JEE Main 25 Jan 2023(First Shift)

Total Time: 180

Total Marks: 300.0

Solution 1

Frequency of modulating wave = 5 kHz
Bandwidth = Twice the frequency of modulating signal
= 2 × 5 kHz
= 10 kHz

Hence, the correct answer is option (3).

Solution 2

$$\lambda_0 = \frac{h}{\sqrt{2m\left[e\left(20\times10^3\right)\right]}}$$

$$\lambda_{\text{new}} = \frac{h}{\sqrt{2m\left[e\left(40\times10^3\right)\right]}} = \frac{\lambda_0}{\sqrt{2}}$$

Hence, the correct answer is option (3).

Solution 3

$$\because\ v_{\text{ms}} = \sqrt{\frac{3RT}{M}}$$

$$\therefore\ v_{\text{ms}} \propto \sqrt{T}$$

Hence, the correct answer is option (1).

Solution 4

$y_5 = 5\ \text{cm},\ D = 1\ \text{m},\ \lambda = 600\ \text{nm}$

$\because\ \frac{5\lambda D}{d} = \frac{5}{100}$

$\therefore\ d = \frac{5\times600\times10^{-9}\times1\times100}{5}$

$= 6\times10^{-5}\ \text{m}$

$= 60\ \mu\text{m}$

Hence, the correct answer is option (2).

Solution 5

(A) Surface tension : kg s^{-2} (IV)
(B) Pressure : kg $m^{-1}s^{-2}$ (III)
(C) Viscosity : kg $m^{-1}s^{-1}$ (I)
(D) Impulse : kg ms^{-1} (II)

Hence, the correct answer is option (3).

Solution 6

From Newton's law of cooling.

$\frac{dT}{dt} = -k(T - T_s)$

$\text{Case I}:\ dT = 12^{\circ}C,\ dt = 2\ \text{min}$

$\frac{12}{2} = -k\left[92 - 22^{\circ}\right] = -k\,70 \ldots (1)$

$\text{Case II}:\ dT = 6^{\circ}\text{C}$

$\frac{6}{dt} = -k[72 - 22] = -k50 \quad \ldots (2)$

$\text{From } (1) \text{ and } (2)$

$dt = 1.4\ \text{min}$

Hence, the correct answer is option (2).

Solution 7

$T=2\pi\sqrt{\frac{l}{g}}$

$g=\text{acceleration due to gravity}$

On earth's surface $g=\frac{Gm}{R^2}$

At height R, $g_R=\frac{Gm}{4R^2}$

$g_R=\frac{g}{4}$

Time period at height $R=2\pi\sqrt{\frac{l}{g_R}}=2\mathrm{T}$

Hence, the correct answer is option (3).

Solution 8

$\eta=1-\frac{T_{\text{sink}}}{T_{\text{source}}}$

50% efficiency $\Rightarrow\frac{1}{2}=1-\frac{T_{\text{sink}}}{T_{\text{source}}}$

$\frac{1}{2}=1-\frac{T_{\text{sink}}}{600}\Rightarrow T_{\text{sink}}=300$

Now, 70% efficiency $\Rightarrow\frac{7}{10}=1-\frac{T_{\text{sink}}}{T_{\text{source}}}$

$\frac{300}{T_{\text{source}}}=\frac{3}{10}$

$T_{\text{source}}=1000\ \mathrm{K}$

Hence, the correct answer is option (4).

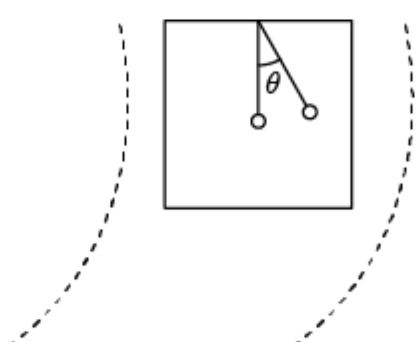
Solution 9

Nuclear density is constant.

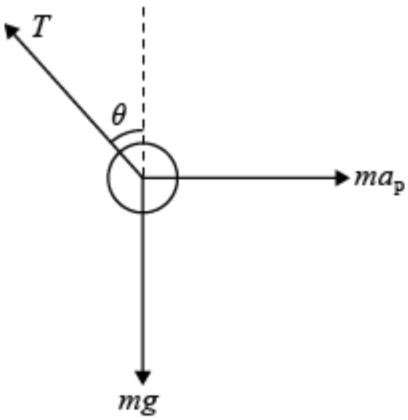
$\frac{\rho_{\text{oxygen}}}{\rho_{\text{Helium}}}=1$

Hence, the correct answer is option (3).

Solution 10



In car's frame, FBD of bob



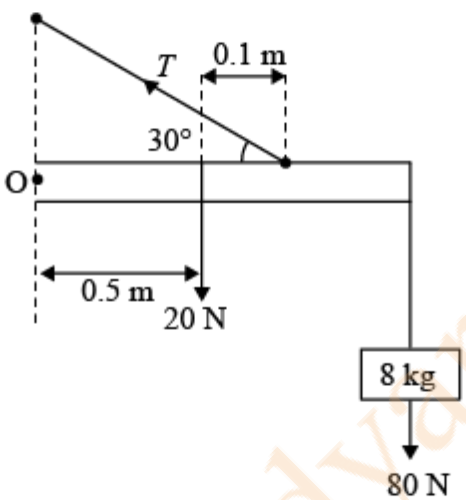
where a_p = Pseudoforce or centrifugal force

$$\theta = \tan^{-1}\left(\frac{a_p}{g}\right) = \tan^{-1}\left(\frac{v^2}{Rg}\right) = \tan^{-1}\left(\frac{400}{40\times10}\right)$$

$$\theta = 45^{\circ} = \frac{\pi}{4}$$

Hence, the correct answer is option (4).

Solution 11



Torque balance about 'O'

$$\frac{T}{2} \times 0.6 = 20 \times 0.5 + 80 \times 1$$

$$T \times 0.3 = 10 + 80 = 90$$

$$T = \frac{900}{3} = 300 \text{ N}$$

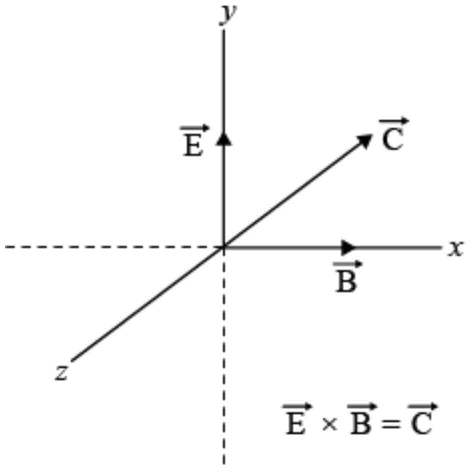
Hence, the correct answer is option (3).

Solution 12

$$v_{\text{avg}} = \frac{2x}{\left(\frac{x}{v_1} + \frac{x}{v_2}\right)} = \left(\frac{2v_1v_2}{v_1+v_2}\right)$$

Hence, the correct answer is option (3).

Solution 13



So, $\overrightarrow{B}$ should be in *x* direction.

Hence, the correct answer is option (4).

Solution 14

$m = 8.92 \times 10^{-3}$ kg
Density = 8.92×10^{3} kg/m^3
$\text{Volume} = \frac{8.92\times10^{-3}}{8.92\times10^{3}} = \left(10^{-6}\right)\text{m}^3$

$\text{Resistance} = \frac{3.4}{2} = 1.7\,\Omega = \frac{\rho l^2}{A}$

$1.7 = \frac{\rho l^2}{(Al)}$

$\Rightarrow 1.7 = \frac{1.7\times10^{-8}\times l^2}{10^{-6}}$

$l^2 = 100$
$l = 10\text{ m}$

Hence, the correct answer is option (3).

Solution 15

$\omega_0 = \frac{1}{\sqrt{LC}}$

If inductance becomes 2*L* and capacitance 8C
$\omega = \frac{1}{\sqrt{2L\times8C}} = \frac{1}{4\sqrt{LC}}$
$\omega = \left(\frac{\omega_0}{4}\right)$

Hence, the correct answer is option (2).

Solution 16

Number of turns per unit length = $\frac{1200}{2} = 600$
Magnetic Intensity $H = nI$
$H = 600 \times 2 = 1200\ \text{A m}^{-1} = 1.2 \times 10^3\ \text{A m}^{-1}$

Hence, the correct answer is option (3).

Solution 17

Photodiodes are used in reverse bias, therefore, the assertion is incorrect.
The reason is correct.
Hence, the correct answer is option (1).

Solution 18

Gravitational acceleration at a distance of r from centre of earth is given by
$g' = \frac{g}{R} r$
Where R is the radius of earth

$$So,\ \frac{d^2 r}{dt^2} = -\frac{g}{R} r$$

$$\Rightarrow T = 2\pi\sqrt{\frac{\text{R}}{\text{g}}} = 2\pi\sqrt{\frac{6400000}{10}}$$

$$= 2\pi \times 800\ \text{sec}$$

$$= 5024\ \text{sec}$$

$$= 1\ \text{hour}\ \ 24\ \text{minutes}\ \left(\text{approx.}\right)$$

Hence, the correct answer is option (4).

Solution 19

$$c = \frac{\varepsilon_0 A}{(d-t)+\frac{t}{K}}$$

$$= \frac{K\varepsilon_0 A}{Kd-t+(K-1)}$$

$$= \frac{5\varepsilon_0 \times 40 \times 10^{-4}}{5\times 2\times 10^{-3} - 1\times 10^{-3}(5-1)}$$

$$= \frac{20\ \varepsilon_0}{6}$$

$$= \frac{10\ \varepsilon_0}{3}$$

Hence, the correct answer is option (2).

Solution 20

$A \rightarrow B_0 = \frac{-\mu_0 I}{4\pi r} + \frac{\mu_0 I}{2r} - \frac{\mu_0 I}{4\pi r}$

$B_0 = \frac{\mu_0 I}{2\pi r}(\pi - 1)$ $\qquad$ $A \rightarrow III$

$B \rightarrow B_0 = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{2r} + \frac{\mu_0 I}{4\pi r}$

$B_0 = \frac{\mu_0 I}{4\pi r}(\pi + 2)$ $\qquad$ $B \rightarrow I$

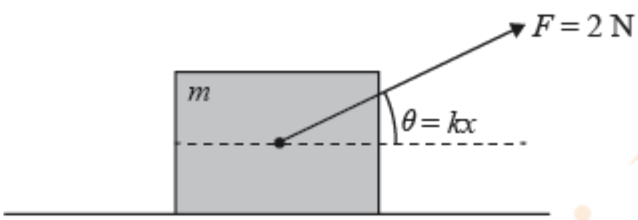
$C \rightarrow B_0 = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r} + 0$

$B_0 = \frac{\mu_0 I}{4\pi r}(\pi + 1)$ $\qquad$ $C \rightarrow IV$

$D. \rightarrow B_0 = \frac{\mu_0 I}{4r}$ $\qquad$ $D \rightarrow II$

Hence, the correct answer is option (3).

Solution 21



Work done = ΔK.E

$\therefore \int F.dx = \frac{1}{2}mv^2 = E$

$\therefore E = \int_0^x 2\cos\left(kx\right)dx$

$E = \frac{2}{k}[\sin kx]_0^x$

$= \frac{2}{k}\sin kx$

$= \frac{2\sin\theta}{k}$

So, the value of *n* is **2**.

Solution 22

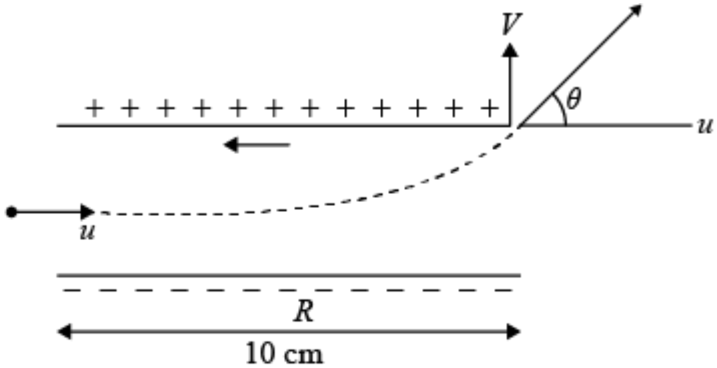
$Y = \frac{F}{\Delta l} \times \left(\frac{l(4)}{\pi d^2}\right)$

$= (\text{slope})\frac{(62.8\times10^{-2})(4)}{\pi(4\times10^{-3})^2}$

$Y = (1) \times 5 \times 10^4\ \text{N/m}^2$

So, the value of *x* is **5**.

Solution 23



Let R is the range and T be the time of motion inside the plate.

$\therefore R = vT$

and, $\tan\theta = \frac{v}{u}$

$= \frac{\left(\frac{eE}{m}\right)T}{u}$

$= \frac{\frac{eE}{m}\left(\frac{R}{u}\right)}{u}$

$= \frac{eER}{mu^2}$

$= \frac{eER}{2(K.E.)}$

$= \frac{(e)\times(10)\times\left(10\times10^{-2}\right)}{2\times(0.5\ \text{eV})}$

$= 1$

$\therefore\ \tan\theta\ =\ 1$

$\theta\ = 45^{\circ}$

Solution 24

$\because$ For maximum amplitude of current, circuit should be at resonance.

$\therefore X_L = X_C$

$\omega L = \frac{1}{\omega C}$

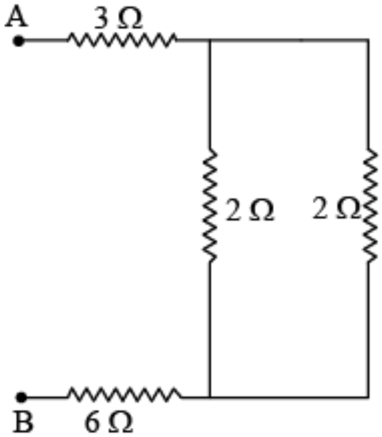
$L = \frac{1}{\omega^2 C}$

$= \frac{1}{\left(2\pi\times2\times10^3\right)^2\times62.5\times10^{-9}}$

$= 100\ \text{mH}$

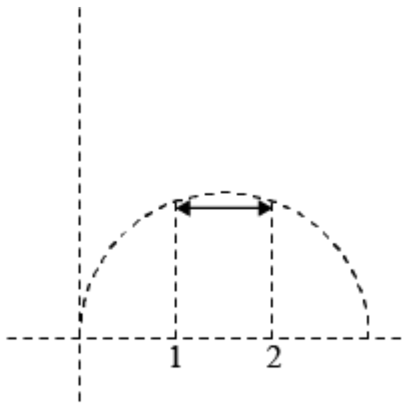
Solution 25

Equivalent circuit can be redrawn as



$\therefore\ R_{AB} = 10\ \Omega$

Solution 26



$\Delta x = \frac{\lambda}{2\pi} \times \left(\frac{\pi}{3}\right) = \left(\frac{\lambda}{6}\right)$

$\Rightarrow \frac{\lambda}{6} = 6\ \text{m}$

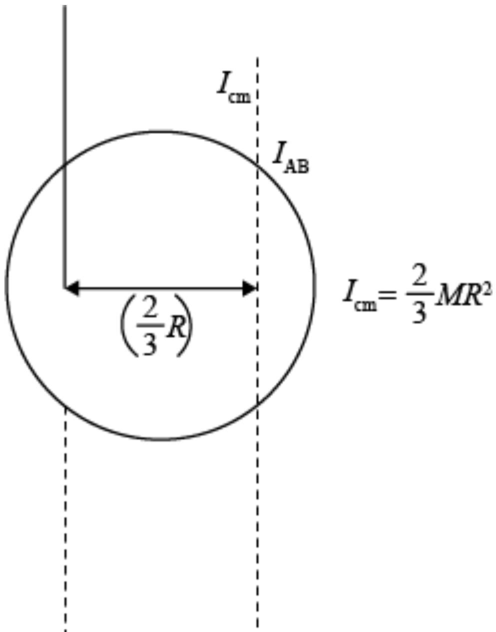
$\lambda\ =\ 36\ \text{m}$

$\text{U}\ =\ \text{f}\lambda\ =\ 500\ \text{Hz}\ \times\ 36$

$= 18000\ \text{m/s}$

$= 18\ \text{km}/\text{s}$

Solution 27



$$I_{AB} = I_{cm} + M \times \left(\frac{2}{3}R\right)^2$$

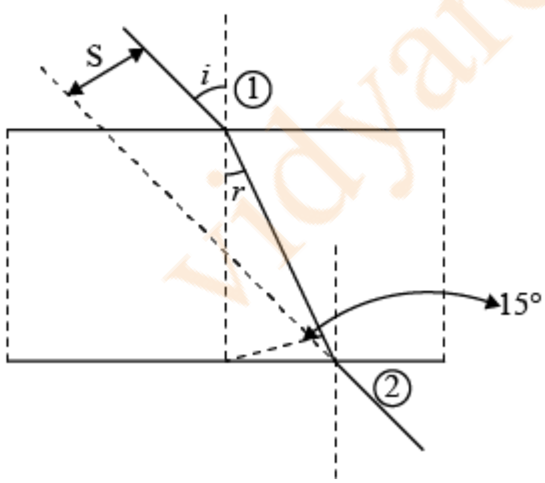
$$= \frac{1}{2}MR^2 + \frac{4}{9}MR^2$$

$$= \frac{(9+8)MR^2}{18} = \left(\frac{17}{18}\right)MR^2$$

$$\frac{l_{AB}}{l_{cm}} = \frac{\frac{17}{18}}{\frac{1}{2}} = \left(\frac{17}{9}\right)$$

Value of $x = 17$

Solution 28



$\sin i = \frac{1}{\sqrt{2}} = 45º$

$\Rightarrow \text{at point} \left(1\right)$

$\mu \sin r = \sin i = \frac{1}{\sqrt{2}}$

$\sin r = \frac{1}{2} \Rightarrow r = 30º$

Lateral displacement

$= \frac{\text{t}}{\text{cos r}} \sin \left(15º\right) = \frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)} \times 0.26$

$= 2 \times 0.26$

$= 0.52 \text{ cm}$

$= 52 \times 10^{-2} \text{ cm}$

Solution 29

$\overrightarrow{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$

$\overrightarrow{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$

$$\overrightarrow{P} \times \overrightarrow{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \sqrt{3} & 2 \\ 4 & \sqrt{3} & 2.5 \end{vmatrix}$$

$= \hat{i}\left(\frac{\sqrt{3}}{2}\right) - \hat{j}\left(-\frac{1}{2}\right) + \hat{k}\left(-\sqrt{3}\right)$

$= \frac{\sqrt{3}}{2}\hat{i} + \frac{\hat{j}}{2} - \sqrt{3}\hat{k}$

$\left|\overrightarrow{P} \times \overrightarrow{Q}\right| = \sqrt{\frac{3}{4} + \frac{1}{4} + 3} = 2$

Unit vector along $\overrightarrow{P} \times \overrightarrow{\text{Q}} = \frac{1}{4}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right)$

$x = 4$

Solution 30

Transition, $n = 3$ to $n = 2$

$\frac{1}{\lambda_0} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \left(\frac{5R}{36}\right) \ \ldots\left(1\right)$

For transition from, $n = 4$ to $n = 2$

$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \left(\frac{3}{16}R\right) \ \ldots\left(2\right)$

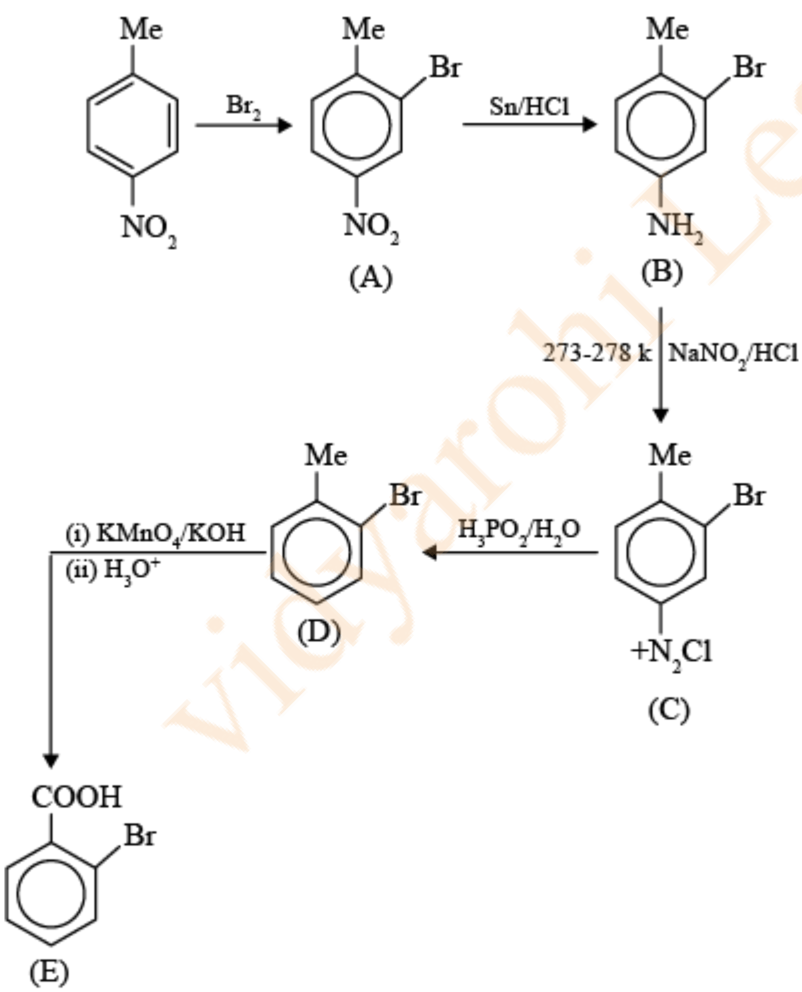
Taking ratio of $\left(1\right)$ and $\left(2\right)$

$\frac{\lambda}{\lambda_0} = \frac{5}{36} \times \frac{16}{3} = \left(\frac{20}{27}\right)$

$\lambda = \frac{20}{27}\lambda_0$

$\mathrm{x} = 27$

Solution 31



Hence, the correct answer is option (2).

Solution 32

Number of X particles = $4 \times \frac{1}{8} + 1 = 1.5$

Number of Y particles = $6 \times \frac{1}{3} \times \frac{1}{2} = 1$

$\therefore$ Empirical formula = $X_{1.5}Y_1 = X_3Y_2$

Disclaimer: *None of the options matches with the correct answer.*

Solution 33

$$P_4 + 8\,SOCl_2 \rightarrow \underset{(A)}{4\,PCl_3} + 4\,SO_2 + 2S_2\,Cl_2$$

$$PCl_3 \xrightarrow{\text{Hydrolysis}} \underset{(B)}{H_3\,PO_3}$$

$$\text{Dibasic acid}$$

Hence, the correct answer is option (2).

Solution 34

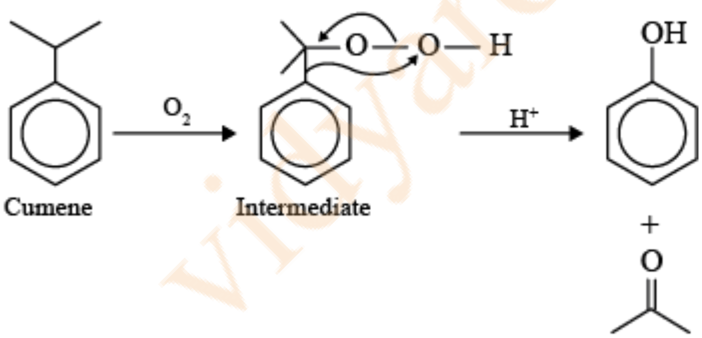
$$\text{Molarity of } H_2O_2 \text{ sol}^n = \frac{\text{volume strength}}{11.2}$$

$$= \frac{25}{11.2} = 2.23\text{ M}$$

$$\therefore \text{ amount of } H_2O_2 \text{ in one litre } = 2.23 \times 34 = 75 \text{ gm}$$

Hence, the correct answer is option (4).

Solution 35



Hence, the correct answer is option (2).

Solution 36

$$r_{Li^{2+}} = r_0 \times \frac{2^2}{3} = x \Rightarrow r_0 = \frac{3x}{4}$$

$$r_{Be^{3+}} = r_0 \times \frac{3^2}{4}$$

$$r_{Be^{3+}} = \frac{3x}{4} \times \frac{3^2}{4} = \frac{27x}{16}$$

Hence, the correct answer is option (3).

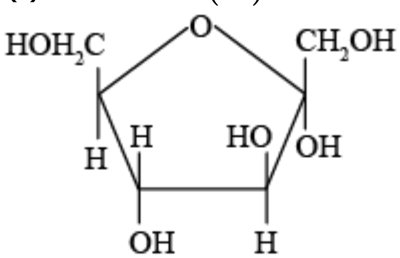
Solution 37

In the extraction of copper FeO is removed as slag $FeSiO_3$
Hence the reaction $CaO + SiO_2 \rightarrow CaSiO_3$ does not occur during extraction of copper.

Hence, the correct answer is option (1).

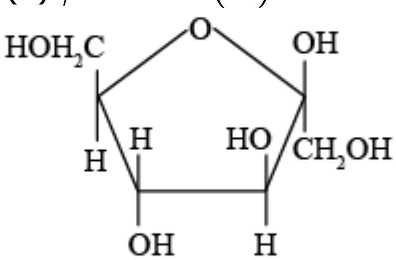
Solution 38

(i) $\alpha - \mathrm{D} - (-) - \mathrm{Fructofuranose} -$



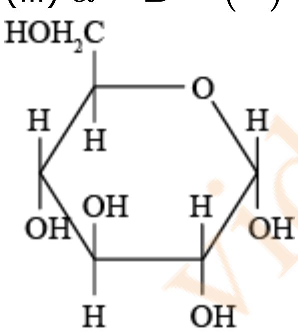
(C)

(ii) $\beta - \mathrm{D} - (-) - \mathrm{Fructofuranose} -$



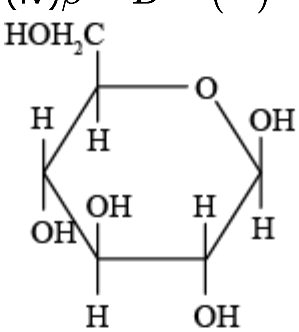
(D)

(iii) $\alpha - \mathrm{D} - (-) - \mathrm{Glucopyranose} -$



(A)

(iv)$\beta - \mathrm{D} - (-) - \mathrm{Glucopyranose} -$



Hence, the correct answer is option (1).

Solution 39

An antibiotic inhibit the growth or survival of microorganism.
Except (1) all the statement are correct.

Hence, the correct answer is option (1).

Solution 40

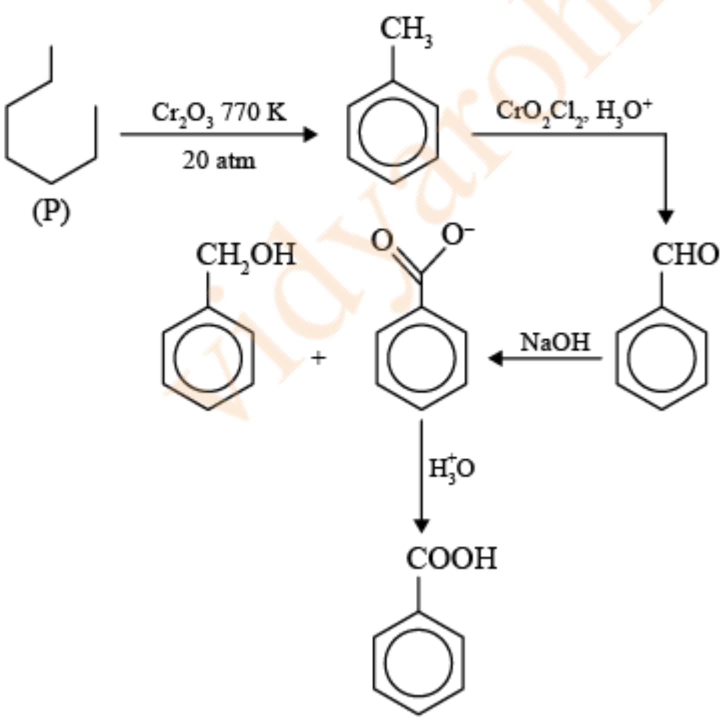
Elements		Colour imparted to flame	
A.	K	II.	Violet
B.	Ca	I.	Brick Red
C.	Sr	IV.	Crimson Red
D.	Ba	III.	Apple Green

Hence, the correct answer is option (1).

Solution 41

Aryl halides having E.W.G at *o*-or *p*-position have greater rate than the *m*-isomers towards nucleophilic aromatic substitution.
Hence, the correct answer is option (2).

Solution 42



Hence, the correct answer is option (2).

Solution 43

Acetal/Ketal are known to be quite stable under basic conditions but readily hydrolyse to the corresponding carbonyl compound (aldehyde/ketone) and alcohol under acidic condition.

Hence, the correct answer is option (4).

Solution 44

Correct stability order of butane is Anti > Gauche > Partially eclipsed > Fully eclipsed.
Hence, the correct answer is option (1).

Solution 45

The correct order of basic strength in aqueous medium is
$Me_2NH > MeNH_2 > Me_3N > NH_3$

Hence, the correct answer is option (1).

Solution 46

$$\underset{(A)}{Ca(OH)_2} + 2\,NH_4Cl \rightarrow CaCl_2 + 2\,\underset{(B)}{NH_3} + 2H_2O$$

$$NH_3 + H_2O + \underset{\text{Excess}}{CO_2} \rightarrow \underset{(C)}{NH_4HCO_3}$$

$$NH_4HCO_3 + NaCl \rightarrow NH_4Cl + NaHCO_3$$

Hence, the correct answer is option (2).

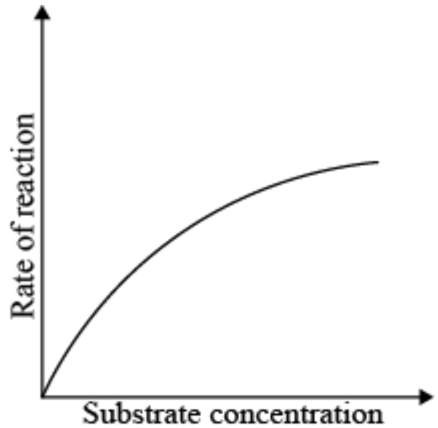
Solution 47

Cations	Group reagents
A. Pb^{2+}, Cu^{2+}	(i) H_2S gas in presence of dilute HCl
B. Al^{3+}, Fe^{3+}	(iii) NH_4OH in presence of NH_4Cl
C. Co^{2+}, Ni^{2+}	(iv) H_2S in presence of NH_4OH
D. Ba^{2+}, Ca^{2+}	(ii) $(NH_4)_2CO_3$ in presence of NH_4OH

Hence, the correct answer is option (1).

Solution 48

The correct plot for enzyme catalysed reaction is



Hence, the correct answer is option (4).

Solution 49

$$NO_2 \xrightarrow{\text{Sunlight}} \underset{(X)}{O} + \underset{(Y)}{NO}$$

$$O \xrightarrow{O_2\,(A)} O_3 \quad (B)$$

Hence, the correct answer is option (2).

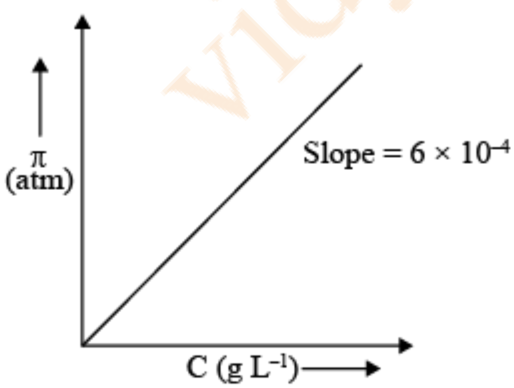
Solution 50

Electron gain	He	Ne	Ar	Kr	Xe
Enthalpy/kJ mol^{-1}	48	116	96	96	77

Hence, correct order of positive electron gain enthalpy is He < Xe < Kr < Ne.

Hence, the correct answer is option (1).

Solution 51



$\pi = CRT$

$\pi = \frac{mole}{volume} \times RT$

$\pi = \frac{mole}{volume} \times \frac{mw}{mw} \times RT$

$\pi = \frac{mass}{volume} \times \frac{RT}{mw}$

$\pi\left(atm\right) = \frac{RT}{mw} \times C\left(gm\ \ lit^{-1}\right)$

$slope = \frac{RT}{mw} = 6 \times 10^{-4}$

$mw = 41500$

Solution 52

m.eq of NaOH = m.eq of monobasic acid 25 × 0.24 × 1 = 1 × V × molarity

Molarity = $\frac{1.21\times10^3}{24.2} = 50\ M$

$\therefore\ V = \frac{25\times0.24}{50} = 0.12\ mL$

$= 12 \times 10^{-2}\ mL$

Solution 53

	NH_3	+	HCl	→	NH_4Cl
At initial	0.1		0		0.1
At time t	0.1− 0.02				0.1 + 0.02

$pOH = pK_b + \log\left[\frac{0.1+0.02}{0.1-0.02}\right]$

$= 4.745 + \log\left(\frac{3}{2}\right) = 4.745 + [0.477 - 0.301]$

$= 4.745 + 0.176$

$pOH = 4.921$

$pH = 14 - pOH$

$= 14 - 4.921 = 9.079$

$pH = 9079 \times 10^{-3}$

Solution 54

Species	**Magnetic property**
$[Ni(CN)_4]^{2-}$	Diamagnetic
$[Ni(CO)_4]$	Diamagnetic
$[NiCl_4]^{2-}$	Paramagnetic
$[FeCN)_6]^{4-}$	Diamagnetic
$[Fe(CN)_6]^{3-}$	Paramagnetic
$Fe(H_2O)_6]^{2+}$	Paramagnetic
$[Cu(NH_3)_4]^{2+}$	Paramagnetic

The number of paramagnetic species is **4**.

Solution 55

$$S\% = \frac{32}{233} \times \frac{1.4439}{0.471} \times 100 = 42\%$$

Solution 56

Time taken for 75% completion = 2 × $t_{\frac{1}{2}}$ = 2 × 30 = 60 min

Solution 57

Ion	Spin only magnetic moment
V^{3+}	$\sqrt{8}$
Cr^{3+}	$\sqrt{15}$
Fe^{2+}	$\sqrt{24}$
Ni^{3+}	$\sqrt{15}$

The number of metal ions which have similar value of spin only magnetic moment in gaseous state is **2**.

Solution 58

$$\text{Reaction at anode}: \frac{1}{2}H_2 \rightarrow H^+ + e^-$$

$$\text{Reaction at cathode}: Fe^{3+}{}_{(aq)} + e^- \rightarrow Fe^{2+}{}_{(aq)}$$

$$E_{cell} = E^0_{cell} - \frac{0.0591}{1}\log\left[\frac{[H^+][Fe^{2+}]}{[Fe^{3+}][pH_2]^{\frac{1}{2}}}\right]$$

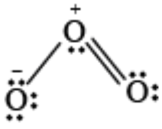
$$0.712 = 0.771 - \frac{0.0591}{1}\log\left(\frac{Fe^{2+}}{Fe^{3+}}\right)$$

$$-0.059 = -0.0591 \log\left(\frac{[Fe^{2+}]}{[Fe^{3+}]}\right)$$

$$\therefore \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10^1 = 10$$

Solution 59

The structure of ozone molecule is drawn below.



The total number of lone pairs of electrons on oxygen atoms of ozone is **6.**

Solution 60

Weight of extra water he would need to perspire

$= \frac{1800}{2} \times \frac{18}{45}$

$= 20 \times 18 = 360 \text{ gm}$

Solution 61

$$\overrightarrow{x} = 10 \text{ and } \sigma^2 = 4, \text{ No. of student } = N\left(\text{let}\right)$$

$$\therefore \frac{\sum x_i}{N} = 10 \text{ and } \frac{\sum x_i^2}{N} - (10)^2 = 4$$

Now if one of x_i is changed from 8 to 12, we have

$$\therefore \frac{\sum x_i + 4}{N} = 10 + \frac{4}{N} = 10.2$$

$$\Rightarrow N = 20$$

$$\text{and } \sigma^2_{\text{new}} = \frac{\sum x_i^2 - (8)^2 + (12)^2}{20} - (10.2)^2$$

$$= \frac{\sum x_i^2}{20} + \frac{144 - 64}{20} - (10.2)^2$$

$$= 104 + 4 - (10.2)^2$$

$$= 108 - 104.04 = 3.96$$

Hence, the correct answer is option (3).

Solution 62

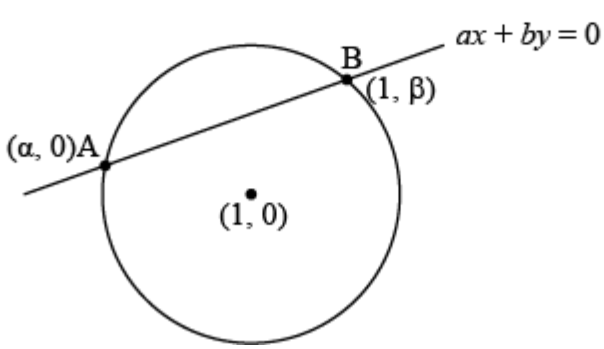
Making truth table (Let$(p \wedge \sim q) \Rightarrow (p \Rightarrow \sim q) = E$)

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$p \Rightarrow \sim q$	E
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$\therefore$ E is a tautology.

Hence, the correct answer is option (4).

Solution 63



As A and B satisfy both line and circle we have α = 0 ⇒ $A(0, 0)$ and β = 1 i.e. $B(1, 1)$

Centre of circle as AB diameter is $\left(\frac{1}{2}, \frac{1}{2}\right)$ and radius = $\frac{1}{\sqrt{2}}$

∴ For image of $\left(\frac{1}{2}; \frac{1}{2}\right)$ in $x + y + z$ we get

$$\frac{x-\frac{1}{2}}{1} = \frac{y-\frac{1}{2}}{1} = \frac{-2(3)}{2}$$

$$\Rightarrow \text{Image } \left(-\frac{5}{2}, -\frac{5}{2}\right)$$

$$\therefore \text{ Equation of required circle}$$

$$\left(x + \frac{5}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{1}{2}$$

$$\Rightarrow x^2 + y^2 + 5x + 5y + \frac{50}{4} - \frac{1}{2} = 0$$

$$\Rightarrow x^2 + y^2 + 5x + 5y + 12 = 0$$

Hence, the correct answer is option (1).

Solution 64

Let,

$$P = (2\lambda + 1, \lambda + 3, 2\lambda + 2) \text{ and } Q\left(\mu + 2, 2\mu + 2, 3\mu + 3\right)$$

$$\text{d.r's of PQ} = < 2\lambda - \mu - 1, \lambda - 2\mu + 1, 2\lambda - 3\mu - 1 >$$

$$\therefore \frac{2\lambda-\mu-1}{1} - \frac{\lambda-2\mu-1}{-1} = \frac{2\lambda-3\mu-1}{-2}$$

$$\therefore -2\lambda + \mu + 1 = \lambda - 2\mu + 1 \text{ and } -2\lambda + 4\mu - 2 = -2\lambda + 3\mu + 1$$

$$\Rightarrow 3\lambda - 3\mu = 0 \text{ and } \mu = 3$$

$$\therefore \lambda = \pm 3 \text{ and } \mu = 3$$

$$\therefore P = \left(7, 6, 8\right) \text{ and } Q\left(5, 8, 12\right)$$

$$\therefore \left|PO\right| = \sqrt{2^2 + 2^2 + 4^2} = \sqrt{24} = 2\sqrt{6}$$

Hence, the correct answer is option (2).

Solution 65

$g(x) = f(-x) - f(x)$

$= \frac{1}{1-e^x} - \frac{1}{1-e^{-x}}$

$= \frac{1}{1-e^x} - \frac{e^x}{e^x-1}$

$= \frac{1+e^x}{1-e^x}$

$g'(x) = \frac{(1-e^x)e^x-(1+e^x)(-e^x)}{(1-e^x)^2}$

$= \frac{e^x-2e^x+e^x+2e^x}{(1-e^x)^2} > 0$

So both the statements are correct.

Hence, the correct answer is option (2).

Solution 66

Given system of equations

ax + 2*ay* – 3*az* = 1

(2*a* + 1)*x* + (2*a* + 3)*y* + (*a* + 1) *z* = 2

(3*a* + 5)*x* + (*a* + 5)*y* + (*a* + 2) *z* = 3

$\text{Let } A = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$

$= a\begin{vmatrix} 1 & 0 & 0 \\ 2a+1 & 1-2a & 7a+4 \\ 3a+5 & -5a-5 & 10a+17 \end{vmatrix}$

$= a\left(15a^2+31a+37\right)$

Now $A = 0$

$\Rightarrow \ a = 0$

So, $S_1 = R - \{0\}$ and at $a = 0$

System has infinite solution but $a \in R - \{0\}$

$\therefore S_2 = \phi$

Hence, the correct answer is option (2).

Solution 67

$f(x) = \int_0^2 e^{|x-t|} dt$

For $x > 2$

$f(x) = \int_0^2 e^{x-t} dt = e^x \left(1 - e^{-2}\right).$

For $x < 0$

$f(x) = \int_0^2 e^{t-x} dt = e^{-x} \left(e^2 - 1\right)$

For $x \in [0, 2]$

$f(x) = \int_0^x e^{x-t} dt \in \int_x^2 e^{t-x} dt$

$= e^{2-x} + e^x - 2$

For $x > 2$

$f(x)|_{\min} = e^2 - 1$

For $x < 0$

$f(x)|_{\min} = e^2 - 1$

For $x \in [0, 2]$

$f(x)|_{\min} = 2(e - 1)$

Solution 68

$y = \frac{1-x^{32}}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^{31}$

$y' = 1 + 2x + 3x^2 + \ldots + 31x^{30}$

$y'(-1) = 1 - 2 + 3 - 4 + \ldots + 31 = 16$

$y'(x) = 2 + 6x + 12x^2 + \ldots + 31.30x^{29}$

$y'(-1) = 2 - 6 + 12 \ldots 31.30 = 480$

$y'(-1) - y'(-1) = -496$

Hence, the correct answer is option (1).

Solution 69

$f'(x) = 8x^3 - 36x + 8$

$= 4\left(2x^3 - 9x + 2\right)$

$= 4\left(x - 2\right)\left(2x^2 + 4x - 1\right)$

$= 4\left(x - 2\right)\left(x - \frac{-2+\sqrt{6}}{2}\right)\left(x - \frac{-2\sqrt{6}}{2}\right)$

Local maxima occurs at $x = \frac{-2+\sqrt{6}}{2} = x_0$

$f(x_0) = 12\sqrt{6} - \frac{33}{2}$

Hence, the correct answer is option (1).

Solution 70

$$I = \lim_{n\to\infty} \frac{(1+2+3+...+3n)-2(3+6+9+..+3n)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$= \lim_{n\to\infty} \frac{\frac{3n(3n+1)}{2}-6\frac{n(n+1)}{2}}{\left(\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}\right)}$$

$$= \lim_{n\to\infty} \frac{3n(n-1)\left[\sqrt{2n^4+4n+3}+\sqrt{n^4+5n+4}\right]}{2[(2n^4+4n-3)-(n^4+5n+4)]}$$

$$\lim_{n\to\infty} \frac{3.1.\left(1-\frac{1}{n}\right)\left[\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}+\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}\right]}{2\left[1-\frac{1}{n^3}-\frac{7}{n^4}\right]}$$

$$= \frac{3\left(\sqrt{2}+1\right)}{2}$$

Hence, the correct answer is option (2).

Solution 71

$y^2 = \frac{x}{2} \Rightarrow$ tangent $y = mx + \frac{1}{8m}$.

$y^2 = x - 1 \Rightarrow$ tangent $y = m\left(x - 1\right) + \frac{1}{4m}$

For common tangent $\frac{1}{8\mathrm{m}} = -\mathrm{m} + \frac{1}{4\mathrm{m}}$

$\Rightarrow 1 = -8\mathrm{m}^2 + 2$

$\because \mathrm{m} > 0 \Rightarrow m = \frac{1}{2\sqrt{2}}$

$\Rightarrow$ Common tangent is $y = \frac{x}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\Rightarrow x - 2\sqrt{2} + 1 = 0$

Distance of point $\left(6, -2\sqrt{2}\right)$ from commin tangent $= 5$

Hence, the correct answer is option (4).

Solution 72

$$|A| = \frac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & 2\log y & \log z \\ \log x & \log y & 3\log z \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 2$$

$$\Rightarrow \left|\text{adj}\left(\text{adj } A^2\right)\right| = \left|\text{adj}\, A\left(^2\right)\right|^2 = \left(\left|A^2\right|^2\right)^2 = |A|^8 = 2^8$$

Hence, the correct answer is option (3).

Solution 73

$f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}\, dx$

Put $x^2 = t \Rightarrow 2xdx = dt$

$f(x) = \int \frac{dt}{(t+1)(t+3)} = \int \frac{dt}{(t+2)^2 - 1} = \frac{1}{2}\log_e \left|\frac{t+1}{t+3} + C\right|$

$f(x) = \frac{1}{2}\log_e\left(\frac{x^2+1}{x^2+3}\right) + C \Rightarrow$

$f(3) = \frac{1}{2}\log_e\left(\frac{10}{12}\right) + C$

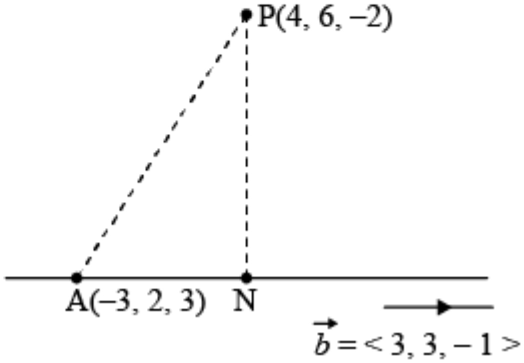
$\because f(3) + \frac{1}{2}(\log_e 5 - \log_e 6) \Rightarrow C = 0$

$f(x) = \frac{1}{2}\log_e\left(\frac{x^2+1}{x^2+3}\right) \Rightarrow$

$f(4) = \frac{1}{2}\left(\log_e 17 - \log_e 19\right)$

Hence, the correct answer is option (3).

Solution 74



$\overrightarrow{AP} = 7\hat{i} + 4\hat{j} - 5\hat{k} \Rightarrow \left|\overrightarrow{AP}\right| = \sqrt{49 + 16 + 25} = \sqrt{90} AN$

$= \text{projection of } \overrightarrow{AP} \text{ on } \overrightarrow{b} = \overrightarrow{AP}.\ \overrightarrow{b} = \frac{21+12+5}{\sqrt{19}} = \frac{38}{\sqrt{19}}$

$(PN)^2 = (AP)^2 - (AN)^2 = 90 - 76 = 14 \Rightarrow PN = \sqrt{14}$

Hence, the correct answer is option (4).

Solution 75

$x + y = 66$

$\frac{x+y}{2} \geq \sqrt{xy}$

$\Rightarrow 33 \geq \sqrt{xy}$

$\Rightarrow xy \leq 1089$

$\therefore\ M = 1089$

$S\ :\ x(66 - x) \geq \frac{5}{9}.1089$

$66x - x^2 \geq 605$

$\Rightarrow x^2 - 66x + 605 \leq 0$

$\Rightarrow \left(x - 61\right)\left(x - 5\right) \leq 0$

$x \in [5,\ 61]$

$A = \{6,\ 9,\ 12,\ ....60\}$

$x\left(A\right) = 19$

$x\left(S\right) = 57$

$\therefore\ P\left(A\right) = \frac{1}{3}$

Hence, the correct answer is option (2).

Solution 76

$$\vec{b}\left(\vec{a}.\vec{c}\right)-\vec{c}\left(\vec{a}.\vec{b}\right)=\frac{\vec{b}-\vec{c}}{2}$$

$$\vec{a}.\vec{c}=\frac{1}{2},\ \vec{a}.\vec{b}=\frac{1}{2}$$

$$\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=\left(\vec{b}.\vec{d}\right)\left(\vec{a}.\vec{c}\right)-\left(\vec{a}.\vec{d}\right)\left(\vec{b}.\vec{c}\right)$$

$$=\left(\vec{a}.\vec{b}\right)\left(\vec{a}.\vec{c}\right)$$

$$=\frac{1}{4}$$

Hence, the correct answer is option (1).

Solution 77

$$T_r={}^{10}C_r x^r$$

$$\text{Coefficient of } x^{10-r}={}^{10}C_{10-r}={}^{10}C_r$$

$$\sum_{r=1}^{10} r^3\left(\frac{{}^{10}C_r}{{}^{10}C_{r-1}}\right)^2$$

$$=\sum_{r=1}^{10} r^3\left(\frac{11-r}{r}\right)^2\Rightarrow\sum r(11-r)^2$$

$$\Rightarrow\sum r\left(121+r^2-22r\right)$$

$$\Rightarrow\sum 121r+\sum r^3-22\sum r^2$$

$$\Rightarrow 121\times\frac{10\times11}{2}+\left(\frac{10\times11}{2}\right)^2-22\times\left(\frac{10\times11\times21}{6}\right)$$

$$=6655+3025-8470$$

$$=1210$$

Hence, the correct answer is option (1).

Solution 78

$$\frac{dy}{dx}=\frac{y}{x}\left(1+xy^2\left(1+\log_e x\right)\right),\ y\left(1\right)=3$$

$$\Rightarrow\ \frac{1}{y^3}\frac{dy}{dx}-\frac{1}{x}.\frac{1}{y^2}=\left(1+\ln x\right)-\frac{1}{y^2}=t$$

$$\Rightarrow\frac{2}{y^3}\frac{dy}{dx}=\frac{dt}{dx}$$

$$\Rightarrow\frac{1}{2}\frac{dt}{dx}+\frac{t}{x}=1+\ln\ x$$

$$\Rightarrow\frac{dt}{dx}+\frac{2t}{x}=2\left(1+\ln x\right)$$

$$\text{IF}=x^2$$

$$t.x^2=\int(1+\ln x)x^2dx$$

$$\Rightarrow-\frac{1}{y^2}.x^2=2\left[\frac{x^3}{3}\left(1+\ln x\right)-\frac{x^3}{9}\right]+c$$

$$y\left(1\right)=3$$

$$\Rightarrow c=-\frac{5}{9}$$

$$\therefore\ \frac{x^2}{y^2}=-2\left(\frac{x^3}{3}\left(1+\ln x\right)-\frac{x^3}{9}\right)+\frac{5}{9}$$

$$\Rightarrow\frac{y^2}{9}=\frac{x^2}{5-2x^3(2+\ln x^3)}$$

Hence, the correct answer is option (3).

Solution 79

$$\left|z-z_1\right|^2-\left|z-z_2\right|^2=\left|z_1-z_2\right|^2$$

$$\Rightarrow\left(x-2\right)^2+\left(y-3\right)^2-\left(x-3\right)^2-\left(y-4\right)^2=1+1$$

$$\Rightarrow-4x+4+9-6y-9+6x-16+8y=2$$

$$\Rightarrow 2x+2y=14$$

$$\Rightarrow x+y=7$$

Hence, the correct answer is option (3).

Solution 80

$$\text{Let } \overrightarrow{b} = \mu\overrightarrow{a} + \lambda\hat{j}$$

$$\text{Now } \overrightarrow{b}\,.\,\overrightarrow{a} = 0$$

$$\Rightarrow \left(\mu\overrightarrow{a} + \lambda\hat{j}\right).\,\overrightarrow{a} = 0$$

$$\Rightarrow \mu\left|\overrightarrow{a}\right|^2 + 2\lambda = 0 \Rightarrow 6\mu + 2\lambda = 0 \quad \text{..... (i)}$$

$$\Rightarrow \overrightarrow{b} = \lambda\left(\overrightarrow{a} - 3\hat{j}\right) = \lambda\left(-\hat{i} - \hat{j} + \hat{k}\right)$$

$$\Rightarrow \left|\overrightarrow{b}\right| = \left|\overrightarrow{a}\right| \Rightarrow \lambda = \pm\sqrt{2}$$

$$\therefore \;\overrightarrow{b} = -\sqrt{2}\left(-\hat{i} - \hat{j} + \hat{k}\right)$$

$$\therefore 3\overrightarrow{a} + \sqrt{2\overrightarrow{b}} = 3\left(-\hat{i} + 2\hat{j} + \hat{k}\right) - 2\left(-\hat{i} - \hat{j} + \hat{k}\right) = -\hat{i} + 8\hat{j} + \hat{k}$$

$$\therefore \text{projection } 3\sqrt{2}$$

Hence, the correct answer is option (4).

Solution 81

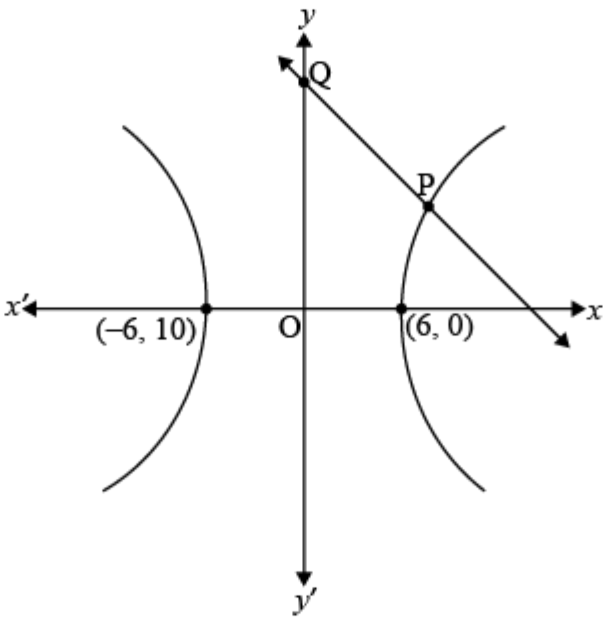
Let the equation of the plane is
$(x - 2y - z - 5) + \wedge(x + y + 3z - 5) = 0$ ...(i)
∵ it's parallel to the line
$x + y + 2z - 7 = 0 = 2x + 3y + z - 2$

So, vector along the line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix}$

$= -5\hat{i} + 3\hat{j} + \hat{k}$
∵ Plane is parallel to line
$\therefore -5(1 + \lambda) + 3(-2 + \lambda) + 1(-1 + 3\lambda) = 0$
$\lambda = 12$

So, by (i)
$13x + 10y + 35z = 65$
∴ $a = 13$, $b = 10$, $c = 35$
and $d = \frac{26+20-35+16}{\sqrt{9}} = 9$

Solution 82



$a = 6,\ e = \frac{\sqrt{5}}{2}$

$\therefore\ \frac{5}{4} = 1 + \frac{b^2}{36} \Rightarrow b^2 = 36 \times \frac{1}{4} = 9\theta$

$\therefore\ H\ :\ \frac{x^2}{36}\ - \frac{y^2}{9} = 1$

$P\left(6\sec\ \theta,\ 3\tan\ \theta\right)$

Slope of tanfent at $P = \frac{6\sec\ \theta}{4\times3\tan\ \theta}$

So, $\frac{1}{2\sin\theta} \times -\sqrt{2} = -1 \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\mathrm{Q} = 45^{\circ}\left(\text{for first quad}\right)$

$\therefore\ P = \left(6\sqrt{2},\ 3\right)$ and $\mathrm{N}\ :\ \sqrt{2\mathrm{x}}\ + y = 15$

$\therefore\ \mathrm{Q}\left(0,\ 15\right)$ Now, $PQ^2 = 72\ +\ 144 = 216$

Solution 83

$S=\left\{a: \log_2\left(9^{2a-4}+13\right)-\log_2\left(\frac{5}{2}.3^{2a-4}+1\right)=2\right\}$

So,

$\frac{9^{2a-4}+13}{\frac{5}{2}3^{2a-4}+1}=4 \Rightarrow 9^{2a-4}+13=103^{2a-4}+4$

Let $3^{2a-4}=t$ then $t^2-10t+9=0$

$(t-9)(t-1)=0$

$\therefore\ 3^{2a-4}=3^2$ or $3^{2a-4}=3^0$

$\therefore\ a=3,\ 2$

Now equation

$x^2-50x+25\beta=0$

$D\geq 0 \Rightarrow (50)^2-4\times 25\beta\geq 0$

$\beta\leq 25$

$\therefore$ Max. $\beta=25$

Solution 84

$f(x)=ax-3$

$g(x)=x^b+c$

$(fog)^{-1}=\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

$(fog)^{-1}(x)=\left(\frac{x+3-ca}{a}\right)^{\frac{1}{b}}=\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

$\Rightarrow a=2,\ b=3,\ c=5$

$fog(ac)+(gof)(b)$

$\because f(x)=2x-3$

$g(x)=x^3+5$

$fog(10)+gof(3)$

$=2007+32$

$=2039$

Solution 85

Type	**Numbers**
$5k$	5, 10, 15, 20, 25
$5k+1$	1, 6, 11, 16, 21
$5k+2$	2, 7, 12, 17, 22
$5k+3$	3, 8, 13, 18, 23

$5k + 4$ 4, 9, 14, 19, 24

To select x and y.

Case I : 1 of ($5k + 1$) and 1 of ($5k + 4$) = 5 × 5 = 25

Case II : 1 of ($5k + 2$) and 1 of ($5k + 3$) = 5 × 5 = 25

Case III : Both of type 5k (both cannot be same) = 5 × 4 = 20

Total = 120

Solution 86

Constant term in the expansion of

$$\left(2x+\frac{1}{x^7}+3x^2\right)^5$$

$$\frac{1}{x^{35}}\left(2x^8+1+3x^9\right)^5$$

$$\frac{1}{x^{35}}\left(1+x^8\left(3x+2\right)\right)^5$$

$$\text{Term independent of } x = \text{coefficient of } x^{35} \text{ in}$$

$$^5C_4\left(x^8\left(3x+2\right)\right)^4$$

$$=^5C_4 \text{ coefficient of } x^3 \text{ in } (2+3x)^4$$

$$=^5\mathrm{C}_4 \times ^4\mathrm{C}_3(2)^1(3)^3$$

$$= 5\times4\times2\times27$$

$$= 1080$$

Solution 87

Out of the given numbers one is ($3k$) type and 3 of ($3k + 1$) type and remaining 3 are ($3k + 2$) type Number of subsets of 1 element = 1

(1 of $3k$ type)

Number of subsets of 2 elements

1 of ($3k$ +1) type + 1 of ($3k$ + 2) type = 9

Number of subsets of 3 elements

1 of $3k$ type + 1 of ($3k$ + 1) type + 1 of ($3k$ +2) type = 9

3 of ($3k$ +1) type = 1

3 of ($3k$ + 2) type = 1

Number of subsets of 4 elements

1 of $3k$ type + 3 of ($3k$ +1) type = 1

1 of $3k$ type + 3 of ($3k$ +2) type = 1

2 of ($3k$ + 1) type + 2 of ($3k$ + 2) type = 9

Number of subsets of 5 elements

1 of $3k$ + 2 of ($3k$ +1) type + 2 of ($3k$ +2) type = 9

Number of subsets of 6 elements

3 of ($3k$ + 1) type + 3 of ($3k$ + 2) type = 1

The set itself = 1

Total = 43

Solution 88

Case-I

−1< x < 0

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$$

$$\tan^{-1}\frac{2x}{1-x^2}=\frac{-\pi}{3}$$

$$2\tan^{-1}x=\frac{-\pi}{3}$$

$$\tan^{-1}x=\frac{-\pi}{6}$$

$$x=\frac{-1}{\sqrt{3}}$$

$\mathbf{Case - II}$

$0<x<1$

$$\tan^{-1}\frac{2x}{1-x^2}+\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$$

$$\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{6}$$

$$2\tan^{-1}x=\frac{\pi}{6}$$

$$\tan^{-1}x=\frac{\pi}{12}$$

$$x=2-\sqrt{3}$$

$$\text{Sum} \;=\; \frac{-1}{\sqrt{3}}+2-\sqrt{3}=2-\frac{4}{\sqrt{3}}$$

$$\Rightarrow \alpha=2$$

Solution 89

a = A + 6d
b = A+8d +1
c = A+16d +2

$$\begin{vmatrix} a & 7 & 1 \\ 26 & 17 & 1 \\ c & 17 & 1 \end{vmatrix} = -70$$

$$\Rightarrow \begin{vmatrix} A+6d & 7 & 1 \\ 2A+16d+2 & 17 & 1 \\ A+16d+2 & 17 & 1 \end{vmatrix} = -70$$

$R_3 \to R_3 - R_2,\ R_2 \to R_2 - R_1$

$$\Rightarrow \begin{vmatrix} A+6d & 7 & 1 \\ A+10d+2 & 10 & 0 \\ -A & 0 & 0 \end{vmatrix} = -70$$

$\Rightarrow A = -7$

$a = A + 6d = 29 \Rightarrow d = 6$

$b = -7 + 48 + 1 = 42$

$c = -7 + 96 + 2 = 91$

$c - a - b = 91 - 29 - 42 = 20$

$\text{Sum} = \frac{20}{2}\left[2 \times 20 + 19 \times \frac{6}{12}\right] = 10\left[40 + \frac{19}{2}\right] = 495$

Solution 90

$x^2 + 6 = \frac{5}{2}x^2 \Rightarrow x = \pm 2$

Area between P_1 and P_2 $\quad$ $[\text{Say} A_1]$

$= \int\limits_{-2}^{2} \left(x^2 + 6\right) - \frac{5}{2}x^2 dx$

$= 2\int\limits_{0}^{2} \left(6 - \frac{3}{2}x^2\right) dx = 2\left[6x - \frac{x^3}{2}\right]_0^2 = 16$

$ax = \frac{5}{2}x^2 \Rightarrow x = 0,\ \frac{2a}{5}$

Area between P_1 and $y = ax$ $\quad$ $[\text{Say } A_2]$

$= \int\limits_{0}^{\frac{2a}{5}} ax - \frac{5}{2}x^2 dx$

$= \left.\frac{ax^2}{2} - \frac{5}{6}x^3\right]_0^{\frac{2a}{5}} : \frac{2a^3}{75}$

$A_1 = A_2 \Rightarrow \frac{2a^3}{75} = 16$

$a^3 = 600$