, f(x) = x and g(x) = 0 so (f − g)(x) = x − 0 = x
For any irrational value of x, f(x) = 0 and g(x) = x so (f − g)(x) = 0 − x = −x
Thus, the function is one-one and onto.

Hence, the correct answer is option (d).

Solution 82

$f\left(x\right) = 3\sin x - 4\sin^3 x = \sin 3x$

$\text{since sinx is increasing on } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$\text{So, } \sin 3\text{x is increasing on } \frac{-\pi}{6} \leq 3\text{x} \leq \frac{\pi}{6}$

$\text{Difference } = \frac{\pi}{6} - \left(\frac{-\pi}{6}\right) = \frac{2\pi}{6} = \frac{\pi}{3}$

Hence, the correct answer is option (a).

Solution 83

$xdy = y(dx + ydy)$
$xdy = ydx + y^2 dy$

$x\frac{dy}{dx} = y + y^2\frac{dy}{dx}$

$-y^2\frac{dy}{dx} = y - x\frac{dy}{dx}$

$-\frac{dy}{dx} = \frac{y - x\frac{dy}{dx}}{y^2}$

$-\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{x}{y}\right)$

$\text{Integrating both sides,}$
$-y + c = \frac{x}{y}$

$\text{Given } y(1) = 1$
$\therefore -1 + c = 1 \Rightarrow c = 2$
$\text{So, the equation is } -y + 2 = \frac{x}{y}$

$\text{when } x = -3, \ -y + 2 = \frac{-3}{y}$

$\Rightarrow y^2 - 2y - 3 = 0$

$\Rightarrow (y+1)\left(y-3\right) = 0$

$\therefore y = -1, \ 3$

Hence, the correct answer is option (a).

Solution 84

$-1 \leq \sin x \leq 1$
That means the maximum value of $\sin^2 x$ is 1
So, maximum value of $f(x) = 4 \times 1 + 1 = 5$
Hence, the correct answer is option (a).

Solution 85

$f\left(x\right) = \int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$

Since the function is not periodic so the 1st statement is incorrect.

$\sin^2(\pi + x) = \left(-\sin x\right)^2 = \sin^2 x$

so, the 2nd statement is correct.
Hence, the correct answer is option (d).

Solution 86

$y=x\left(\frac{dy}{dx}\right)^2+\left(\frac{dx}{dy}\right)^2$

$y=x\left(\frac{dy}{dx}\right)^2+\frac{1}{\left(\frac{dy}{dx}\right)^2}$

$y\left(\frac{dy}{dx}\right)^2=x\left(\frac{dy}{dx}\right)^4+1$

Highest order derivative is $\frac{dy}{dx}$
So, order is 1 and degree is 4
Hence, the correct answer is option (d).

Solution 87

$y^2-2ay+x^2=a^2$
$\text{Differentiating w.r.t. x.}$
$2y\frac{dy}{dx}-2a\frac{dy}{dx}+2x=0$
$\text{Solving for } a,$

$a=\frac{y\frac{dy}{dx}+x}{\frac{dy}{dx}}=\frac{py+x}{p} \qquad \left(p=\frac{dy}{dx}\right)$

$\text{Substituting a in the original equation,}$

$y^2-2y\left(\frac{py+x}{p}\right)+x^2=\left(\frac{py+x}{p}\right)^2$

$y^2-\left(\frac{2py^2+2xy}{p}\right)+x^2=\frac{p^2y^2+x^2+2pxy}{p^2}$

$p^2y^2-2p^2y^2-2pxy+p^2x^2=p^2y^2+x^2+2pxy$

$p^2x^2-2pxy-2p^2y^2=x^2+2pxy$

$p^2\left(x^2-2y^2\right)-4pxy-x^2=0$
Hence, the correct answer is option (a).

Solution 88

$ydx-\left(x+2y^2\right)dy=0$

$\frac{dx}{dy}=\frac{x}{y}+2y$

$\frac{dx}{dy}-\frac{x}{y}=2y$

$I.F.=e^{\int\frac{-1}{y}dy}=e^{\ln\left(\frac{1}{y}\right)}=\frac{1}{y}$

$\therefore \frac{1}{y}\frac{dx}{dy}-\frac{x}{y^2}=2$

$\frac{d}{dy}\left(x.\frac{1}{y}\right)=2$

$\text{Integrating both sides with respect to y,}$

$\frac{x}{y}=2y+c$

$x=2y^2+cy$

Hence, the correct answer is option (c).

Solution 89

$f(x+y) = f(x).\, f(y)$

Taking $x = 1\ and\ y = 0,$

$f(1+0) = f(1).\, f(0)$

$f(1) = f(1).\, f(0)$

$f(0) = 1.$

Now,

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \frac{f(x) f(h) - f(x)}{h}$

$= f(x) \lim\limits_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$= f(x)\, f'(0)$

$\therefore f'(5) = f(5)\, f'(0)$

Hence, the correct answer is option (a).

Solution 90

$\text{Given}: \ f(x) = f(a-x)\ and\ g(x) + g(a-x) = 2$

$f(x) = f(a-x)$

$f(x).\, g(x) = f(a-x).\, g(x)$

$\Rightarrow f(x).\, g(x) = f(a-x).\, \{2 - g(a-x)\}$

$\Rightarrow f(x).\, g(x) = 2f(a-x) - f(a-x).\, g(a-x)$

$\Rightarrow f(x).\, g(x) + f(a-x).\, g(a-x) = 2f(a-x)$

$\Rightarrow f(x).\, g(x) + f(a-x).\, g(a-x) = 2f(x) \quad \ldots \text{(i)}$

$\text{let } I = \int_0^a f(x).\, g(x) dx \qquad \ldots \text{(ii)}$

$\text{then by property of definte integral}$

$I = \int_0^a f(a-x).\, g(a-x) dx \quad \ldots \text{(iii)}$

$\text{Adding (ii) and (iii)},$

$2I = \int_0^a [f(x).\, g(x) + f(a-x).\, g(a-x)] dx$

$2I = 2\int_0^a f(x) dx \qquad [\text{from (i)}]$

$I = \int_0^a f(x) dx$

Hence, the correct answer is option (b).

Solution 91

$\ln\left(\frac{dy}{dx}\right) - a = 0$

$\Rightarrow \ln\left(\frac{dy}{dx}\right) = a$

$\Rightarrow \frac{d}{dx}\left(y\right) = e^a$

integrating both sides with respect to x,

$\Rightarrow y = xe^a + c$

Hence, the correct answer is option (a).

Solution 92

$f\left(-2^-\right) = 2\left(-2\right) + 1 = -3,\ f\left(-2^+\right) = -2 - 1 = -3$ and $f\left(-1\right) = -2 - 1 = -3$

$\therefore f\left(x\right)$ is continuous at $x = -2$

$f\left(0^-\right) = -1$ and $f\left(0^+\right) = 2$

$\therefore\ f\left(x\right)$ is discontinuous at $x = 0$.

Thus, $f\left(x\right)$ is discontinuous at $x = 0$ and continuous at every other point.

Hence, the correct answer is option (c).

Solution 93

By the property of limits,

$\lim\limits_{x\to a} \left\{f\left(x\right).g\left(x\right)\right\} = \lim\limits_{x\to a} f\left(x\right).\ \lim\limits_{x\to a} g\left(x\right)$

So, If $\lim\limits_{x\to a} f\left(x\right)$ and $\lim\limits_{x\to a} g\left(x\right)$ exist then $\lim\limits_{x\to a} f\left(x\right)g\left(x\right)$ also exists.

But the converse of the above property is not necessarily true always.

So, if $\lim\limits_{x\to a} f\left(x\right)g\left(x\right)$ exists then $\lim\limits_{x\to a} f\left(x\right)$ and $\lim\limits_{x\to a} g\left(x\right)$ may or may not exist.

Hence, the correct answer is option (a).

Solution 94

1st choice,

$f\left(-x\right) = (-x)^2 - 1 = x^2 - 1 = f\left(x\right)$, so it's even

2nd choice,

$f\left(-x\right) = \left(-x\right) - \frac{3}{x} = -\left(x + \frac{3}{x}\right) = -f\left(x\right)$, so it's odd

3rd choice,

$f\left(-x\right) = |-x| = |x| = f\left(x\right)$, so it's even

4th choice,

$f\left(-x\right) = \left(-x\right)^2\left(-x - 3\right) = -x^2\left(x + 3\right)$, neither equal to $f\left(x\right)$ nor equal to $-f\left(x\right)$

Hence, the correct answer is option (d).

Solution 95

let $y = \log_{10}\left(5x^2 + 3\right) = \log_e\left(5x^2 + 3\right).\log_{10} e$

$\frac{dy}{dx} = \frac{10x}{5x^2+3}.\log_{10} e$

Hence, the correct answer is option (c).

Solution 96

Given: $f\left(a\right)\ =\frac{a-1}{a+1}$

1st statement: $f\left(2a\right)\ =\frac{2a-1}{2a+1}\neq f\left(a\right)\ +1$

2nd statement: $f\left(\frac{1}{a}\right)=\frac{\frac{1}{a}-1}{\frac{1}{a}+1}=\frac{1-a}{1+a}=-\frac{a-1}{a+1}=-f\left(a\right)$

Hence, the correct answer is option (b).

Solution 97

The largest triangle that can be inscribed in a circle is an equilateral triangle.
Since the angle subtended by an arc at the centre is twice the angle subtended at the alternate segment.
So, ∠BOC=2∠BAC=60°=120°
Area of triangle BOC=$\frac{1}{2}a^2\sin 120^{\circ}\ =\ \frac{1}{2}a^2\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{4}a^2$
$\therefore ar\left(ABC\right)=3\times ar\left(BOC\right)=\frac{3\sqrt{3}}{4}a^2$
Hence, the correct answer is option (c).

Solution 98

$f\left(x\right)=x+\frac{1}{x}$

$f'\left(x\right)=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}=\frac{(x-1)(x+1)}{x^2}$

In the interval (0, 1), *f* '(*x*) is negative that means the *f*(*x*) decreases in the given interval.
Hence, the correct answer is option (c).

Solution 99

$f\left(x\right)=x^n,\ n\neq 0$

$f'\left(x\right)=nx^{n-1}$

Here, $n-1\in R$
So, *n* can be any real number except zero.
Hence, the correct answer is option (d).

Solution 100

$\int_{e^{-1}}^{e^2}\left|\frac{\ln x}{x}\right|dx=\int_{e^{-1}}^{e^0}-\frac{\ln x}{x}dx+\int_{e^0}^{e^2}\frac{\ln x}{x}dx$

$\text{let }\ \ln x=t\ \Rightarrow\frac{1}{x}dx=dt$

$\text{when }\ x=e^{-1},\ t=-1;\ \text{ when }\ x=e^0,\ t=0;\ \text{ when }\ x=e^2,\ t=2$

$\therefore\int_{-1}^{0}-t\,dt\ +\ \int_0^2 t\,dt$

$=\ -\left[\frac{t^2}{2}\right]_{-1}^{0}\ +\ \left[\frac{t^2}{2}\right]_0^2$

$=\ \frac{1}{2}+2$

$=\frac{5}{2}$

Hence, the correct answer is option (b).

Solution 101

New variance = $k^2 \times$ old variance (k is a constant which is multiplied by each observation)
⇒New variance = $3^2 \times 5$ = 45.
Hence, the correct answer is option (d).

Solution 102

Sum of observations = number of observations x mean = 100 x 20 = 2000
Sum of incorrect observations = 21+21+18+20=80
After omitting the incorrect observations,
New sum of observations = 2000 - 80 = 1920
Number of observations = 100 - 4 = 96
New mean=$\frac{\text{sum of observations}}{\text{number of observations}} = \frac{1920}{96} = 20$
Hence, the correct answer is option (b).

Solution 103

Total number of ways to make a committee of 2 persons out of 4 = ${}^4C_2$ = 6
Number of ways to choose only women = ${}^2C_0 \times {}^2C_2$ = 1
Thus, the probability of making a committee of only women = $\frac{1}{6}$
Hence, the correct answer is option (a).

Solution 104

Probability that no one can solve the question =
$P\left(\overline{A}\right).P\left(\overline{B}\right).P\left(\overline{C}\right) = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$

Probability that the question will be solved = $1 - \frac{1}{4} = \frac{3}{4}$
Hence, the correct answer is option (c).

Solution 105

Let the number of boys to be *x* and number of girls to be *y*
$\therefore x + y = 150 \ldots (i)$
Weight of all the students = Weight of all the boys + Weight of all the girls
$\Rightarrow 60 \times 150 = 70x + 55y$
$\Rightarrow 1800 = 14x + 11y \ldots (ii)$

Solving (i) and (ii), we have *x*=50 and *y*=100
So, the number of boys = 50
Hence, the correct answer is option (a).

Solution 106

Since $A \subseteq B$, *so* $A \cap B = A$
$P\left({}^{A}/_{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{2}{5}$
$P\left({}^{B}/_{A}\right) = \frac{P(A\cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1$

Hence, the correct answer is option (b).

Solution 107

For the point to be closer to the center than to the boundary, it must lie in the circular region having radius half of the radius of the original circle.

Probability $= \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$

Hence, the correct answer is option (b).

Solution 108

Correlation coefficient of x and y = $\sqrt{b_{xy} \times b_{yx}}$

$4y - 15x + 410 = 0 \Rightarrow y = \frac{15}{4}x - \frac{410}{4}$

$\therefore b_{yx} = \frac{15}{4}$

$30x - 2y - 825 = 0 \Rightarrow x = \frac{2}{30}y + \frac{825}{30}$

$\therefore b_{xy} = \frac{2}{30}$

Correlation coeeificient = $\sqrt{\frac{15}{4} \times \frac{2}{30}} = \frac{1}{2}$

Hence, the correct answer is option (b).

Solution 109

Central angle for 40% candidates of second class = $\frac{40}{100} \times 360^{\circ} = 144^{\circ}$

Hence, the correct answer is option (c).

Solution 110

Range = Highest observation − lowest observation

So, range doesn't give any idea about the dispersion of the data as it depends on the two extreme values(highest and lowest). Thus, both the statements are correct and 2nd is the correct explanation of the 1st.

Hence, the correct answer is option (a).

Solution 111

Total number of aces = 4

Total number of cards = 52

So, the probability of getting an ace = $\frac{\text{total number of aces}}{\text{total number of cards}} = \frac{4}{52} = \frac{1}{13}$

Hence, the correct answer is option (a).

Solution 112

$\frac{4}{5} \times$S.D. = M.D.

4 $\times$ S.D. = 5 $\times$M.D.

Hence, the correct answer is option (c).

Solution 113

Tabular and Graphical methods are used to represent a data.

Hence, the correct answer is option (b).

Solution 114

Abscissa of the point of intersection of 'more than ogive' and 'less than ogive' gives median of the data.

Hence, the correct answer is option (a).

Solution 115

Mutually exclusive events don't occur together so occurance of one prevents the occurance of other.

So, 1st statement is correct.

For two mutually exclusive events, $P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$

So, 2nd statement is correct.

Hence, the correct answer is option (c).

Solution 116

$$\text{Correlation coefficient} = \sqrt{(\text{regression coefficient of x on y}) \times (\text{regression coefficient of y on x})}$$

$$= \sqrt{\left(\frac{-1}{2}\right) \times \left(\frac{-1}{8}\right)}$$

$$= \sqrt{\frac{1}{16}} = \frac{1}{4}$$

But both the regression coefficients are negative so, answer should be negative.
Therefore, correlation coefficient = $\frac{-1}{4}$
Hence, the correct answer is option (a).

Solution 117

Let the observations are x_1, x_2, x_3, x_4 and x_5.
33 is median or middle observation so observations are x_1, x_2, 33, x_4 and x_5
Entries 40 (incorrect) and 35 (correct) are both more than 33 so both are on the right side of 33
Thus, median remains same.
Sum of observations earlier = 32 × 5 = 160
Sum of observations now = 160 - 40 + 35 = 155
Since the sum decreases so the mean also decreases.

Thus, the median will remain same but mean will decrease.

Hence, the correct answer is option (b).

Solution 118

Total number of probable outcomes = 36
Let A be the event of getting the sum 8 or 9.
Favourable outcomes = {(2,6), (3, 5), (4, 4), (5, 3), (6, 2), (3, 6), (4, 5), (5, 4), (6, 3)}
n(A) = 9
Therefore, the probability of getting the sum either 8 or 9 = P(A) = $\frac{9}{36} = \frac{1}{4}$
Probability of getting the sum neither 8 nor 9 = $\mathrm{P}\left(\overline{\mathrm{A}}\right) = 1 - \frac{1}{4} = \frac{3}{4}$
Hence, the correct answer is option C.

Solution 119

A and B are mutually exclusive so $P(A \cap B) = 0$

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

$$P\left(\overline{A} \cap \overline{B}\right) = P\left(\overline{A \cup B}\right) = 1 - P\left(A \cup B\right) = 1 - \frac{7}{12} = \frac{5}{12}.$$

Hence, the correct answer is option (d).

Solution 120

Given: mean ($\bar{x}$) = 12$\Rightarrow np = 12$ ...(i)
S.D. = 2 $\Rightarrow \sqrt{npq} = 2 \ \Rightarrow npq = 4$ ... (ii)
From (i) and (ii), q = $\frac{1}{3}$
Therefore, p = 1 − q = $1 - \frac{1}{3} = \frac{2}{3}$
From (ii), $n \times \frac{1}{3} \times \frac{2}{3} = 4 \ \Rightarrow n = 18$
Thus, number of trials is 18.
Hence, the correct answer is option (c).