JEE Main 24 June 2022(Second Shift)

Total Time: 180

Total Marks: 300.0

Solution 1

$\text{Velocity gradient} = \frac{dv}{dx}$

$\Rightarrow \text{Dimensions are} \frac{[LT^{-1}]}{[L]} = [T^{-1}]$

Decay constant λ has dimensions of $[T^{-1}]$ because of the relation $\frac{dN}{dt} = -\lambda N$
⇒ Velocity gradient and decay constant have same dimensions.

Hence, the correct answer is option A.

Solution 2

We know that

$T^2 \propto R^3$

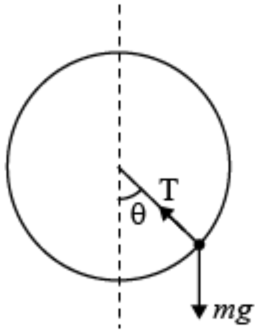
$\Rightarrow \left(\frac{T'}{T}\right)^2 = \left(\frac{3R}{R}\right)^3$

$\Rightarrow \frac{T'}{T} = 3\sqrt{3}$

$\Rightarrow T' = 3\sqrt{3} \text{ years}$

Hence, the correct answer is option D.

Solution 3



At any θ : $T - mg\ \cos\theta = \frac{mv^2}{R}$

$\Rightarrow T = mg\ \cos\theta + \frac{mv^2}{R}$

Since *v* is constant,

⇒ *T* will be minimum when cosθ is minimum.

⇒ θ = 180° corresponds to $T_{minimum}$.

Hence, the correct answer is option B.

Solution 4

According to given information :

$$\frac{kQ^2}{L^2} = \mu mg$$

Putting the values, we get

L = 12 cm

Hence, the correct answer is option A.

Solution 5

$$\begin{aligned}\text{Efficiency } \eta &= 1 - \frac{T}{T_H} \\ &= 1 - \frac{400}{1000} \\ &= 0.6\end{aligned}$$

$$\Rightarrow 0.6 = \frac{W}{Q}$$

⇒ $W = 0.6Q = 3000$ kcal = 12.6×10^6 J

Hence, the correct answer is option C.

Solution 6

$\omega_1 A_1 = \omega_2 A_2$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1}$$

$$= \sqrt{\frac{k_2}{m_2}} \times \sqrt{\frac{m_1}{k_1}} = \sqrt{\frac{9k}{100} \times \frac{50}{2\,k}} = \frac{3}{2}$$

Hence, the correct answer is option B.

Solution 7

$$\text{Req} = \frac{2\times4}{2+6} + 6 = \frac{22}{3}$$

⇒ A and B are in parallel and C is in series.

Hence, the correct answer is option B.

Solution 8

Electromagnet requires high permeability and low retentivity.

Hence, the correct answer is option C.

Solution 9

$$\therefore\ r = \frac{mv}{qB} = \frac{\sqrt{2m(KE)}}{qB}$$

$$\Rightarrow r_1 : r_2 : r_3 = \frac{\sqrt{m_1}}{q_1} : \frac{\sqrt{m_2}}{q_2} : \frac{\sqrt{m_3}}{q_3}$$

$$= \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2}$$

$$= 1 : \sqrt{2} : 1$$

Hence, the correct answer is option D.

Solution 10

$X = |X_C - X_L|$
So, it can be zero if $X_C = X_L$
And, average power in ac circuit can be zero.

Hence, the correct answer is option C.

Solution 11

For equilibrium

$$-\frac{dU}{dr} = 0 = \frac{10A}{r^{11}} - \frac{5B}{r^6}$$

$$\Rightarrow r^5 = \frac{2A}{B}$$

$$\text{And } r = \left(\frac{2A}{B}\right)^{\frac{1}{5}}$$

Hence, the correct answer is option C.

Solution 12

Let time taken to ascent is t_1 and that to descent is t_2. Height will be same so

$$H = \frac{1}{2} \times 12t_1^2 = \frac{1}{2} \times 8t_2^2$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{\sqrt{2}}{\sqrt{3}}$$

Hence, the correct answer is option B.

Solution 13

$$\theta_1 = \frac{1}{2}\alpha\left(2 \times 1 - 1\right) = 5 \text{ rad}$$

⇒ α = 10 rad/sec^2

$$\text{So } \theta_2 = \frac{1}{2} \times \alpha \ (2 \times 2 - 1) = 15 \text{ rad}$$

Hence, the correct answer is option B.

Solution 14

$$\frac{1}{2} \times 1.5 \times 60^2 \times \frac{1}{4} = 100 \times 0.42 \times \triangle T$$

$$\triangle T = \frac{1.5 \times 60^2}{8 \times 100 \times 0.42} = 16.07^\circ \text{C}$$

Hence, the correct answer is option C.

Solution 15

Let initially the charge is q so

$$\frac{1}{2}\frac{q^2}{C} = U_i$$

$$\text{And } \frac{1}{2}\frac{(q+2)^2}{C} = U_f$$

$$\text{Given } \frac{U_f - U_i}{U_i} \times 100 = 44$$

$$\frac{(q+2)^2 - q^2}{q} = .44$$

$$\Rightarrow q = 10\text{C}$$

Hence, the correct answer is option A.

Solution 16

$$\frac{mv^2}{r} = \frac{2k\rho \times \pi R^2 q}{r}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{\rho R^2 q}{4\varepsilon_0}$$

Hence, the correct answer is option A.

Solution 17

$$200 \times \frac{1}{4\pi \times 16} \times \frac{3.5}{100} = \frac{B_0^2}{2\mu_0}C$$

$\mu_0 = 4\pi \times 10^{-7}$ Tm/A

$C = 3 \times 10^8$ m/sec

$\Rightarrow B_0 = 1.71 \times 10^{-8}$ T

Hence, the correct answer is option B.

Solution 18

$$3.8 = 0.6 + \frac{1}{2}mv_1^2$$

$$1.4 = 0.6 + \frac{1}{2}mv_2^2$$

$$\Rightarrow \frac{v_1^2}{v_2^2} = \frac{3.2}{0.8} = \frac{4}{1}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{2}{1}$$

Hence, the correct answer is option B.

Solution 19

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{5}{1}\right)^2$$

$$= \frac{25}{1}$$

Hence, the correct answer is option D.

Solution 20

$T.E. = \frac{-Z^2 m e^4}{8(nh\varepsilon_0)^2}$

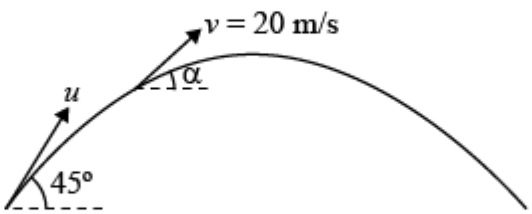
$P.E. = \frac{-Z^2 m e^4}{4(nh\varepsilon_0)^2}$

$K.E. = \frac{Z^2 m e^4}{8(nh\varepsilon_0)^2}$

As electron makes transition to higher level, total energy and potential energy increases (due to negative sign) while the kinetic energy reduces.

Hence, the correct answer is option B.

Solution 21



⇒ $v \cos\alpha = u \cos 45°$ ...(i)

& $v \sin\alpha = u \sin 45° - gt$ ...(ii)

Solve for u we get

$u = 20\sqrt{2}\ \mathrm{m/s}$

$\Rightarrow H = \frac{u^2 \sin^2 45^\circ}{20} = 20\ \mathrm{m}$

Hence, the correct answer is 20.

Solution 22

We know that $v = f\lambda$

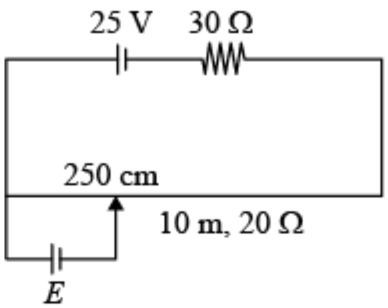
Putting the values,

$\frac{3\times10^8}{\sqrt{6.25}} = f \times 20 \times 10^{-3}$

⇒ $f = 6 \times 10^9$ Hz

Hence, the correct answer is 6.

Solution 23



$$\therefore\ E = l \times \left(\frac{20}{4}\right) = \frac{25}{(30+20)} \times \left(\frac{20}{4}\right)$$

$$= \frac{1}{2} \times 5 = 2.5\ \text{volts}$$

$$= \frac{25}{10}\ \text{volts}$$

Hence, the correct answer is 25.

Solution 24

$A = |10 \cos(\pi x)|$

At $x = \frac{4}{3}$

$$A = \left|10\ \cos\ \left(\pi \times \frac{4}{3}\right)\right|$$

$$= |-5\ \text{cm}|$$

$\therefore$ Amp = 5 cm

Hence, the correct answer is 5.

Solution 25

V_L = 5 V as V_Z = 5 V

$$\therefore\ I_L = \frac{V_L}{R_L} = \frac{5}{10^3} = 5\ \text{mA}$$

Hence, the correct answer is 5.

Solution 26

$$N_1 = \frac{\left(\frac{10^{-2}}{1}\right)}{2^4}$$

$$N_2 = \frac{\left(\frac{10^{-2}}{2}\right)}{2^2}$$

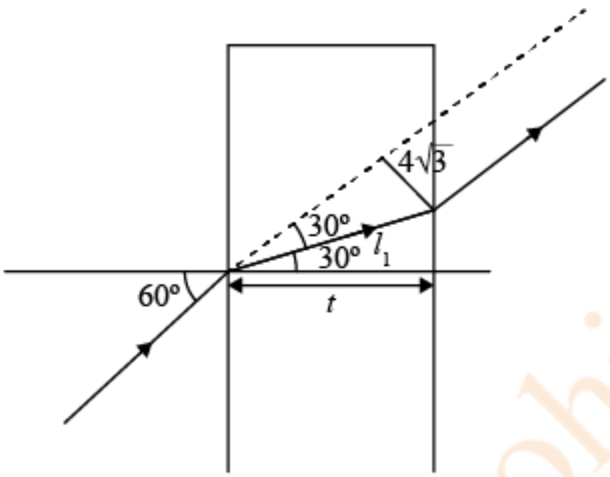
$$\Rightarrow \frac{N_1}{N_2} = \frac{1}{2}$$

∴ Mass ratio of *A* and *B*,

$$\frac{m_1}{m_2} = \frac{N_1}{N_2} \times \left(\frac{M_1}{M_2}\right)$$
$$= \frac{1}{2} \times \left(\frac{1}{2}\right)$$
$$= \frac{1}{4}$$
$$= \frac{25}{100}$$
$$\therefore\ x = 25$$

Hence, the correct answer is 25.

Solution 27



$$1 \times \sin 60^\circ = \sqrt{3} \times \ \sin\ r$$
$$\Rightarrow r\ =\ 30º$$
$$\therefore\ l_1 = 4\sqrt{3} \times 2$$
$$= 8\sqrt{3}\ \text{cm}$$

∴ Thickness, $t = l_1 \cos 30º$

$$= 8\sqrt{3} \times \frac{\sqrt{3}}{2}$$
$$= 4 \times 3$$
$$= 12\ \text{cm}$$

Hence, the correct answer is 12.

Solution 28

V_{max} = NABω

$= 1000 \times 1 \times 0.07 \times (2\pi \times 1)$

$\cong 440$ volts

Hence, the correct answer is 440.

Solution 29

By 1^{st} law,

$$\Delta U = \Delta Q - \frac{\Delta Q}{4} = \frac{3}{4}\,\Delta\,Q$$
$$\Rightarrow nC_v\,\Delta\,T = \frac{3}{4}nC\,\Delta\,T$$
$$\Rightarrow C = \frac{4C_v}{3} = 2R$$

Hence, the correct answer is 2.

Solution 30

Temperature of surrounding = 20ºC

For 0 → 6 minutes, average temp. = 70ºC

→ Rate of cooling ∝ 70ºC – 20ºC = 50ºC

For 6 → t_2 minutes, average temp. = 50ºC

→ Rate of cooling ∝ 30ºC

⇒ $t_2 - 6 = \frac{5}{3}$ = (6 minutes)

⇒ $t_2 = 16$ minutes

Hence, the correct answer is 16.

Solution 31

Mass of organic compound = 120 g

Mass of CO_2 = 330 g

Moles of CO_2 = $\frac{330}{44} = 7.5$

Mass of carbon = 7.5 × 12 = 90 gm

Percentage of C = $\frac{90 \times 100}{120} = 75\%$

Mass of H_2O = 270 g

Moles of H_2O = $\frac{270}{18} = 15$

Mass of hydrogen = 15 × 2 = 30 gm

Percentage of H = $\frac{30\times100}{120} = 25\%$

Hence, the correct answer is option D.

Solution 32

Wavelength of radiation = 300 nm

$$\text{Photon energy} = \frac{\text{hc}}{\lambda}$$
$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{300\times10^{-9}}$$
$$= 6.63 \times 10^{-19}\ \text{J}$$

Energy of 1 mole of photons

= $6.63 \times 10^{-19} \times 6.02 \times 10^{23} \times 10^{-3}$

= 399 kJ

Hence, the correct answer is option C.

Solution 33

$$C_2^{2-} : \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \pi_{2p_x}^2 = \pi_{2p_y}^2 \sigma_{2p_z}^2$$
$$N_2^{2-} : \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \sigma_{2p_z}^2\ \pi_{2p_x}^2 = \pi_{2p_y}^2 \pi_{2p_x}^{*1} = \pi_{2p_y}^{*1}$$
$$O_2^{2-} : \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \sigma_{2p_z}^2\ \pi_{2p_x}^2 = \pi_{2p_y}^2 \pi_{2p_x}^{*2} = \pi_{2p_y}^{*2}$$

$$\text{B.O.}\ (C_2^{2-}) = 3;\ \text{B.O.}\ (N_2^{2-}) = 2;\ \text{B.O.}\ (O_2^{2-}) = 1$$

Hence, the correct answer is option B.

Solution 34

2C (graphite) + $3H_2(g) \rightarrow C_2H_6(g)$
ΔH_r = +1560 + 2(−394) +3(−286)
= −86.0 kJ mol^{-1}
Enthalpy of formation of $C_2H_6(g)$ = −86.0 kJ mol^{-1}

Hence, the correct answer is option C.

Solution 35

A → Products

For a first order reaction,

$$t_{\frac{1}{2}} = \frac{\ln 2}{k} = \frac{0.693}{k}$$

Time for 90% conversion,

$$t_{90\%} = \frac{1}{k}\ln\frac{100}{10} = \frac{\ln 10}{k} = \frac{2.303}{k}$$
$$t_{90\%} = \frac{2.303}{0.693}t_{\frac{1}{2}} = 3.32t_{\frac{1}{2}}$$

Hence, the correct answer is option C.

Solution 36

Melting points of the given metals

Hg : −38.83° C

Ag : 961.8° C

Ga : 29.76° C

Cs : 28.44° C

∴ Metal having highest melting point is Ag.

Hence, the correct answer is option B.

Solution 37

Hall-Herault process is used for the extraction of aluminium by electrolysis molten $Al2O_3$
$2Al2O_3 + 3C \rightarrow 4Al + 3CO_2$

Hence, the correct answer is option B.

Solution 38

Molecular hydrogen is produced as a byproduct in the industrial production of NaOH by electrolysis of *aq* NaCl solution

$NaCl \rightarrow Na^+ + Cl^-$

$$H_2O \rightleftharpoons H^+ + OH^-$$

Cathode : $2H_2O + 2e \rightarrow H_2 + 2OH^-$

Anode : $2Cl^- \rightarrow Cl_2 + 2e$

NaOH is crystallised from the remaining part of electrolyte.

Hence, the correct answer is option A.

Solution 39

Baking soda ($NaHCO_3$) is used in certain type of fire extinguishers because it decomposes at high temperature to produce CO_2 which extinguishes fire

$$2\,NaHCO_3\left(s\right) \xrightarrow{\Delta} Na_2\,CO_3\,(s) + H_2O \uparrow + CO_2 \uparrow$$

Hence, the correct answer is option A.

Solution 40

PCl_5 is well known but NCl_5 is not because nitrogen does not have vacant *d*-orbitals in its valence shell.
So, nitrogen cannot expand its octet. On the other hand phosphorus has vacant *d*-orbitals in its valence shell which enables it to expand its octet.

Hence, the correct answer is option B.

Solution 41

Crystal field splitting (Δ_0) for octahedral complexes depends on oxidation state of the metal as well as to which transition series the metal belongs. For the same oxidation state, the crystal field splitting (Δ_0) increases as we move from $3d \rightarrow 4d \rightarrow 5d$. Cr^{3+} and Fe^{3+} belong to 3*d* series, Mo^{3+} belongs to 4*d* series and Os^{3+} belongs to 5*d* series. Therefore crystal field splitting (Δ_0) is highest for $[Os(H_2O)_6]^{3+}$.

Hence, the correct answer is option D.

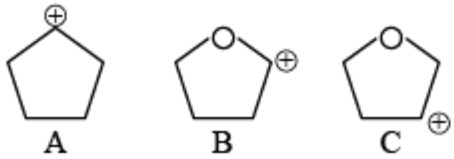
Solution 42

Among the given gases, the green house gases which are responsible for heating the atmosphere are CH_4, water vapour and ozone. Nitrogen is not a green house gas.

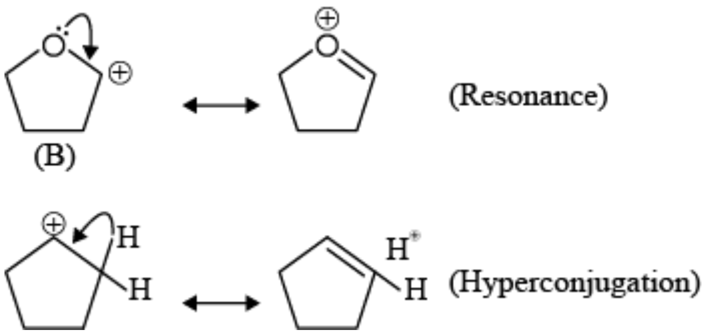
Hence, the correct answer is option D.

Solution 43

The given carbocations are



Carbocation (A) is stabilised by hyperconjugation due to 4 α hydrogen atoms. Carbocation (C) is also stabilised by hyperconjugation due to 4 α hydrogen atoms but destabilised by –I effect of O-atom. Carbocation (B) is most stable as it is stabilised by resonance.



∴ Correct decreasing order of stability is

B > A > C

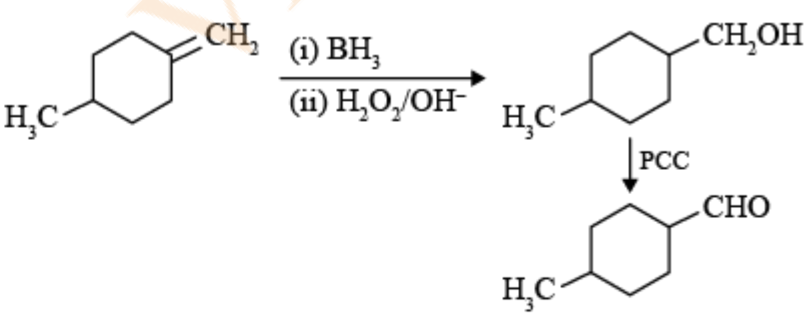
Disclaimer: None of the given options is correct.

Solution 44

The π-bond present is alkenes is weaker than σ-bond present in alkanes. That makes alkenes less stable than alkanes. Therefore, statement-I is correct. Carbon-carbon double bond is stronger than Carbon-carbon single bond because more energy is required to break 1 sigma and 1 pi bond than to break 1 sigma bond only. Therefore, statement-II is also correct.

Hence, the correct answer is option A.

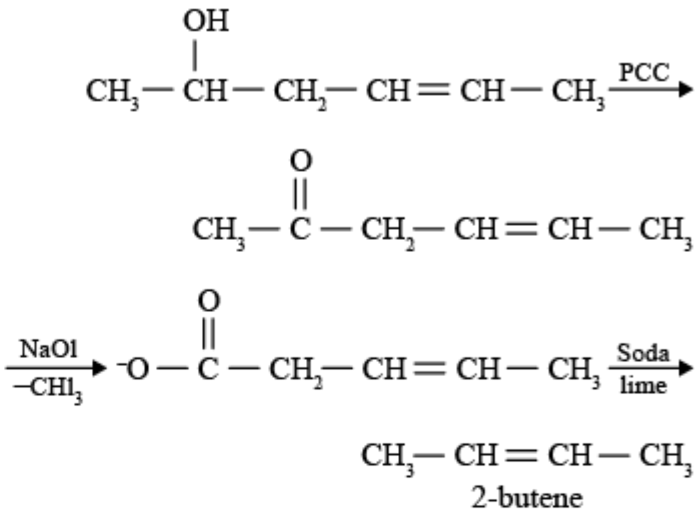
Solution 45



The first step involves addition of H_2O to alkene according to anti-markownikoff's rule while the second step involves oxidation of 1° alcohol to aldehyde.

Hence, the correct answer is option C.

Solution 46



Hence, the correct answer is option C.

Solution 47

$$\underset{\text{Propan}-1-\text{ol}}{CH_3-CH_2-CH_2-OH} \xrightarrow{SOCl_2}$$

$$CH_3-CH_2-CH_2-Cl \xrightarrow{KCN}$$

$$CH_3-CH_2-CH_2-CN \xrightarrow[Na(Hg)/C_2H_5OH]{H_2/Ni}$$

$$\underset{n-\text{butylamine}}{CH_3-CH_2-CH_2-CH_2-NH_2}$$

Hence, the correct answer is option A.

Solution 48

Nylon 6, 6 is a condensation polymer of hexamethylene diamine and adipic acid
Dacron is a condensation polymer of terephthalic acid and ethylene glycol.
Buna-N is an addition polymer of 1, 3-butadiene and acrylonitrile
Silicone is a condensation polymer of dialkyl silanediol.

Hence, the correct answer is option C.

Solution 49

The given structure is that of cimetidine which is well known antacid.

Hence, the correct answer is option C.

Solution 50

Cupric salts give green flame with blue centre. The colour of other salts are

Sr^{2+}	Crimson red
Ca^{2+}	Brick red
Ba^{2+}	Green

Hence, the correct answer is option A.

Solution 51

V_1, Volume of 0.2 g H_2 at 200 K = $\frac{0.2\times R\times 200}{2\times P}$

V_2, Volume of 3.0 g of gas A at 300 K = $\frac{3.0\times R\times 300}{M\times P}$

$V_1 = V_2$ (Given)

$$\frac{0.2\times R\times 200}{2\times P} = \frac{3.0\times R\times 300}{M\times P}$$

$\therefore M = 45 \text{ g mol}^{-1}$

Hence, the correct answer is 45.

Solution 52

According to Henry's law, partial pressure of a gas is given by

$P_g = (K_H)\, X_g$

where X_g is mole fraction of gas in solution

$$0.835 = 1.67 \times 10^3\ (X_{CO_2})$$

$$X_{CO_2} = 5 \times 10^{-4}$$

Mass of CO_2 in 1 L water = 1221×10^{-3} g

Hence, the correct answer is 1221.

Solution 53

$$PCl_5\,(g) \rightleftharpoons PCl_3\,(g) + Cl_2\,(g)$$

Initial moles	5		
Equilibrium moles	$5 - x$	x	x

Number of moles of N_2 = 2

Equilibrium pressure = 2.46 atm

$$P_{eq} = \frac{(7+x)\times0.082\times600}{200} = 2.46$$

On solving, x = 3

$$\therefore\ KP = \frac{\left(\frac{3P}{10}\right)\left(\frac{3P}{10}\right)}{\left(\frac{2P}{10}\right)} = \frac{9\times2.46}{20}$$

$$= 1107 \times 10^{-3}\ \text{atm}$$

Hence, the correct answer is 1107.

Solution 54

Molarity of KCl solution	= 0.1 M
Resistance	= 1750 ohm
Conductivity	= 0.152×10^{-3} S cm^{-1}
Conductivity	= $\frac{\text{Cell constant}}{\text{Resistance}}$
∴ Cell constant	= $0.152 \times 10^{-3} \times 1750$
	= 266×10^{-3} cm^{-1}

Hence, the correct answer is 266.

Solution 55

Mass of wood charcoal = 0.6 g

Initial moles of acetic acid = 0.2 × 0.2 = 0.04

Final moles of acetic acid = 0.1 × 0.2 = 0.02

Moles of acetic acid adsorbed = 0.04 – 0.02

= 0.02

Mass of acetic acid adsorbed per gm of charcoal $= \frac{0.02\times60}{0.6} = 2.0\ g$

Hence, the correct answer is 2.

Solution 56

Baryte	$BaSO_4$
Galena	PbS
Zinc blende	ZnS
Copper pyrites	$CuFeS_2$

Of the given minerals, only 3 are sulphide based.

Hence, the correct answer is 3.

Solution 57

Manganese (VI) disproportionates in acidic medium as

$$3\,MnO_4^{2-} + 4H^+ \longrightarrow 2\,MnO_4^- + MnO_2{}^+ 2H_2O$$

Difference in oxidation states of Mn in the products formed = 7 – 4 = 3

Hence, the correct answer is 3.

Solution 58

Mass of organic compound = 0.2 g

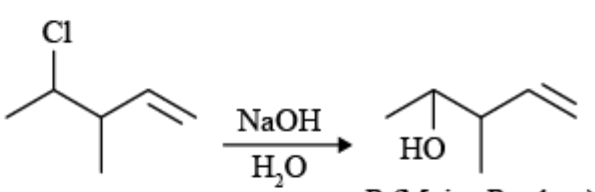
Volume of N_2 gas evolved at STP = 22.4 mL

$$\text{Mass of } N_2 \text{ gas evolved} = \frac{22.4\times10^{-3}\times28}{22.4} = 0.028 \text{ g}$$

Percentage of nitrogen in the compound $= \frac{0.028\times100}{0.2} = 14\%$

Hence, the correct answer is 14.

Solution 59



P (Major Product)

The given reaction undergoes nucleophilic substitution by SN_2 mechanism at room temperature

∴ No. of π electrons present in P = 2

Hence, the correct answer is 2.

Solution 60

The given pentapeptide is

ALA – GLY – LEU – ALA – VAL

It has 4 peptide linkages.

Hence, the correct answer is 4.

Solution 61

Given $x * y = x^2 + y^3$ and $(x * 1) * 1 = x * (1 * 1)$

So, $(x^2 + 1) * 1 = x * 2$

$\Rightarrow (x^2 + 1)^2 + 1 = x^2 + 8$

$\Rightarrow x^4 + 2x^2 + 2 = x^2 + 8$

$\Rightarrow (x^2)^2 + x^2 - 6 = 0$

$\therefore (x^2 + 3)(x^2 - 2) = 0$

$\therefore x^2 = 2$

$$\text{Now, } 2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)=2\sin^{-1}\left(\frac{4}{8}\right)$$
$$=2\cdot\frac{\pi}{6}$$
$$=\frac{\pi}{3}$$

Hence, the correct answer is option B.

Solution 62

Given equation : $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$
$\Rightarrow e^{2x} - 4 = 0$ or $6e^{2x} - 5e^x + 1 = 0$
$\Rightarrow e^{2x} = 4$ or $6(e^x)^2 - 3e^x - 2e^x + 1 = 0$
$\Rightarrow 2x = \ln 4$ or $(3e^x - 1)(2e^x - 1) = 0$

$$\Rightarrow x = \ln 2 \quad \text{or } e^x = \frac{1}{3} \text{ or } e^x = \frac{1}{2}$$
$$\text{or } x = \ln\left(\frac{1}{3}\right), \ -\ln 2$$

Sum of all real roots = ln2 – ln3 – ln2

= $-\ln 3$

Hence, the correct answer is option B.

Solution 63

Given system of equations

$x + y + az = 2$ .....(i)

$3x + y + z = 4$ .....(ii)

$x + 2z = 1$ .....(iii)

Solving (i), (ii) and (iii), we get

$x = 1$, $y = 1$, $z = 0$ (and for unique solution $a \neq -3$)

Now, $(\alpha, 1)$, $(1, \alpha)$ and $(1, -1)$ are collinear

$$\therefore \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \alpha(\alpha + 1) - 1(0) + 1(-1 - \alpha) = 0$

$\Rightarrow \alpha^2 - 1 = 0$

$\therefore \alpha = \pm 1$

$\therefore$ Sum of absolute values of $\alpha = 1 + 1 = 2$

Hence, the correct answer is option C.

Solution 64

$x, y > 0$ and $x^3y^2 = 2^{15}$

Now, $3x + 2y = (x + x + x) + (y + y)$

So, by A.M. ≥ G.M. inequality

$$\frac{3x+2y}{5} \geq \sqrt[5]{x^3 \cdot y^2}$$

$$\therefore\ 3x + 2y \geq 5\sqrt[5]{2^{15}}$$
$$\geq 40$$

$\therefore$ Least value of $3x + 4y = 40$

Hence, the correct answer is option D.

Solution 65

$$f(x)=\begin{cases}\frac{\sin(x-[x])}{x-[x]}, & x\in(-2,-1)\\ \max\{2x,\ 3[|x|]\}, & |x|<1\\ 1, & \text{otherwise}\end{cases}$$

$$f(x)=\begin{cases}\frac{\sin(x+2)}{x+2}, & x\varepsilon(-2,-1)\\ 0, & x\varepsilon(-1,\ 0]\\ 2x, & x\varepsilon(0,\ 1)\\ 1, & \text{otherwise}\end{cases}$$

It clearly shows that *f*(*x*) is discontinuous

At *x* = −1, 1 also non differentiable

and at *x* = 0, L.H.D = $\lim\limits_{h\to0}\frac{f(0-h)-f(0)}{-h}=0$

R.H.D = $\lim\limits_{h\to0}\frac{f(0+h)-f(0)}{h}=2$

∴ *f*(*x*) is not differentiable at *x* = 0

∴ *m* = 2, *n* = 3

Hence, the correct answer is option C.

Solution 66

$$I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{dx}{(1+e^{x})(\sin^6 x+\cos^6 x)}\qquad\ldots\ldots(\text{i})$$

$$I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{dx}{(1+e^{-x})(\sin^6 x+\cos^6 x)}\qquad\ldots\ldots(\text{ii})$$

(i) and (ii)

From equation (i) & (ii)

$$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{\sin^6 x+\cos^6 x}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{dx}{\sin^6 x+\cos^6 x} = \int_0^{\frac{\pi}{2}} \frac{dx}{1-\frac{3}{4}\sin^2 2x}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{4\ \sec^2\ 2xdx}{4+\tan^2\ 2x} = 2\int_0^{\frac{\pi}{4}} \frac{4\ \sec^2\ 2x}{4+\tan^2\ 2x}dx$$

when x = 0, t = 0

Now, tan2x = t

when, $x = \frac{\pi}{4},\ t \to \infty$

2sec^{2}2x dx = dt

$$\therefore\ I=2\int_0^{\infty} \frac{2dt}{4+t^2}$$
$$=2\left(\tan^{-1}\ \frac{t}{2}\right)_0^{\infty}$$
$$=2 \cdot \frac{\pi}{2}$$
$$=\pi$$

Hence, the correct answer is option C.

Solution 67

$$\lim_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+\ldots+\frac{n^2}{(n^2+n^2)(n+n)}\right)$$
$$=\lim_{n\to\infty}\ \sum_{r=1}^{n}\frac{n^2}{(n^2+r^2)(n+r)}$$
$$=\lim_{n\to\infty}\ \sum_{r=1}^{n}\frac{1}{n}\frac{1}{\left[1+\left(\frac{r}{n}\right)^2\right]\left[1+\left(\frac{r}{n}\right)\right]}$$
$$=\int_0^1\frac{1}{(1+x^2)(1+x)}dx$$
$$=\frac{1}{2}\int_0^1\left[\frac{1}{1+x}-\frac{(x-1)}{(1+x^2)}\right]dx$$
$$=\frac{1}{2}\left[\ln(1+x)-\frac{1}{2}\ln\left(1+x^2\right)+\tan^{-1}x\right]_0^1$$
$$=\frac{1}{2}\left[\frac{\pi}{4}+\frac{1}{2}\ln 2\right]$$
$$=\frac{\pi}{8}+\frac{1}{4}\ln 2$$

Hence, the correct answer is option A.

Solution 68

According to the question (Let $P(x, y)$)

$$2x-y\frac{dx}{dy}=0 \qquad \left(\begin{matrix}\because \text{ equation of tangent at} \\ P : y-y=\frac{dy}{dx}(y-x)\end{matrix}\right)$$

$$\therefore\ 2\frac{dy}{y}=\frac{dx}{x}$$

⇒ $2\ln y = \ln x + \ln c$

⇒ $y^2 = cx$

∵ this curve passes through (3, 3)

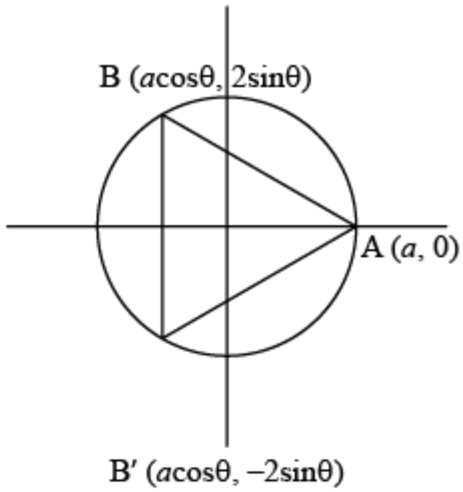
∴ $c = 3$

∴ required parabola is $y^2 = 3x$ and L.R = 3

Hence, the correct answer is option A.

Solution 69

Given ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1,\ a>2$



∴ Let $A(\theta)$ be the area of $\Delta ABB'$

$$\text{Then } A(\theta) = \frac{1}{2} 4\sin\theta \left(a + a\cos\theta\right)$$
$$A'(\theta) = a\left(2\cos\theta + 2\cos^2\theta\right)$$

For maxima $A'(\theta) = 0$

$$\Rightarrow \cos\theta = -1,\ \cos\theta = \frac{1}{2}$$

But for maximum area $\cos\theta = \frac{1}{2}$

$$\therefore\ A(\theta) = 6\sqrt{3}$$
$$\Rightarrow 2\frac{\sqrt{3}}{2}\left(a + \frac{a}{2}\right) = 6\sqrt{3}$$
$$\Rightarrow a = 4$$
$$\therefore\ e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{16}} = \frac{\sqrt{3}}{2}$$

Hence, the correct answer is option A.

Solution 70

∵ $A(1, \alpha)$, $B(\alpha, 0)$ and $C(0, \alpha)$ are the vertices of ΔABC and area of $\Delta ABC = 4$

$$\therefore \left|\frac{1}{2}\begin{vmatrix} 1 & \alpha & 1 \\ \alpha & 0 & 1 \\ 0 & \alpha & 1 \end{vmatrix}\right| = 4$$

$$\Rightarrow \left|1(1-\alpha) - \alpha(\alpha) + \alpha^2\right| = 8$$

$$\Rightarrow \alpha = \pm 8$$

Now, $(\alpha, -\alpha)$, $(-\alpha, \alpha)$ and (α^2, β) are collinear

$$\therefore \begin{vmatrix} 8 & -8 & 1 \\ -8 & 8 & 1 \\ 64 & \beta & 1 \end{vmatrix} = 0 = \begin{vmatrix} -8 & 8 & 1 \\ 8 & -8 & 1 \\ 64 & \beta & 1 \end{vmatrix}$$

$\Rightarrow 8(8 - \beta) + 8(-8 - 64) + 1(-8\beta - 8 \times 64) = 0$

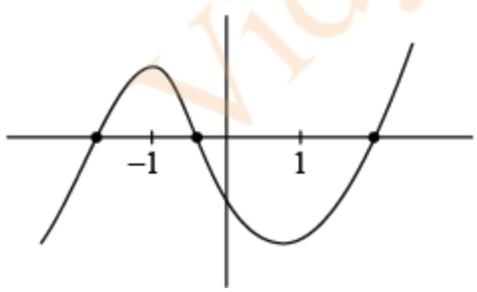
$\Rightarrow 8 - \beta - 72 - \beta - 64 = 0$

$\Rightarrow \beta = -64$

Hence, the correct answer is option C.

Solution 71

Given equation $x^7 - 7x - 2 = 0$
Let $f(x) = x^7 - 7x - 2$
$f'(x) = 7x^6 - 7 = 7(x^6 - 1)$
and $f'(x) = 0 \Rightarrow x = +1$
and $f(-1) = -1 + 7 - 2 = 5 > 0$
$f(1) = 1 - 7 - 2 = -8 < 0$
So, roughly sketch of $f(x)$ will be



So, number of real roots of $f(x) = 0$ and 3

Hence, the correct answer is option D.

Solution 72

$\because$ x is a random variable

$\therefore k + 2k + 4k + 6k + 8k = 1$

$\therefore\ k = \frac{1}{21}$

Now $P(1 < X < 4 | x \leq 2) = \frac{4k}{7k} = \frac{4}{7}$

Hence, the correct answer is option A.

Solution 73

$$\cos\left(x + \frac{\pi}{3}\right)\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4}\cos^2 2x,\ x \in [-3\pi,\ 3\pi]$$

$$\Rightarrow \cos 2x + \cos\frac{2\pi}{3} = \frac{1}{2}\cos^2 2x$$

$$\Rightarrow \cos^2 2x - 2\cos 2x - 1 = 0$$

$$\Rightarrow \cos 2x = 1$$

$\therefore x = -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi$

$\therefore$ Number of solutions = 7

Hence, the correct answer is option D.

Solution 74

$$\text{Let } \vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\vec{p} = 2\hat{i} + 3\hat{j} + \lambda\hat{k},\quad \vec{q} = \hat{i} + 4\hat{j} + 5\hat{k}$$

$$\therefore\ \vec{p} \times \vec{q} = (15 - 4\lambda)\hat{i} - (10 - \lambda)\hat{j} + 5\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$$

$\therefore$ Shortest distance

$$= \left|\frac{(15-4\lambda)-2(10-\lambda)+10}{\sqrt{(15-4\lambda)^2+(10-\lambda)^2+25}}\right| = \frac{1}{\sqrt{3}}$$

$\Rightarrow 3(5 - 2\lambda)^2 = (15 - 4\lambda)^2 + (10 - \lambda)^2 + 25$

$\Rightarrow 5\lambda^2 - 80\lambda + 275 = 0$

$\therefore$ Sum of values of $\lambda = \frac{80}{5} = 16$

Hence, the correct answer is option A.

Solution 75

Let $P(x, y, z)$ be any point on plane P_1

Then $(x + 4)^2 + (y - 2)^2 + (z - 1)^2 = (x - 2)^2 + (y + 2)^2 + (z - 3)^2$

$\Rightarrow 12x - 8y + 4z + 4 = 0$

$\Rightarrow 3x - 2y + z + 1 = 0$

And $P_2 : 2x + y + 3z = 1$

$\therefore$ angle between P_1 and P_2

$\cos\theta\left|\frac{6-2+3}{14}\right| \Rightarrow \theta = \frac{\pi}{3}$

Hence, the correct answer is option C.

Solution 76

$\because \quad \left|\widehat{a} + \hat{b} + 2\left(\widehat{a} \times \hat{b}\right)\right| = 2,\ \theta \in (0,\ \pi)$

$\Rightarrow \left|\widehat{a} + \hat{b} + 2\left(\widehat{a} \times \hat{b}\right)\right|^2 = 4.$

$\Rightarrow |\widehat{a}|^2 + \left|\hat{b}\right|^2 + 4\left|\widehat{a} \times \hat{b}\right|^2 + 2\widehat{a} \cdot \hat{b} = 4.$

$\therefore\ \cos\theta = \cos 2\theta$

$\therefore\ \theta = \frac{2\pi}{3}$

where θ is angle between $\widehat{a}$ and $\hat{b}$.

$\therefore\ 2\left|\widehat{a} \times \hat{b}\right| = \sqrt{3} = \left|\widehat{a} - \hat{b}\right|$

(S1) is correct

And projection of $\widehat{a}$ on $\left(\widehat{a} + \hat{b}\right) = \left|\frac{\widehat{a} \cdot \left(\widehat{a} + \hat{b}\right)}{\left|\widehat{a} + \hat{b}\right|}\right| = \frac{1}{2}.$

(S2) is correct.

Hence, the correct answer is option C.

Solution 77

Let $x^3 = \theta \Rightarrow \frac{\theta}{2} \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\therefore\ y = \tan^{-1}\ (\sec\theta - \tan\theta)$

$= \tan^{-1}\ \left(\frac{1-\sin\theta}{\cos\theta}\right)$

$\therefore\ y = \frac{\pi}{4} - \frac{\theta}{2}.$

$y = \frac{\pi}{4} - \frac{x^3}{2}$

$\therefore\ y' = \frac{-3x^2}{2}$

$y'' = -3x$

$\therefore\ x^2y'' - 6y + \frac{3\pi}{2} = 0.$

Hence, the correct answer is option B.

Solution 78

∵ given statement is

$(A \wedge C) \to B$

Then its negation is

$\sim \{(A \wedge C) \to B\}$

or $\sim \{\sim (A \wedge C) \vee B\}$

$\therefore (A \wedge C) \wedge (\sim B)$

or $(\sim B) \wedge (A \wedge C)$

Hence, the correct answer is option B.

Solution 79

$\because -\frac{dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1}$

$\therefore\ \frac{dy}{dx} = \frac{x^2y^2 - xy + 1}{x^2}$

Let $xy = v \Rightarrow y + x\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore\ \frac{dv}{dx} - y = \frac{(v^2 - v + 1)y}{v}$

$\therefore\ \frac{dv}{dx} = \frac{v^2 + 1}{x}$

$\therefore$ $y(1) = 1 \Rightarrow \tan^{-1}(xy) = \ln x + \tan^{-1}(1)$

Put $x = e$ and $y = y(e)$ we get

$\tan^{-1}(e \cdot y(e)) = 1 + \tan^{-1} 1.$

$\tan^{-1}(e \cdot y(e)) - \tan^{-1} 1 = 1$

$\therefore\ e\left(y\left(e\right)\right) = \frac{1+\tan(1)}{1-\tan(1)}$

Hence, the correct answer is option D.

Solution 80

$\because f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$

$\therefore f'_\lambda(x) = 12(\lambda x^2 - 6\lambda x + 3)$

For $f_\lambda(x)$ increasing : $(6\lambda)^2 - 12\lambda \le 0$

$\therefore\ \lambda \in \left[0, \frac{1}{3}\right]$

$\therefore\ \lambda^* = \frac{1}{3}$

Now, $f_\lambda{}^*\left(x\right) = \frac{4}{3}x^3 - 12x^2 + 36x + 48$

$$\therefore\ f_\lambda{}^*\left(1\right) + f_\lambda{}^*\left(-1\right) = 73\tfrac{1}{2} - 1\tfrac{1}{2}$$
$$= 72.$$

Hence, the correct answer is option D.

Solution 81

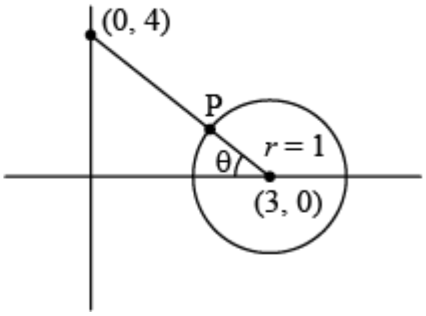
Here $|z - 3| \le 1$

$\Rightarrow (x - 3)^2 + y^2 < 1$

and $z\left(4 + 3i\right) + \bar{z}\left(4 - 3i\right) \le 24$

$\Rightarrow 4x - 3y \le 12$

$\tan\theta = \frac{4}{3}$



$$\therefore \text{ Coordinate of } P = (3 - \cos\theta,\ \sin\theta)$$

$$= \left(3 - \frac{3}{5},\ \frac{4}{5}\right)$$

$$\therefore\ \alpha + i\beta = \frac{12}{5} + \frac{4}{5}i$$

$$\therefore\ 25\,(\alpha + \beta) = 80$$

Hence, the correct answer is 80.

Solution 82

$$S = \left\{ \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix};\ a,\ b\ \in \{1,\ 2,\ 3,\ \ldots,\ 100\} \right\}$$

$\because\ A = \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}$ then even powers of

A as $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, if $b = 1$ and $a \in \{1, \ldots, 100\}$

Here, $n(n + 1)$ is always even.

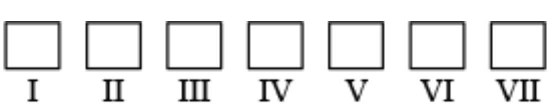
$\therefore T_1, T_2, T_3, \ldots, T_n$ are all I for $b = 1$ and each value of a.

$$\therefore\ \bigcap_{n=1}^{100} T_n = 100$$

Hence, the correct answer is 100.

Solution 83

Sum of all given numbers = 31



Difference between odd and even positions must be 0, 11 or 22, but 0 and 22 are not possible.

∴ Only difference 11 is possible

This is possible only when either 1, 2, 3, 4 is filled in odd position in some order and remaining in other order. Similar arrangements of 2, 3, 5 or 7, 2, 1 or 4, 5, 1 at even positions.

∴ Total possible arrangements = (4! × 3!) × 4

= 576

Hence, the correct answer is 576.

Solution 84

The numbers upto 24 which gives g.c.d. with 24 equals to 1 are 1, 5, 7, 11, 13, 17, 19 and 23.
Sum of these numbers = 96
There are four such blocks and a number 97 is there upto 100.
∴ Complete sum
= 96 + (24 × 8 + 96) + (48 × 8 + 96) + (72 × 8 + 96) + 97
= 1633

Hence, the correct answer is 1633.

Solution 85

$1+3+3^2+....+3^{2021}=\frac{3^{2022}-1}{2}$

$=\frac{1}{2}\left\{(10-1)^{1011}-1\right\}$

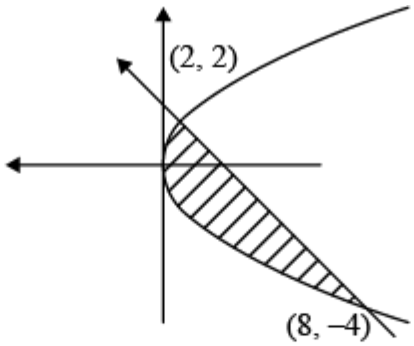
$=\frac{1}{2}\{100k+10110-1-1\}$

$=50\ k_1+4$

∴ Remainder = 4

Hence, the correct answer is 4.

Solution 86



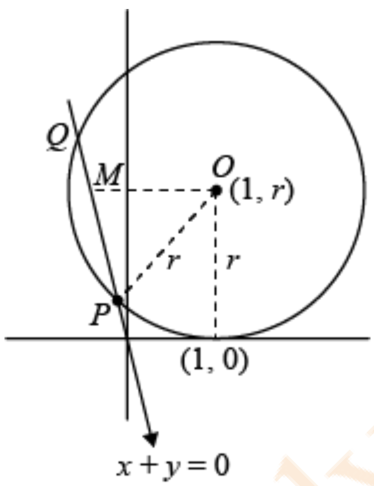
$$\text{The required area} = \int_{-4}^{2}\left(4 - y - \frac{y^2}{2}\right)dy$$

$$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6}\right]_{-4}^{2}$$

$$= 18 \text{ square units}$$

Hence, the correct answer is 18.

Solution 87



Here, $OM^2 = OP^2 - PM^2$

$$\left(\frac{|1+r|}{\sqrt{2}}\right)^2 = r^2 - 1$$

$$\therefore\ r^2 - 2r - 3 = 0$$

$$\therefore\ r = 3$$

∴ Equation of circle is

$(x - 1)^2 + (y - 3)^2 = 3^2$

∴ $h = 1, k = 3, r = 3$

$\therefore h + k + r = 7$

Hence, the correct answer is 7.

Solution 88

Student guesses only two wrong. So there are three possibilities
(i) Student guesses both wrong from 1st section
(ii) Student guesses both wrong from 2nd section
(iii) Student guesses two wrong one from each section

$$\text{Required probabilities} = {}^4C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2\left(\frac{1}{4}\right)^6 + {}^6C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^4 + {}^4C_1 \cdot {}^6C_1 \left(\right.$$

$$= \frac{1}{4^{10}}\left[6 \times 9 + 15 \times 9^3 + 24 \times 9^2\right]$$

$$= \frac{27}{4^{10}}\left[2 + 27 \times 15 + 72\right]$$

$$= \frac{27 \times 479}{4^{10}}$$

Hence, the correct answer is 479.

Solution 89

$$\because\ H : \frac{x^2}{a^2} - \frac{y^2}{1} = 1$$

$$\therefore\ \text{Length of latus rectum} = \frac{2}{a}$$

$$E : \frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$\text{Length of latus rectum} = \frac{6}{2} = 3$$

$$\because \frac{2}{a} = 3 \Rightarrow a = \frac{2}{3}$$

$$\therefore 12\left(e_H^2 + e_E^2\right) = 12\left(1 + \frac{9}{4}\right) + \left(1 - \frac{3}{4}\right) = 42$$

Hence, the correct answer is 42.

Solution 90

Focus = (4, 4) and vertex = (3, 2)

$\therefore$ Point of intersection of directrix with axis of parabola = A = (2, 0)

Image of $A(2, 0)$ with respect to line

$x + 2y = 6$ is $B(x_2, y_2)$

$\therefore \frac{x_2-2}{1} = \frac{y_2-0}{2} = \frac{-2(2+0-6)}{5}$

$\therefore\ B(x_2,\ y_2) = \left(\frac{18}{5},\ \frac{16}{5}\right).$

Point B is point of intersection of direction with axes of parabola P_2.

$\therefore x + 2y = \lambda$ must have point $\left(\frac{18}{5},\ \frac{16}{5}\right)$

$\therefore x + 2y = 10$

Hence, the correct answer is 10.