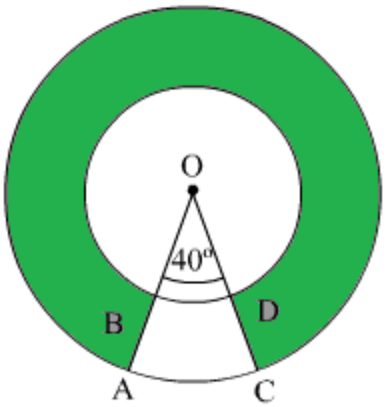
Case Based Questions

Solution 1

We have



(i) Area of region ABDC = Area of sector AOC − Area of sector BOD

(ii) The formula for finding the area of the sector if the degree measure of the angle at the centre of the circle (of radius 'r') is 'θ'

is $\frac{\theta}{360^\circ} \times \pi r^2$

(iii) The area of the sector AOC is given by

$\frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2 \quad \left(\text{Here } \theta = 40^\circ,\ r = 14 \text{ cm}\right)$$

$$= \frac{1}{9} \times 22 \times 14 \times 2 \text{ cm}^2$$

$$= \frac{22}{9} \times 28 \text{ cm}^2$$

$$= 68.44 \text{ cm}^2$$

(iv) Area of the circular ring

$= \left(\frac{22}{7} \times 14 \times 14 - \frac{22}{7} \times 7 \times 7\right) \mathrm{cm}^2$

$= \left(22 \times 14 \times 2 - 22 \times 7\right) \mathrm{cm}^2$

$= 22\left(28 - 7\right) \mathrm{cm}^2$

$= 22 \times 21 \mathrm{~cm}^2$

$= 462 \mathrm{~cm}^2$

(v) Length of an arc of a sector of angle 40°

$= \frac{40^{\circ}}{360^{\circ}} \times 2\pi r$

$= \frac{40^{\circ}}{360^{\circ}} \times 2 \times \frac{22}{7} \times 14 \quad \left(r = 14 \mathrm{~cm}\right)$

$\approx 9.78 \mathrm{~cm}$

Solution 2

(a) The coordinates of the post office are (−3, −4).

(b) Let the coordinates of the food market be (−6, *y*).

$\therefore \sqrt{(-6-0)^2 + (y-0)^2} = 2\sqrt{10}$

$\Rightarrow 36 + y^2 = 40$

$\Rightarrow y^2 = 40 - 36 = 4$

$\Rightarrow y = \pm 2$

Thus, the ordinate of the food market is 2 or −2.

(c) The coordinates of the post office and school are (−3, −4) and (6, −2) respectively. Let (*x*, 0) be the point on the *x*-axis which is equidistant from the post office and school.

$\therefore \sqrt{[x-(-3)]^2 + [0-(-4)]^2} = \sqrt{(x-6)^2 + [0-(-2)]^2}$

$\Rightarrow (x+3)^2 + 16 = (x-6)^2 + 4$

$\Rightarrow x^2 + 6x + 25 = x^2 - 12x + 40$

$$\Rightarrow 18x = 15$$
$$\Rightarrow x = \frac{5}{6}$$

Thus, the required coordinates are $\left(\frac{5}{6}, 0\right)$.

(d) The coordinates of the temple and park are (−4, 2) and (8, 6) respectively. Let P(x, y) be the coordinates of the point which divides the line segment joining the temple and park in the ratio 3 : 1 internally.

$$\therefore (x, y) = \left(\frac{3\times8+1\times(-4)}{3+1}, \frac{3\times6+1\times2}{3+1}\right)$$
$$\Rightarrow (x, y) = (5, 5)$$

Thus, the required coordinates are (5, 5).

(e) The coordinates of fire station, food corner and school are (5, 4), (3, 1) and (6, −2). Let the coordinates of point P be (x, y).
We know that the diagonals of parallelogram bisect each other. So, the coordinates of mid-point of line segment joining the point P and food corner are same as the coordinates of the mid-point joining the line segment fire station and school.

$$\therefore \left(\frac{3+x}{2}, \frac{1+y}{2}\right) = \left(\frac{5+6}{2}, \frac{4+(-2)}{2}\right)$$
$$\Rightarrow \frac{3+x}{2} = \frac{11}{2} \text{ and } \frac{1+y}{2} = 1$$
$$\Rightarrow x = 8 \text{ and } y = 1$$

Thus, the coordinates of point P are (8, 1).

Solution 3

PQRS is a trapezium with PQ || RS. PR and QS intersect at O.

(a) In ΔPOQ and ΔROS, we have
∠OPQ = ∠ORS (Alternate angles)
∠OQP = ∠OSR (Alternate angles)
∴ΔPOQ~ΔROS (AA similarity criterion)

(b) In ΔPOQ, XY || PQ.
Using basic proportionality theorem, we have

$$\frac{\text{OX}}{\text{XP}} = \frac{\text{OY}}{\text{YQ}}$$
$$\Rightarrow \frac{2.4 \text{ cm}}{3.6 \text{ cm}} = \frac{3 \text{ cm}}{\text{YQ}}$$
$$\Rightarrow \text{YQ} = \frac{3\times3.6}{2.4} = 4.5 \text{ cm}$$

(c) We know that if two triangles are similar, then their corresponding sides are proportional and they are proportional to the corresponding perimeters.

$$\Rightarrow \frac{\text{Perimeter of } \Delta\text{POQ}}{\text{Perimeter of } \Delta\text{ROS}} = \frac{\text{PQ}}{\text{RS}} = \frac{\text{OQ}}{\text{OS}} = \frac{\text{OP}}{\text{OR}}$$

$$\Rightarrow \frac{\text{Perimeter of } \Delta\text{POQ}}{\text{Perimeter of } \Delta\text{ROS}} = \frac{8 \text{ cm}}{16 \text{ cm}}$$

$$\Rightarrow \frac{\text{Perimeter of } \Delta\text{POQ}}{\text{Perimeter of } \Delta\text{ROS}} = \frac{1}{2}$$

⇒ Perimeter of ΔPOQ : Perimeter of ΔROS = 1 : 2

(d) ΔPOQ~ΔROS (AA similarity criterion)

$\frac{\text{PQ}}{\text{RS}} = \frac{\text{OQ}}{\text{OS}} = \frac{\text{OP}}{\text{OR}}$ (If two triangles are similar, then their corresponding sides are proportional)

$$\Rightarrow \frac{x+1}{2x+4} = \frac{4}{4x-2}$$

$$\Rightarrow (x+1)(2x-1) = 4(x+2)$$

$$\Rightarrow 2x^2 - 3x - 9 = 0$$

$$\Rightarrow (2x+3)(x-3) = 0$$

$$\Rightarrow x = 3, -\frac{3}{2}$$

Since x cannot be negative, so x = 3.

(e) We know that the internal bisector of an angle of a triangle divide the opposite side internally in the ratio of sides containing the angle.

In ΔROS, OT is the bisector of ∠ROS.

$$\therefore \frac{\text{OS}}{\text{OR}} = \frac{\text{ST}}{\text{TR}}$$

$\Rightarrow \frac{\text{OS}}{7.2 \text{ cm}} = \frac{10 \text{ cm}}{6 \text{ cm}}$ (ST = 16 − 6 = 10 cm)

$\Rightarrow \text{OS} = \frac{10\times7.2}{6}$ = 12 cm

Solution 4

(a) The shape formed by the garland is parabola.

(b)

$$x^2 + 2x - 15$$

$$= x^2 + 5x - 3x - 15$$

$$= x(x+5) - 3(x+5)$$

$$= (x+5)(x-3)$$

For the zeroes of the curve, put y = 0.

$(x + 5)(x - 3) = 0$

$\Rightarrow x = 3, -5$

(c) The curve represented by the garland is a parabola. The parabola represents a quadratic polynomial which has at most two zeroes.

(d) The polynomial represented by the curve is
x^2 − (Sum of the zeroes)x + Product of the zeroes
$= x^2 - 7x - 18$

(e) One root of the quadratic polynomial = −1
∴ Other root of the quadratic polynomial = $-\frac{2}{3}$
Sum of the roots of the quadratic polynomial = $-1 + \left(-\frac{2}{3}\right) = -\frac{5}{3}$
So, the required quadratic polynomial is
$k[x^2$ − (Sum of the zeroes)x + Product of the zeroes]
$= k\left(x^2 + \frac{5}{3}x + \frac{2}{3}\right)$
$= \frac{k}{3}\left(3x^2 + 5x + 2\right)$

$= 3x^2 + 5x + 2$, for $k = 3$

Solution 5

(a) The class 15–20 has the maximum frequency 9. So, the modal class is 15–20.

(b) Here, $l = 15$, $h = 5$, $f = 9$, $f_1 = 6$ and $f_2 = 7$
$\therefore \text{Mode} = l + \frac{f-f_1}{2f-f_1-f_2} \times h = 15 + \frac{3}{5} \times 5 = 18$

(c)

Number of Pair of Shoes Sold	10–15	15–20	20–25	25–30	30–35
Number of Days	6	9	7	3	5
Cumulative Frequency	6	15	22	25	30

$N = 30 \Rightarrow \frac{N}{2} = 15$
The cumulative frequency just greater than $\frac{N}{2}$ is 22 and the corresponding class is 20–25. So, 20–25 is the median class.
The upper limit of the median class is 25.

(d)

Number of Pair of Shoes Sold	10–15	15–20	20–25	25–30	30–35

Number of Days	6	9	7	3	5
Cumulative Frequency	6	15	22	25	30

N = 30 $\Rightarrow \frac{N}{2} = 15$

The cumulative frequency just greater than $\frac{N}{2}$ is 22 and the corresponding class is 20–25. So, 20–25 is the median class.

Here, l = 20, f = 7, F = 15 and h = 5

$\therefore \text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h = 20 + \frac{15-15}{7} \times 5 = 20$

(e) Mode = 3Median − 2Mean

⇒ 18 = 3 × 20 − 2Mean

⇒ 2Mean = 60 − 18 = 42

⇒ Mean = 21