JEE Main 1 Feb 2023(First Shift)

Total Time: 180

Total Marks: 300.0

Solution 1

$$l_{\text{final}} = \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{6^4} = \frac{1}{2^n}$$

$$n = 6$$

Solution 2

$$a = \frac{30 - 50\mu}{5}$$

$$\because s = ut + \frac{1}{2}at^2$$

$$50 = \frac{1}{2}\left(\frac{30-50\mu}{5}\right) \times 100$$

$$5 = 30 – 50\mu$$

$$\mu = \frac{25}{15} = 0.5$$

Solution 3

Statement-I is correct as g′ = g – $\omega^2 Rcos^2\phi$
Statement-II is clearly incorrect

Solution 4

(Theoretical)
(A) Intrinsic semiconductor → II
(B) n-type semiconductor → III
(C) p-type semiconductor → I
(D) Metals → IV

Solution 5

$$[a] = \left[\mathrm{ML}^5\,\mathrm{T}^{-2}\right]$$

$$[b] = \left[L^3\right]$$

$$\left[\frac{b^2}{a}\right] = \left[\frac{L^6}{\mathrm{ML}^5\,\mathrm{T}^2}\right] = \left[\mathrm{M}^{-1}\,\mathrm{LT}^{-2}\right]$$

$$= [\text{Compressibility}]$$

Solution 6

$$Bp = \frac{\mu_0 i}{4\pi r} + \frac{1}{2}\left(\frac{\mu_0 i}{2r}\right)$$

$$\frac{\mu_0 i}{4r}\left[\frac{1}{\pi} + 1\right]$$

Solution 7

Speed of transverse wave = $\sqrt{\frac{T}{M}}$

$= \sqrt{\frac{70}{7\times10^{-3}}} = 100\ \mathrm{m/s}$

Solution 8

$$\gamma = \frac{3}{2}$$

$$\omega = \frac{nR\Delta T}{1-\gamma} = \frac{nRT_f - nRT_i}{1-\gamma}$$

$$= \frac{(PV)_f - (PV_i)}{1-\gamma} \quad ...(1)$$

$$PV^{\gamma} = \text{constant}$$

$$P_i V_i^{\gamma} = P_f (2V_i)^{\gamma} \Rightarrow P_f = \frac{P_i}{2^{\gamma}} = \frac{P_i}{2\sqrt{2}} \quad ...(2)$$

From (1) and (2)

$$\omega = \frac{\frac{P_i}{2\sqrt{2}} 2V_i - P_i V_i}{1-\gamma} = \frac{P_i V_i}{\frac{-1}{2}}\left(\frac{1}{\sqrt{2}} - 1\right)$$

$$= -nRT\left(\sqrt{2} - 2\right)$$

$$= nRT\left(2 - \sqrt{2}\right)$$

Solution 9

Average kinetic energy of a molecule of gas $= \frac{f}{2} k_B T$

f is degree of freedom.

Solution 10

AC generator works on EMZ principle (A-II) Transformer uses Mutual induction (B-IV)

Resonance occurs when both L and C are present (C–Z) and quality factor determines sharpness of resonance (D-III)

Solution 11

FM broadcast varies from 89 Hz to 108 Hz

Solution 12

$$M_E = 9M_P$$

$$R_E = 2R_P$$

$$\text{Escape velocity} = \sqrt{\frac{2mG}{R}}$$

$$\text{For earth } v_e = \sqrt{\frac{2GM_E}{R_E}}$$

$$\text{For P, } v_{\text{e}} = \sqrt{\frac{\frac{2GM_E}{9}}{\frac{R_E}{2}}} = \sqrt{\frac{2GM_E}{R_E} \times \frac{2}{9}}$$

$$= \frac{v_e\sqrt{2}}{3}$$

Solution 13

Mass defect = 2 (Mass of *p* + mass of *n*) – mass of He nucleus
Δm = 0.0305*u*
B.E = 931.5 × Δm = 931.5 × 0.0305
= 28.4 MeV

Solution 14

For same λ_1 momentum should be same,
$(P)_p = (P)_a$

$$\Rightarrow \sqrt{2k_p m_p} = \sqrt{2k_a m_a}$$

$$\Rightarrow k_P m_P = k_a m_a$$

$$\frac{k_P}{k_a} = \left(\frac{m_a}{m_p}\right) = \frac{4}{1} = 4:1$$

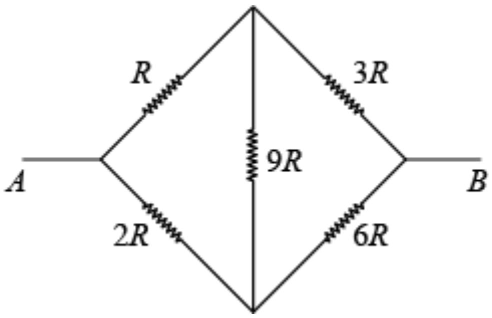
Solution 15

Initial volume = Final volume
So, $R = 5r$
Gain in surface energy = $[125 \times 4\pi r^2 \times T - 4\pi R^2 T]$
$= 4\pi T[125r^2 - R^2]$
$= 16\pi R^2 T$
$= 16\pi \times (10^{-3})^2 \times 0.45$
$= 22.6 \times 10^{-6}$ J
$= 2.26 \times 10^{-5}$ J

Solution 16

1. Klystron valve used to produce Microwave
2. Gamma ray → Radioactive decay
3. Radio wave → Rapid acceleration and deacceleration of electrons in aerials
4. X-ray → Inner shell electrons

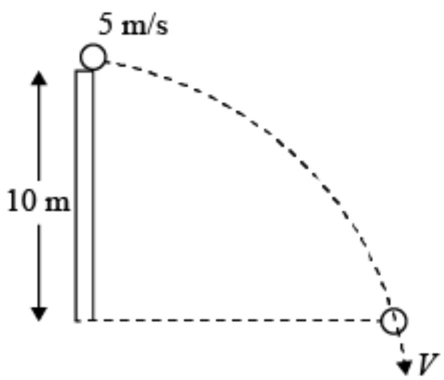
Solution 17



This is balanced Wheatstone bridge,

$$R_{eq} = \frac{4R \times 8R}{12R} = \left(\frac{8R}{3}\right)$$

Solution 18



$$v = \sqrt{u^2 + 2gh}$$
$$= \sqrt{25 + 2 \times 10 \times 10}$$
$$= \sqrt{225} = 15 \text{ m/s}$$

Solution 19

From the figure:

$$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} \qquad \text{(Leftward)}$$

$$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

$$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} \qquad \text{(Rightward)}$$

Solution 20

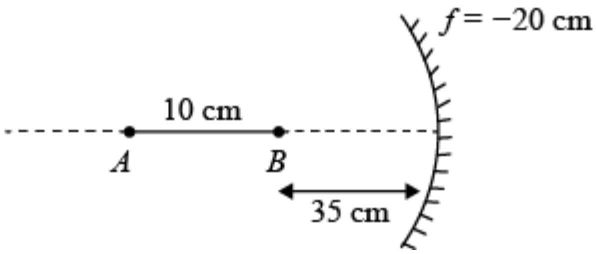
AB = BC = CD

$$\Rightarrow \text{Average speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{AD}{\frac{AB}{V_1}+\frac{AB}{V_2}+\frac{AB}{V_3}}$$

$$= \frac{3V_1V_2V_3}{V_1V_2+V_2V_3+V_1V_3}$$

Solution 21



$$\text{A: } \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}+\frac{1}{-45}=\frac{1}{-20}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{45}-\frac{1}{20}=\frac{4-9}{180}==-\frac{1}{36}$$

$$\Rightarrow v=-36 \text{ cm}$$

$$\text{B: } \frac{1}{v}+\frac{1}{-35}=\frac{1}{-20}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{35}-\frac{1}{20}=\frac{4-7}{140}$$

$$\Rightarrow v=-\frac{140}{3}$$

$$\Rightarrow \text{ length of image} = \frac{140}{3}-36=\frac{32}{3}\text{ cm}$$

$$\Rightarrow x=32$$

Solution 22

A = 3 cm
K = 1.25 U

$$\Rightarrow K + \frac{K}{1.25} = K_{\max}$$

$$\Rightarrow \frac{9}{5}K = K_{\max}$$

$$\Rightarrow \frac{9}{5}\frac{1}{2}mv^2 = \frac{1}{2}mv^2{}_{\max}$$

$$\Rightarrow \frac{9}{5}\left[\omega\sqrt{A^2 - x^2}\right]^2 = \omega^2 A^2$$

$$\Rightarrow 9\left(A^2 - x^2\right) = 5A^2$$

$$\Rightarrow x^2 = \frac{4A^2}{9}$$

$$\Rightarrow x = \frac{2A}{3}$$

$$\Rightarrow x = 2 \text{ cm}$$

Solution 23

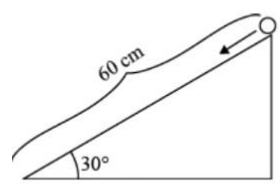
$$B = \frac{\frac{-dp}{dv}}{v}$$

$$\Rightarrow \frac{B_{\text{water}}}{B_{\text{Liquid}}} = \frac{\left(\frac{dv}{v}\right)_{\text{liquid}}}{\left(\frac{dv}{v}\right)_{\text{water}}}$$

$$= \frac{0.03}{0.01} = 3$$

$$\Rightarrow x = 1$$

Solution 24



$$\Rightarrow mg\left[\frac{30}{100}\right] = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mv^2}{2}$$

$$\Rightarrow 0.3 \times 10 = \frac{3}{4}v^2$$

$$\Rightarrow v^2 = 4$$

$$\Rightarrow v = 2 \text{ m/s}$$

Solution 25

Let the electron jumps to n^{th} orbit so

$$12.75 = 13.6\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

$$\Rightarrow n = 4$$

$$\text{So } L = \frac{nh}{2\pi} = \frac{2h}{\pi}$$

$$= \frac{2 \times 4.14 \times 10^{-15}}{\pi}$$

$$= 8.28 \times 10^{-15}$$

$$= 828 \times 10^{-17} \text{ eVs}$$

Solution 26

$$W=\overrightarrow{F}.\left(\overrightarrow{r}_2-\overrightarrow{r}_1\right)$$

$$=\left(5\hat{i}+2\hat{j}+7\hat{k}\right).\left(3\hat{i}-5\hat{j}+5\hat{k}\right)$$

$$=15–10+35$$

$$=40\text{ J}$$

Solution 27

Average rate of energy is maximum at resonance.

$\therefore X_L = X_C$

$$79.6=\frac{1}{2\pi(50)\times C}$$

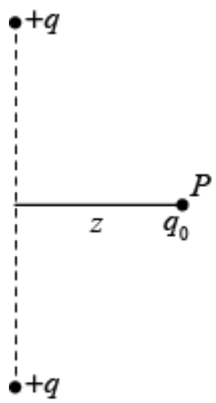
$$C=\frac{1}{79.6\times 2\pi(50)}$$

$$\approx 40\ \mu\text{F}$$

Solution 28

$$F_p=q_0E_p=q_0\frac{kqz}{\left(a^2+z^2\right)^{\frac{3}{2}}}$$

$$\text{or } F_p=\frac{kqq_0z}{\left(a^2+z^2\right)^{\frac{3}{2}}}$$



$$\text{To maximize }\frac{dF_p}{dz}=0$$

$$\text{or } kqq_0\frac{\left(a^2+z^2\right)^{\frac{3}{2}}-z\frac{3}{2}\times 2z\left(a^2+z^2\right)^{\frac{1}{2}}}{\left(a^2+z^2\right)^3}=0$$

$$\Rightarrow z=\frac{a}{\sqrt{2}}$$

Solution 29

$R=\sqrt{\frac{2mqV}{qB}}$

$R=\frac{1}{B}\sqrt{\frac{2mV}{q}}$

or $m=\frac{R^2B^2q}{2V}$

$=\frac{\left(3\times10^{-2}\right)^2\times\left(4\times10^{-3}\right)^2\times2\times10^{-6}}{2\times100}$

$=144\times10^{-18}$ kg

Solution 30

$E\propto l$

$\frac{E_1}{E_2}=\frac{l_1}{l_2}$

$\frac{1.5}{E}=\frac{60}{100}$

$E=\frac{150}{60}=\frac{5}{2}=\frac{25}{10}$

So $x=25$

Solution 31

- Chlorine can easily combine with oxygen to form oxides, which can explode
- Chemical reactivity of an element can be determined by its reaction with oxygen and Halogens

Hence, the correct answer is option (2).

Solution 32

$\mathrm{A}_{0.95}\mathrm{O}$

$\%$ of $\mathrm{A}^{2+}=\frac{85}{95}\times100\approx90\%$

$\%$ of $\mathrm{A}^{3+}=\frac{10}{95}\times100\approx10\%$

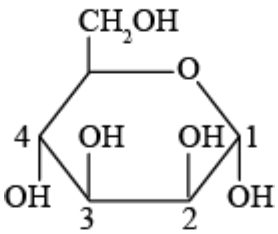
Option (A) satisfies this condition

Hence, the correct answer is option (1).

Solution 33

Hydrogen is an environment friendly fuel as its combustion produces only water vapours.
Hence, the correct answer is option (3).

Solution 34



C_2 and C_3OH are cis
C_3 and C_4 are anti to each other.

Hence, the correct answer is option (3).

Solution 35

(A) Tranquilizers are antidepressant drugs
(B) Aspirin prevents blood clotting and hence Anti blood clotting
(C) Salvarsan is an antibiotic
(D) Soframicine is antiseptic

Hence, the correct answer is option (4).

Solution 36

$$C\,(s) + \frac{1}{2}O_2\,(g) \rightarrow CO\,(g) \qquad (\Delta S > 0)$$

$$\text{Slope} = (-ve)$$

CO doesn't get decompose at high temperature.

Hence, the correct answer is option (2).

Solution 37

(A) Molisch test is for carbohydrates
(B) Biuret test is for proteins/peptide
(C) Carbylamine test is for primary amine
(D) Schiff's test is for aldehyde

Hence, the correct answer is option (3).

Solution 38

$$Fe^{3+} + \left[Fe\,(CN)_6\right]^{4-} \rightarrow Fe_4\left[Fe(CN)_6\right]_3$$

prussian blue

Hence, the correct answer is option (2).

Solution 39

MW order, Kr > Ar > Ne > He
Z (at critical point)$= \frac{3}{8}$

Hence, the correct answer is option (2).

Solution 40

CN^- is strongest field ligand among given ligands.
Hence, the correct answer is option (3).

Solution 41

A = $FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$ – double salt
B. $CuSO_4.4NH_3.H_2O = [Cu(NH_3)_4]SO_4.H_2O$ – complex salt
C. $K_2SO_4.Al_2(SO_4)_3.24H_2O$ – double salt
D. $Fe(CN)_2.4KCN$
$K_4[Fe(CN)_6]$ – complex salt

Hence, the correct answer is option (2).

Solution 42

Photochemical smog is caused by Nitrogen oxides which can be prevented by using catalytic convertors in the automobiles/industy.
Hence, the correct answer is option (3).

Solution 43

A: Cis – But-2-ene
B: Trans-But-2-ene
BP: A > B
MP: B > A
μ-order = A > B (μ of B = 0)
Addition of Br_2 is easy in B.
Thus, only statement (C) is incorrect.

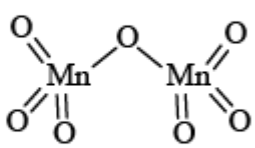
Disclaimer: *In this question, none of the options is correct.*

Solution 44

• BeO is amphoteric
• $BeCO_3 \rightleftharpoons BeO + CO_2$
To shift equilibrium in backward direction, It is kept in atmosphere of CO_2
• $BeSO_4$ is readily soluble in water
• Be shows anomalous behaviour

Hence, the correct answer is option (4).

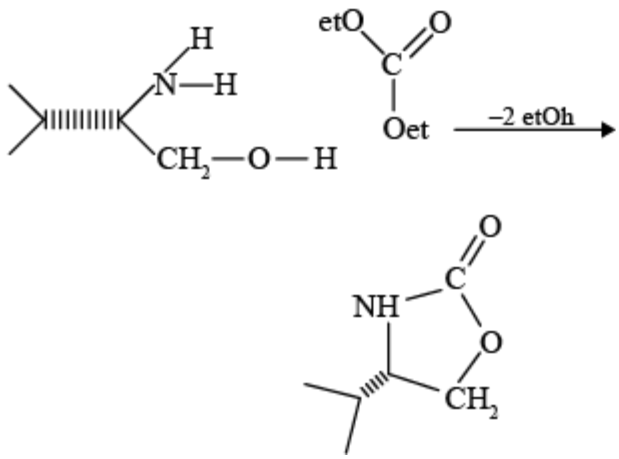
Solution 45



Mn is surrounded tetrahedrally by O-atoms.
Mn_2O_7, contains Mn-O-Mn Bridge.

Hence, the correct answer is option (1).

Solution 46



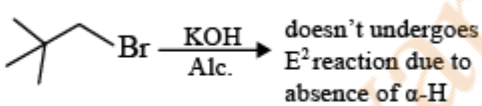
Hence, the correct answer is option (2).

Solution 47

A : Slaked lime : $Ca(OH)_2$
B : Dead burnt plaster : $CaSO_4$
C : Caustic Soda : NaOH
D : Washing Soda : $Na_2CO_3 . 10H_2O$

Hence, the correct answer is option (1).

Solution 48



Hence, the correct answer is option (1).

Solution 49

b > d > c > a
b will form Aromatic Benzene on dehydration
d will form conjugated alkene
a will not undergo dehydration easily

Hence, the correct answer is option (3).

Solution 50

Resonating structures are hypothetical and are assumed to explain properties of Real hybrid.

Hence, the correct answer is option (1).

Solution 51

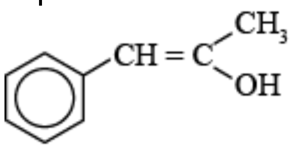
Bromic Acid $\left(\mathrm{HB}\underset{+5}{\mathrm{r}}\mathrm{O}_3\right)$

Perbromic Acid $\left(\mathrm{HB}\underset{+7}{\mathrm{r}}\mathrm{O}_4\right)$

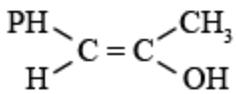
Sum of oxidation states of bromine in bromic acid and perbromic acid is **12**.

Solution 52

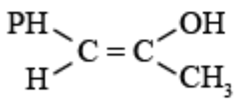
2 possibilities



(E/Z Isomers)



(E)



(Z)

Solution 53

25 × M = 20 × 1

$\mathrm{M}=\frac{20}{25}=\frac{4}{5}=0.8$

$\Delta\,\mathrm{T_f}=(\mathrm{i})\,(\mathrm{K_f})\,(\mathrm{m})$

$=(2)\,(2)\left(\frac{4}{5}\right)=\frac{16}{5}=3.2$

$\text{Nearest Integer}=3$

Solution 54

$\mathbf{A}\ \mathrm{C_0}\xrightarrow{15\ \mathrm{min}}\frac{\mathrm{C_0}}{2}$

$\mathbf{B}\ 4\mathrm{C_0}\xrightarrow{5\ \mathrm{min}}2\mathrm{C_0}\xrightarrow{5\ \mathrm{min}}\mathrm{C_0}\xrightarrow{5\ \mathrm{min}}\frac{\mathrm{C_0}}{2}$

Solution 55

$\frac{(\mathrm{i})+(\mathrm{iii})}{2}-\left(\mathrm{ii}\right)$ gives desired reaction

$\Delta\,H_r=\frac{436+78}{2}-\left(-242\right)$

$\Delta\,H_r=\frac{436+78}{2}+242=499\ \mathrm{kJ\ \ mol^{-1}}$

Solution 56

$m=\frac{1000\ \mathrm{M}}{1000\ \rho-\mathrm{Mmw}}=\frac{1000\times3}{1000-3\times(58.5)}$

$=\frac{3000}{(1000-175.5)}=3.638$

$=363.8\times10^{-2}$

$\text{Nearest integer}=364$

Solution 57

- Characteristics of electrons emitted doesn't depend upon material of electrode, nature of gas present.
- Cathode rays start from cathode
- $\lambda=\frac{\mathrm{h}}{\mathrm{mv}}=\frac{6\times10^{-34}}{(9\times10^{-31})(10^{3})}=0.666\times10^{-6}\ \mathrm{m}$

$\lambda=666.67\ \mathrm{nm}$

$\text{A \& C are correct}$

Solution 58

$$\begin{array}{cccc} \mathrm{X_{(g)}} & \leftrightharpoons \mathrm{Y_{(g)}} & + \mathrm{Z_{(g)}} & \qquad \mathrm{K_{p1}}=3 \\ 1 & - & - & \\ \downarrow & \downarrow & \downarrow & \\ (1-\mathrm{a}) & \mathrm{a} & \mathrm{a} & \end{array}$$

$\text{mole fraction}\ \left(\frac{1-\mathrm{a}}{1+\mathrm{a}}\right)\ \left(\frac{\mathrm{a}}{1+\mathrm{a}}\right)\left(\frac{\mathrm{a}}{1+\mathrm{a}}\right)$

$\mathrm{K_{p_1}}=3=\frac{\alpha}{(1+\alpha)}\frac{\alpha}{(1+\alpha)}\frac{(1+\alpha)}{(1-\alpha)}(p_1)^1$

$3=\frac{\alpha^2}{1-\alpha^2}\cdot p_1$

$$\begin{array}{ccc} \mathrm{A_{(g)}} & \leftrightharpoons 2\mathrm{B_{(g)}} & \qquad \mathrm{K_{p_2}}=1 \\ 1 & - & \\ \downarrow & \downarrow & \\ (1-\mathrm{a}) & 2\mathrm{a} & \end{array}$$

$\text{mole fraction}\ \left(\frac{1-\mathrm{a}}{1+\mathrm{a}}\right)\ \left(\frac{2\mathrm{a}}{1+\mathrm{a}}\right)$

$1=\frac{4\alpha^2}{(1+\alpha)^2}\frac{(1+\alpha)}{(1-\alpha)}\cdot p_2$

$1=\frac{4\alpha^2}{1-\alpha^2}\cdot p_2$

$\frac{Kp_1}{Kp_2}=\frac{3}{1}=\frac{p_1}{4p_2}$

$\Rightarrow\frac{p_1}{p_2}=12$

Solution 59

$$5e^-+8\mathrm{H}^++\mathrm{MnO}_4^-\to\mathrm{Mn}^{2+}+4\mathrm{H}_2\mathrm{O}$$

$$10^{-1}\qquad 10^{-3}$$

$1.282=1.54-\frac{.059}{5}\log\frac{10^{-3}}{10^{-1}(\mathrm{H}^+)^8}$

$-.258=\frac{-.059}{5}(-2+8\,\mathrm{pH})$

$21.8644=(-2+8\,\mathrm{pH})$

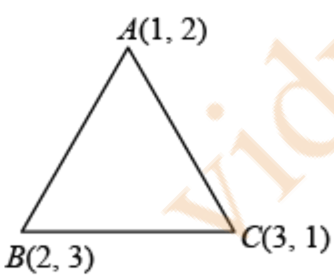
$23.8644=8\,\mathrm{pH}$

$\mathrm{pH}=2.98\approx3$

Solution 60

Compound I – achiral
Compound II – chiral
Compound III – achiral
Compound IV – chiral
Compound V – achiral

Solution 61



Altitude of BC is $y-2=\frac{1}{2}(x-1)\Rightarrow x-2y+3=0$
Altitude of AB is $y-1=(-1)(x-3)\Rightarrow x+y=4$
$\therefore$ Orthocentre $\left(\frac{5}{3},\frac{7}{3}\right)$
$\therefore\alpha+4\beta=11$ and $4\alpha+\beta=9$
Equation is $x^2-20x+99=0$

Hence, the correct answer is option (1).

Solution 62

Let observations 1, 3, 5, a, b

$\Rightarrow \frac{9+a+b}{5} = 5 \,\&\, \frac{a^2+b^2+35}{5} - 25 = 8$

$\Rightarrow a + b = 16 \,\&\, a^2 + b^2 = 130$

$\therefore$ a & b are 7 & 9

$\therefore a^3 + b^3 = 7^3 + 9^3 = 1072$

Hence, the correct answer is option (4).

Solution 63

$(x - 2)^2 + y^2 = 4(x - 3)^2 + 4y^2$

$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$

$\therefore\ C \equiv \left(\frac{10}{3}, 0\right) \,\&\, r = \sqrt{\left(\frac{10}{3}\right)^2 - \frac{32}{3}} = \frac{2}{3}$

$\therefore 3(\alpha + \beta + \gamma) = 3\left(\frac{12}{3}\right) = 12$

Solution 64

$\frac{dy}{dx} + y\ \tan\ x = x\ \sec\ x$

$\therefore$ I.F $= e^{\int \tan x dx} = \sec x$

$\Rightarrow y \sec x = \int x \sec^2 x\, dx$

$\Rightarrow y \sec x = x \tan x - \ln |\sec x| + c \cos x$

$\downarrow y(0) = 1$

$\Rightarrow 1 = e$

$\therefore y = x \sin x - \cos x \ln |\sec x| + \cos x$

$\therefore\ y\left(\frac{\pi}{6}\right) = \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ln\left(\frac{2}{\sqrt{3}e}\right)$

Hence, the correct answer is option (4).

Solution 65

$S = \sum_{r=1}^{10} \frac{r}{1+r^2+r^4} = \frac{1}{2}\sum\left(\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right)$

$T_1 = \frac{1}{2}\left(\frac{1}{1^2-1+1} - \frac{1}{1^2+1+1}\right)$

$T_2 = \frac{1}{2}\left(\frac{1}{2^2-2+1} - \frac{1}{2^2+2+1}\right)$

$T_3 = \frac{1}{2}\left(\frac{1}{3^2-3+1} - \frac{1}{3^2+3+1}\right)$

$\vdots$

$\vdots$

$T_{10}=\frac{1}{2}\left(\frac{1}{10^2-10+1}-\frac{1}{10^2+10+1}\right)$
$S=\frac{1}{2}\left(1-\frac{1}{111}\right)=\frac{55}{111}$

Hence, the correct answer is option (3).

Solution 66

$\frac{x^2-y^2}{2-(-3)}=\frac{xy}{\frac{1}{2}}$
OR $x^2 - y^2 = 10xy$

Hence, the correct answer is option (4).

Solution 67

Disclaimer: None of the options is correct.
$\cos^{-1}\left(2x\right)-2\cos^{-1}\left(\sqrt{1-x^2}\right)=\pi$
This is possible only when
$\cos^{-1}(2x) = \pi$ ...(i)
And $2\cos^{-1}\sqrt{1-x^2}=0$ ...(ii)
From (i)
$x=-\frac{1}{2}$
Which does not satisfy (ii)
So no such x exist

Solution 68

$\frac{1}{(51)!}\left({}^{51}C_1+{}^{51}C_3+\ldots+{}^{51}C_{51}\right)$
$=\frac{2^{50}}{(51)!}$

Hence, the correct answer is option (1).

Solution 69

$$\begin{vmatrix}\lambda & 1 & 1\\ 1 & \lambda & 1\\ 1 & 1 & \lambda\end{vmatrix}=0$$
$\lambda(\lambda^2 - 1) - 1(\lambda - 1) + 1(1 - \lambda) = 0$
$\lambda^3 - \lambda - \lambda + 1 + 1 - \lambda = 0$
$\lambda^3 - 3\lambda + 2 = 0$
$(\lambda - 1)(\lambda^2 + \lambda - 2) = 0$

λ = 1, −2
For λ = 1 ⇒ ∞ solution
λ = −2 ⇒ no solution
$\sum_{\lambda \in S} |\lambda|^2 + |\lambda| = 6$

Hence, the correct answer is option (3).

Solution 70

We know that
$\cos 2A + \cos 2B + \cos 2C \geq \frac{-3}{2}$ where equality holds for equilateral triangle

$$r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}}{4}a^2}{\frac{3}{2}a} = \frac{a}{2\sqrt{3}}$$

$$a = 2\sqrt{3r} = 6\sqrt{3}$$

$$\text{Area} = \frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$$

Hence, the correct answer is option (3).

Solution 71

$$C_1 \to = C_1 + C_2 + C_3$$

$$\left(2 + \sin 2x\right) \begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 1 & 1 + \cos^2 x & \sin 2x \\ 1 & \cos^2 x & 1 + \sin 2x \end{vmatrix}$$

$R_2 \to R_2 \to R_1$; $R_3 \to R_3 \to R_1$

$$\left(2 + \sin 2x\right) \begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$f\left(x\right) = 2 + \sin 2x;\ x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$$

$$f(x)_{\max} = 2 + 1 = 3\ for\ x = \frac{\pi}{4}$$

$$f(x)_{\min} = 2 + \frac{\sqrt{3}}{2} \text{for}\ x = \frac{\pi}{6}, \frac{\pi}{3}$$

$$\beta^2 - 2\sqrt{\alpha} = 4 + \frac{3}{4} + 2\sqrt{3} - 2\sqrt{3}$$

$$= \frac{19}{4}$$

Hence, the correct answer is option (4).

Solution 72

$\frac{dy}{dx}+\frac{x+a}{y-2}=0$

(y – 2)dy + (x + a)dx = 0

Integrating

$\frac{y^2}{2}-2y+\frac{x^2}{2}+ax=C$

Or $x^2 + 2ax + y^2 - 4y = C$

At x = 1, y = 0

1 + 2a = C

Equation of circle

$x^2 + 2ax + y^2 - 4y = 1 + 2a$

$x^2 + y^2 + 2ax - 4y - (1 + 2a) = 0$

$r=\sqrt{a^2+4+1+2a}=2$

$a^2 + 2a + 5 = 4 \Rightarrow \boxed{a=-1}$

Curve is $x^2 + y^2 - 2x - 4y + 1 = 0$

Intersection with y-axis

$P=\left(0,\ 2+\sqrt{3}\right)\ Q\equiv 0,\ 2-\sqrt{3}\big)$

For normal at P & Q

$R=\left(1+\frac{2}{\sqrt{3}},0\right),\ S=\left(1-\frac{2}{\sqrt{3}},\ 0\right)$

$RS=\frac{4\sqrt{3}}{3}$

Hence, the correct answer is option (4).

Solution 73

$f'\left(x\right)=2+\frac{1}{1+x^2},\ g'\left(x\right)=\frac{1}{\sqrt{x^2+1}}$

$f''\left(x\right)=-\frac{2x}{\left(1+x^2\right)^2}<0$

$g''\left(x\right)=-\frac{1}{2}\left(x^2+1\right)^{-3/2}\cdot 2x<0$

$f'\left(x\right)\Big|_{\min}=f'\left(3\right)=2+\frac{1}{10}=\frac{21}{10}$

$g'(x)|_{\max} = g'(0) = 1$

$f'(x)|_{max} = f(3) = 2 + \tan^{-1} 3$

$g(x)|_{max} = g(3) = \ln\left(3 + \sqrt{10}\right) < \ln < 7 < 2$

Hence, the correct answer is option (4).

Solution 74

$np + npq = 5$
$np(1 + q) = 5$ ...(i)
$np(npq) = 6$ ...(ii)
$\Rightarrow np = 3, npq = 2$

$\Rightarrow q = \frac{2}{3},\ p = \frac{1}{3},\ n = 9$

$$6\left(n + p - q\right) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right) = 6\left(9 - \frac{1}{3}\right)$$
$$= 52$$

Hence, the correct answer is option (1).

Solution 75

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix} = \hat{i}\begin{pmatrix} 2 \end{pmatrix} - \hat{j}\begin{pmatrix} -2 \end{pmatrix} + \hat{k}\begin{pmatrix} 2 \end{pmatrix}$$

$\therefore\ \overrightarrow{b_1} \times \overrightarrow{b_2} = \hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{a_1} - \overrightarrow{a_2} = 8\hat{i} + 7\hat{j} + 3\hat{k}$

$$d = \frac{\left(\overrightarrow{a_1} - \overrightarrow{a_2}\right) \cdot \left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} = \left|\frac{8+7+3}{\sqrt{3}}\right| = \frac{18}{\sqrt{3}} = 6\sqrt{3}$$

Hence, the correct answer is option (2).

Solution 76

$$\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+3}\cdots\frac{1}{n+n}\right)$$
$$=\lim_{n\to\infty}\ \sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\left(\frac{r}{n}\right)}\right)$$
$$=\int_0^1\frac{dx}{1+x}=\log\Big(1+x)_0^1=\log 2$$

Hence, the correct answer is option (3).

Solution 77

For reflexive:
$3a-3a+\sqrt{7}$ is an irrational number $\forall a\ \in\ R\ R$ is reflexive
For symmetric
Let $3a-3b+\sqrt{7}$ is an irrational number
⇒ $3b-3a+\sqrt{7}$ is an irrational number
For e.g., Let $3a-3b=\sqrt{7}$
$\sqrt{7}+\sqrt{7}$ is irrational but $-\sqrt{7}+\sqrt{7}$ is not.
∴ *R* is not symmetric
For transitive:
Let $3a-3b+\sqrt{7}$ is irrational and $3b-3c+\sqrt{7}$ is irrational
⇒ $3a-3c+\sqrt{7}$ is irrational
For e.g., take *a* = 0, $b=-\sqrt{7},\ c=\frac{\sqrt{7}}{3}$
R is not transitive.

Hence, the correct answer is option (1).

Solution 78

q ∨ (~ *q* ∧ *p*)
⇒ (*q* ∨ ~ *q*) ∧ (*q* ∨ *p*)
⇒ T ∧ (*q* ∨ *p*)
⇒ *q* ∨ *p*
Now,
~ (*q* ∨ *p*)
= ~ *q* ∧ ~ *p*

Hence, the correct answer is option (4).

Solution 79

Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=t$

$t+\frac{1}{t}=10$

$\Rightarrow t^2-10t+1=0$

$\Rightarrow t=\frac{10\pm\sqrt{100-4}}{2}=5\pm2\sqrt{6}$

Case-I

$t=5+2\sqrt{6}$

$\Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\sqrt{3}+\sqrt{2}\right)^2$

$\Rightarrow x^2-4=2\Rightarrow 6=x=\pm\sqrt{6}$

Case-II

$t=5-2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\sqrt{3}-\sqrt{2}\right)^2$

$=\left(\left(\sqrt{3}-\sqrt{2}\right)^{-1}\right)^{x^2-4}=\left(\sqrt{3}-\sqrt{2}\right)^2$

$=4-x^2=2$

$=x^2=2$

$=x=\pm\sqrt{2}$

Hence, the correct answer is option (4).

Solution 80

P(2, –1, 3) Plane: *x* + 2*y* – *z* = 0

Let *Q*(α, β γ)

Then,

$\frac{\alpha-2}{1}=\frac{\beta+1}{2}=\frac{\gamma-3}{-1}=\frac{-2(-3)}{6}$

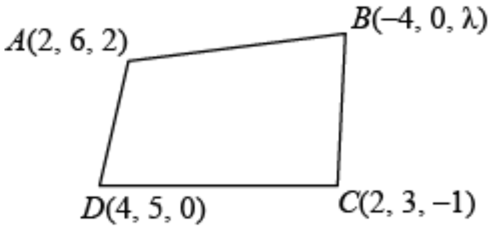
∴ α = 3, β = 1, γ = 2

Now distance of *Q* from the plane 3*x* + 2*y* + *z* + 29 = 0

$\left(d=\frac{9+2+2+29}{\sqrt{14}}=\frac{42}{\sqrt{14}}=3\sqrt{14}\right)$

Hence, the correct answer is option (4).

Solution 81



$\overrightarrow{d}_1 = 3\hat{j} + 3\hat{k}$

$\overrightarrow{d}_2 = 8\hat{i} + 5\hat{j} - \lambda\hat{k}$

$\overrightarrow{d_1} \times \overrightarrow{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 3 \\ 8 & 5 & -\lambda \end{vmatrix}$

$= (-3\lambda - 15)\hat{i} + 24\hat{j} - 24\hat{k}$

$\frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right| = 18$

$\sqrt{(3\lambda + 15)^2 + 24^2 + 24^2} = 36$

$(3\lambda + 15)^2 = 1296 - 1152$

$3\lambda + 15 = \pm 12$

$3\lambda = -3$

$\lambda = -1$

$3\lambda + 15 = -12$

$\lambda = -\frac{27}{3}$

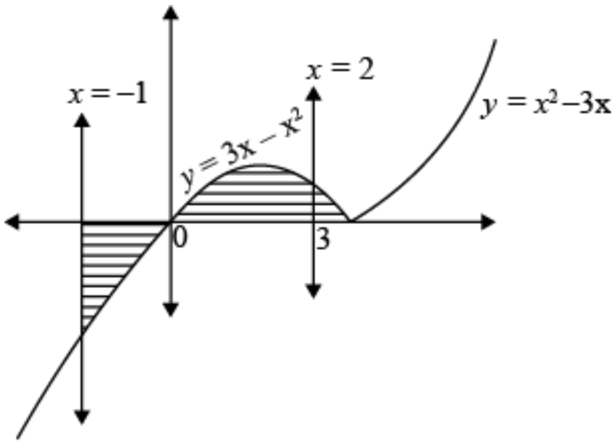
$\lambda = -9$

$\because \lambda \in [-5,\ 5]$

$\therefore \lambda = -1$

$5 - 6(-1) = 11$

Solution 82



$$\text{Area} = \int_{-1}^{2} \left|3x - x^2\right|$$

$$A = \int_{-1}^{0} x^2 - 3x \ dx + \int_{0}^{2} 3x - x^2 dx$$

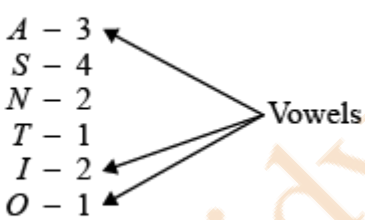
$$= \frac{x^3}{3} - \frac{3x^2}{2}\Big]_{-1}^{0} + \frac{3x^2}{2} - \frac{x^3}{3}\Big]_{0}^{2}$$

$$= 0 - \left(\frac{-1}{3} - \frac{3}{2}\right) + \left(6 - \frac{8}{3}\right) - 0$$

$$= \frac{31}{6}$$

$$\therefore \ 12A = 62$$

Solution 83



Number of arrangements = $\frac{8!}{4!2!} \times \frac{6!}{3!2!} = 50400$

Solution 84

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix} = \hat{i} - 5a\hat{j} - 3a\hat{k}$$

$$\left[\vec{u}\,\vec{v}\,\vec{w}\right] = \vec{u}\,.\left(\vec{v} \times \vec{w}\right)$$

$$= \left|\vec{u}\right|\left|\vec{v} \times \vec{w}\right| \times \cos\theta$$

$$= \alpha\sqrt{34\alpha^2 + 1\cos}\theta$$

$$\left[\vec{u}\,\vec{v}\,\vec{w}\right]_{\min} =-\alpha\sqrt{3401}$$

$$\alpha\sqrt{34\alpha^2 + 1} \times (-1) =-\alpha\sqrt{3401}$$

$$(\text{taking }\cos\theta = 1)$$

$$\Rightarrow \alpha = 10$$

$$\vec{v} \times \vec{w} = \hat{i} - 50\hat{j} - 30\hat{k}$$

$$\cos\theta =-1 \Rightarrow \vec{u} \text{ is antiparallel to } \vec{\text{v}} \times \vec{w}$$

$$\vec{u} = -\left|\vec{u}\right|.\frac{\vec{v} \times \vec{w}}{\left|\vec{v} \times \vec{w}\right|} = \frac{-10\left(\hat{i}-50\hat{j}-30\hat{k}\right)}{\sqrt{3401}}$$

$$\left|\vec{u}\,.\,\hat{i}\right|^2 = \left|\frac{-10}{\sqrt{3401}}\right|^2 = \frac{100}{3401} = \frac{m}{n}$$

$$m + n = 3501$$

Solution 85

Given, a_1 = 8, a_2, a_3...an are in A.P.
Now 2(16 + 3d) = 50
3d = 9 ⇒ $\boxed{d = 3}$
Now 2($2a_n$ – 9) = 170
a_n = 47
8 + (n – 1) 3 = 47
$\boxed{n = 14}$
Product of middle two terms = $a_7 \times a_8$
= (8 + 18) (8 + 21)
= 26 × 29
= 754

Solution 86

$$I=\int_0^1\left(x^{21}+x^{14}+x^7\right)\left(2x^{14}+3x^7+6\right)^{1/7}dx$$

$$I=\int_0^1\left(x^{20}+x^{13}+x^6\right)\left(2x^{21}+3x^{14}+6x^7\right)^{1/7}dx$$

Let $2x^{21} + 3x^{14} + 6x^7 = t$
$\Rightarrow 42(x^{20} + x^{13} + x^6)dx = dt$
$I=\frac{1}{42}\int_0^{11}t^{1/7}\,dt=\frac{1}{42}\;\frac{7}{8}\left[t^{8/7}\right]_0^{11}$
$=\frac{1}{48}11^{817}$
$\therefore l = 48, m = 8, n = 7$
$\therefore l + m + n = 63$

Solution 87

$f'(x) + f(x) = k$
$\Rightarrow e^x f(x) = ke^x + c$
$f(x) = k + ce^{-x}$
$k=\int_0^2\left(k+ce^{-t}\right)dt$

$k=2k+c\cdot\left.\frac{e^{-t}}{-1}\right|_0^2$

$k=2k+c\left(\frac{e^{-2}}{-1}+1\right)$

$-k=c\left(1-\frac{1}{e^2}\right)$

$f\left(x\right)=ce^{-x}-c\left(1-\frac{1}{e^2}\right)$

$f\left(0\right)=c-c+\frac{c}{e^2}=\frac{1}{e^2}\Rightarrow c=1$

$f\left(2\right)=e^{-2}-r\left(1-e^{-2}\right)$

$= 2e^{-2} - 1$
$2f(0) - f(2) = 1$

Solution 88

Let $g'(1) = a$ and $g''(2) = b$
$\Rightarrow f(x) = x^2 + ax + b$
Now, $f(1) = 1 + a + b$; $f'(x) = 2x + a$; $f''(x) = 2$
$g(x) = (1 + a + b)x^2 + x(2x + a) + 2$
$\Rightarrow g(x) = (a + b + 3)x^2 + ax + 2$
$\Rightarrow g'(x) = 2x(a + b + 3) + a \Rightarrow g'(1) = 2(a + b + 3 + a = a$
$\Rightarrow a + b + 3 = 0$ ...(i)

$g''(x) = 2(a + b + 3) = b$

$\Rightarrow 2a + b + 6 = 0$ ...(ii)

Solving (i) and (ii), we get

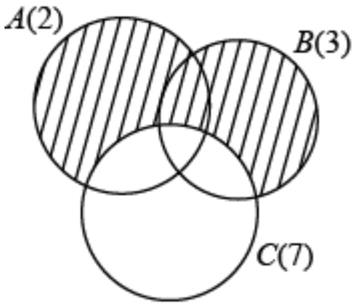
$a = -3$ and $b = 0$

$f(x) = x^2 - 3x$ and $g(x) = -3x + 2$

$f(4) = 4$ and $g(4) = -12 + 2 = -10$

$\Rightarrow f(4) - g(4) = 16 - 2 = 14$

Solution 89



A = Numbers divisible by 2

B = Numbers divisible by 3

C = Numbers divisible by 7

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= n(2) + n(3) - n(6)$

$n(A) = n(2) = 100, 102...., 998, = 450$

$n(B) = n(3) = 102, 105, ..., 999 = 30$

$n(A \cap B) = n(6) = 102, 108, ..., 996 = 150$

$n(2 \text{ or } 3) = 450 + 300 - 150 = 600$

Now,

$n(A \cap C) = n(14) = 112, 126, ..., 994 = 64$

$n(A \cap B \cap C) = n(42) = 126, 168, ..., 966 = 21$

$n(B \cap C) = n(21) = 105, 126, ..., 987, = 43$

$n(2 \text{ or } 3 \text{ not by } 7) = 600 - [64 + 43 - 21]$

$= 514$

Solution 90

$19^{200} + 23^{200}$

$(21 - 2)^{200} + (21 + 2)^{200} = 49\lambda + 2^{201}$

$2^{201} = 8^{67} = (7 + 1)^{67} = 49\lambda + 7 \times 67 + 1$

$= 49\lambda + 470$

$= 49(\lambda + 9) + 29$

Remainder = 29