NDA II 2017_Mathematics

Total Time: 150

Total Marks: 300.0

Q.No.1: If $x + \log_{10}(1 + 2^x) = x\log_{10}5 + \log_{10}6$ then x is equal to

A. 2, −3
B. 2 only
C. 1
D. 3

Marks:[2.50]

Q.No.2: The remainder and the quotient of the binary division $(101110)_2 \div (110)_2$ are respectively

A. $(111)_2$ and $(100)_2$
B. $(100)_2$ and $(111)_2$
C. $(101)_2$ and $(101)_2$
D. $(100)_2$ and $(100)_2$

Marks:[2.50]

Q.No.3: The matrix A has x rows and $x + 5$ columns. The matrix B has y rows and $11 - y$ columns. Both AB and BA exist. What are the values of x and y respectively?

A. 8 and 3
B. 3 and 4
C. 3 and 8
D. 8 and 8

Marks:[2.50]

Typesetting math: 100%

Q.No.4: If $S_n = nP + \frac{n(n-1)Q}{2}$, where S_n denotes the sum of the first n terms of an AP, then the common difference is

A. $P + Q$

B. $2P + 3Q$

C. $2Q$

D. Q

Marks:[2.50]

Q.No.5: The roots of the equation
$(q - r)x^2 + (r - p)x + (p - q) = 0$ are

A. $(r - p)/(q - r)$, 1/2

B. $(p - q)/(q - r)$, 1

C. $(q - r)/(p - q)$, 1

D. $(r - p)/(p - q)$, 1/2

Marks:[2.50]

Q.No.6: If E is the universal set and $A = B \cup C$, then the set $E - (E - (E - (E - (E - A))))$ is the same as the set

A. $B' \cup C'$

B. $B \cup C$

C. $B' \cap C'$

D. $B \cap C$

Marks:[2.50]

Q.No.7: If $A = \{x : x$ is a multiple of $2\}$, $B = \{x : x$ is a multiple of $5\}$ and $C = \{x : x$ is a multiple of $10\}$, then $A \cap (B \cap C)$ is equal to

A. A

B. B

C. C

D. $\{x : x$ is a multiple of $100\}$

Marks:[2.50]

Q.No.8: If α and β are the roots of equation $1 + x + x^2 = 0$, then the matrix product $\begin{bmatrix} 1 & \beta \\ \alpha & \alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ 1 & \beta \end{bmatrix}$ is equal to

A. $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

Typesetting math: 100%

B. $\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

D. $\begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix}$

Marks:[2.50]

Q.No.9: If |*a*| denotes the absolute value of an integer, then which of the following are correct?
1. |*ab*| = |*a*| |*b*|
2. |*a* + *b*| ≤ |*a*| + |*b*|
3. |*a* − *b*| ≥ ||*a*| − |*b*||

Select the correct answer using the code given below.

A. 1 and 2 only

B. 2 and 3 only

C. 1 and 3 only

D. 1, 2 and 3

Marks:[2.50]

Q.No.10: How many different permutations can be made out of the letters of the word 'PERMUTATION'?

A. 19958400

B. 19954800

C. 19952400

D. 39916800

Marks:[2.50]

Q.No.11: If $A = \begin{bmatrix} 4i-6 & 10i \\ 14i & 6+4i \end{bmatrix}$ and $k = \frac{1}{2i}$, where $i = \sqrt{-1}$, then *kA* is equal to

A. $\begin{bmatrix} 2+3i & 5 \\ 7 & 2-3i \end{bmatrix}$

B. $\begin{bmatrix} 2-3i & 5 \\ 7 & 2+3i \end{bmatrix}$

C. $\begin{bmatrix} 2-3i & 7 \\ 5 & 2+3i \end{bmatrix}$

D. $\begin{bmatrix} 2+3i & 5 \\ 7 & 2+3i \end{bmatrix}$

Marks:[2.50]

Q.No.12: The sum of all real roots of equation $|x-3|^2 + |x-3| - 2 = 0$ is

A. 2
B. 3
C. 4
D. 6

Marks:[2.50]

Q.No.13: It is given that the roots of equation $x^2 - 4x - \log_3 P = 0$ are real. For this, the minimum value of P is

A. $\frac{1}{27}$
B. $\frac{1}{64}$
C. $\frac{1}{81}$
D. 1

Marks:[2.50]

Q.No.14: If A is a square matrix, then the value of adjA^T – (adj $A)^T$ is equal to

A. A
B. $2|A|I$, where I is the identity matrix
C. null matrix whose order is the same as that of A
D. unit matrix whose order is the same as that of A

Marks:[2.50]

Q.No.15: The value of the product $6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times 6^{\frac{1}{16}} \times ...$ up to infinite terms is

A. 6
B. 36
C. 216
D. 512

Marks:[2.50]

Q.No.16: The value of determinant

$$\begin{vmatrix} \cos^2 \frac{\theta}{2} & \sin^2 \frac{\theta}{2} \\ \sin^2 \frac{\theta}{2} & \cos^2 \frac{\theta}{2} \end{vmatrix}$$

for all the values of θ, is

A. 1
B. $\cos \theta$
C. $\sin \theta$
D. $\cos 2\theta$

Marks:[2.50]

Q.No.17: The number of terms in the expansion of $(x+a)^{100} + (x-a)^{100}$ after simplification is

A. 202
B. 101
C. 51
D. 50

Marks:[2.50]

Q.No.18: In the expansion of $(1 + x)^{50}$, the sum of coefficients of odd powers of x is

A. 2^{26}
B. 2^{49}
C. 2^{50}
D. 2^{51}

Marks:[2.50]

Q.No.19: If a, b, c are non-zero real numbers, then the inverse of matrix

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

is equal to

A. $\begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$

B. $\frac{1}{abc} \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$

C. $\frac{1}{abc}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

D. $\frac{1}{abc}\begin{bmatrix}a & 0 & 0\\0 & b & 0\\0 & 0 & c\end{bmatrix}$

Marks:[2.50]

Q.No.20: A person is to count 4500 notes. Let a_n denote the number of notes that he counts in the n^{th} minute. If $a_1 = a_2 = a_3 = \ldots = a_{10} = 150$, and a_{10}, a_{11}, a_{12}, ... are in AP with the common difference −2, then the time taken by him to count all the notes is

A. 24 minutes
B. 34 minutes
C. 125 minutes
D. 135 minutes

Marks:[2.50]

Q.No.21: The smallest positive integer n, which $\left(\frac{1+i}{1-i}\right)^n = 1$, is

A. 1
B. 4
C. 8
D. 16

Marks:[2.50]

Q.No.22: If we define a relation R on the set N × M as $(a, b)\ R\ (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$, then the relation is

A. symmetric only
B. symmetric and transitive only
C. equivalence relation
D. reflexive only

Marks:[2.50]

Q.No.23: If $y = x + x^2 + x^3 + \ldots$ up to infinite terms, where $x < 1$, then which of the following is correct?

A. $x = \frac{y}{1+y}$

Typesetting math: 100%

B. $x = \frac{y}{1-y}$

C. $x = \frac{1+y}{y}$

D. $x = \frac{1-y}{y}$

Marks:[2.50]

Q.No.24: If α and β are the roots of equation $3x^2 + 2x + 1 = 0$, then the equation whose roots are $\alpha + \beta^{-1}$ and $\beta + \alpha^{-1}$ is

A. $3x^2 + 8x + 16 = 0$

B. $3x^2 - 8x - 16 = 0$

C. $3x^2 + 8x - 16 = 0$

D. $x^2 + 8x + 16 = 0$

Marks:[2.50]

Q.No.25: The value of $\frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + \ldots$ up to infinite terms is

A. $\log_e 9$

B. 0

C. 1

D. $\log_e 3$

Marks:[2.50]

Q.No.26: A tea party is arranged for 16 people along the two sides of a long table with eight chairs on each side. Four particular men wish to sit on one particular side and two particular men on the other side. The number of ways they can be seated is

A. $24 \times 8! \times 8!$

B. $(8!)^3$

C. $210 \times 8! \times 8!$

D. $16!$

Marks:[2.50]

Typesetting math: 100%

Q.No.27: The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + kz = k^2$ has no solution if k equals

A. 0

B. 1

C. −1

D. −2

Marks:[2.50]

Q.No.28: If $1\ .3\ +2\ .3^2\ +3\ .3^3\ + \ldots + n.\,3^n = \frac{(2n-1)3^a+b}{4}$ then a and b are respectively

A. n, 2

B. n, 3

C. $n + 1$, 2

D. $n + 1$, 3

Marks:[2.50]

Q.No.29: In ΔPQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of equation $ax^2 + bx + c = 0$, then which of the following is correct?

A. $a = b + c$

B. $b = c + a$

C. $c = a + b$

D. $b = c$

Marks:[2.50]

Q.No.30: If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to

A. $1 + \sqrt{3}$

B. $1 + \sqrt{5}$

C. $1 - \sqrt{5}$

D. $\sqrt{5} - 1$

Marks:[2.50]

Q.No.31: The angle of elevation of stationary cloud from the point 25 m above a lake is 15° and the angle of depression of its image in the lake is 45°. The height of the cloud above the lake level is

A. 25 m

B. $25\sqrt{3}$

C. 50 m

D. $50\sqrt{3}$ m

Marks:[2.50]

Q.No.32: The value of tan 9° – tan 27° – tan 63° + tan 81° is equals to

A. −1

B. 0

C. 1

D. 4

Marks:[2.50]

Q.No.33: The value of $\sqrt{3}$ cosec 20° – sec 20° is equal to

A. 4

B. 2

C. 1

D. −4

Marks:[2.50]

Q.No.34: Angle α is divided into two parts *A* and *B* such that *A* – *B* = *x* and tan *A* : tan *B* = *p* : *q*. The value of sin *x* is equal to

A. $\frac{(p+q)\ \sin\alpha}{p-q}$

B. $\frac{p\sin\ \alpha}{p+q}$

C. $\frac{p\sin\ \alpha}{p-q}$

D. $\frac{(p-q)\ \sin\alpha}{p+q}$

Marks:[2.50]

Q.No.35: The value of $\sin^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)$ is equal to

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Marks:[2.50]

Q.No.36: The angles of the elevation of the top of a tower from the top and the foot of a pole are respectively 30° and 45°. If h_T is the height of the tower and h_P is the height of the pole, then which of the following are correct?

1. $\frac{2h_P h_T}{3+\sqrt{3}} = h_P^2$
2. $\frac{h_T - h_P}{\sqrt{3}+1} = \frac{h_P}{2}$
3. $\frac{2(h_{P+}h_T)}{h_P} = 4 + \sqrt{3}$

Select the correct answer using the code given below

A. 1 and 3 only
B. 2 and 3 only
C. 1 and 2 only
D. 1, 2 and 3

Marks:[2.50]

Q.No.37: In a triangle *ABC*, *a* – 2*b* + *c* = 0. The value of cot $\left(\frac{A}{2}\right)$ cot $\left(\frac{C}{2}\right)$ is

A. $\frac{9}{2}$
B. 3
C. $\frac{3}{2}$
D. 1

Marks:[2.50]

Q.No.38: $\sqrt{1 + sin\ A} = -\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)$ is true if

A. $\frac{3\pi}{2} < A < \frac{5\pi}{2}$ only
B. $\frac{\pi}{2} < A < \frac{3\pi}{2}$ only
C. $\frac{3\pi}{2} < A < \frac{7\pi}{2}$
D. $0 < A < \frac{3\pi}{2}$

Marks:[2.50]

Q.No.39: In triangle *ABC,* if $\frac{\sin^2 A+\sin^2 B+\sin^2 C}{\cos^2 A+\cos^2 B+\cos^2 C} = 2$ then the triangle is

A. right-angled
B. equilateral
C. isosceles
D. obtuse-angled

Marks:[2.50]

Q.No.40: The principal value of $\sin^{-1} x$ lies in the interval

A. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C. $\left[0, \frac{\pi}{2}\right]$

D. [0, π]

Marks:[2.50]

Q.No.41: The points (a, b), $(0, 0)$, $(-a, -b)$ and (ab, b^2) are

A. the vertices of parallelogram

B. the vertices of a rectangle

C. the vertices of a square

D. collinear

Marks:[2.50]

Q.No.42: The length of the normal from origin to the plane $x + 2y - 2z = 9$ is equal to

A. 2 units

B. 3 units

C. 4 units

D. 5 units

Marks:[2.50]

Q.No.43: If α, β and γ are the angles which the vector $\overrightarrow{OP}$ (O being the origin) makes with positive direction of coordinate axes, then which of the following are correct?

1. $\cos^2\alpha + \cos^2\beta = \sin^2\gamma$
2. $\sin^2\alpha + \sin^2\beta = \cos^2\gamma$
3. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Select the correct answer using the code given below

A. 1 and 2 only

B. 2 and 3 only

C. 1 and 3 only

D. 1, 2 and 3

Marks:[2.50]

Q.No.44: The angle between the lines $x + y - 3 = 0$ and $x - y + 3 = 0$ is α and the acute angle between the lines $x - \sqrt{3}y + 2\sqrt{3} = 0$ and $\sqrt{3}x - y + 1 = 0$ is

β. Which one of the following is correct?

A. $\alpha = \beta$

B. $\alpha > \beta$

C. $\alpha < \beta$

D. $\alpha = 2\beta$

Marks:[2.50]

Q.No.45: Let $\overrightarrow{\alpha} = \hat{i} + 2\hat{j} - \hat{k}$, $\overrightarrow{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\overrightarrow{\gamma} = 2\hat{i} + \hat{j} + 6\hat{k}$ be three vectors. If $\overrightarrow{\alpha}$ and $\overrightarrow{\beta}$ are both perpendicular to the vector $\overrightarrow{\delta}$ and $\overrightarrow{\delta} \cdot \overrightarrow{\gamma} = 10$, then what is the magnitude of $\overrightarrow{\delta}$?

A. $\sqrt{3}$ units

B. $2\sqrt{3}$ units

C. $\frac{\sqrt{3}}{2}$ unit

D. $\frac{1}{\sqrt{3}}$ unit

Marks:[2.50]

Q.No.46: If $\widehat{a}$ and $\hat{b}$ are two unit vectors, then the vector $\left(\widehat{a} + \hat{b}\right) \times \left(\widehat{a} \times \hat{b}\right)$ is parallel to

A. $\left(\widehat{a} - \hat{b}\right)$

B. $\left(\widehat{a} + \hat{b}\right)$

C. $\left(2\widehat{a} - \hat{b}\right)$

D. $\left(2\widehat{a} + \hat{b}\right)$

Marks:[2.50]

Typesetting math: 100%

Q.No.47: A force $\overrightarrow{F} = \hat{i} + 3\hat{j} + 2\hat{k}$ acts on a particle to displace it from the point $A\left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ to the point $B\left(3\hat{i} - \hat{j} + 5\hat{k}\right)$. The work done by the force will be

A. 5 units
B. 7 units
C. 9 units
D. 10 units

Marks:[2.50]

Q.No.48: For any vector $\overrightarrow{a}$ $\left|\overrightarrow{a} \times \hat{i}\right|^2 + \left|\overrightarrow{a} \times \hat{j}\right|^2 + \left|\overrightarrow{a} \times \hat{k}\right|^2$ is equal to

A. $\left|\overrightarrow{a}\right|^2$
B. $2\left|\overrightarrow{a}\right|^2$
C. $3\left|\overrightarrow{a}\right|^2$
D. $4\left|\overrightarrow{a}\right|^2$

Marks:[2.50]

Q.No.49: A man running around a race course notes that the sum of the distance of two flag-posts from him is always 10 m and the distance between the flag-posts is 8 m. The area of the path enclosed is

A. 18п square metres
B. 15п square metres
C. 12п square metres
D. 8п square metres

Marks:[2.50]

Q.No.50: The distance of point (1,3) from the line $2x + 3y = 6$, measured parallel to the line $4x + y = 4$, is

A. $\frac{5}{\sqrt{13}}$ units

B. $\frac{3}{\sqrt{17}}$ units

C. $\sqrt{17}$ units

D. $\frac{\sqrt{17}}{2}$ units

Marks:[2.50]

Q.No.51: If the vectors $a\hat{i} + \hat{j} + \hat{k},\ \hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}\ (a,\ b,\ c \neq 1)$ are coplanar, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

A. 0

B. 1

C. $a + b + c$

D. abc

Marks:[2.50]

Q.No.52: The point of intersection of the line joining the points (−3, 4, −8) and (5, −6, 4) with the XY-plane is

A. $\left(\frac{7}{3},\ -\frac{8}{3},\ 0\right)$

B. $\left(-\frac{7}{3},\ -\frac{8}{3},\ 0\right)$

C. $\left(-\frac{7}{3},\ \frac{8}{3},\ 0\right)$

D. $\left(\frac{7}{3},\ \frac{8}{3},\ 0\right)$

Marks:[2.50]

Q.No.53: If the angle between the lines whose direction ratios are (2, −1, 2) and (x, 3, 5) is $\frac{\pi}{4}$, then the smaller value of x is

A. 52

B. 4

C. 2

D. 1

Marks:[2.50]

Typesetting math: 100%

Q.No.54: The position of the point (1, 2) relative to the ellipse $2x^2 + 7y^2 = 20$ is

A. outside the ellipse
B. inside the ellipse but not at the focus
C. on the ellipse
D. at the focus

Marks:[2.50]

Q.No.55: The equation of a straight line which cuts off an intercept of 5 units on negative direction of *y*-axis and makes an angle 120° with positive direction of *x*-axis is

A. $y + \sqrt{3}x + 5 = 0$
B. $y - \sqrt{3}x + 5 = 0$
C. $y + \sqrt{3}x - 5 = 0$
D. $y - \sqrt{3}x - 5 = 0$

Marks:[2.50]

Q.No.56: The equation of the line passing through the point (2, 3) and the point of intersection of lines 2*x* – 3*y* + 7 = 0 and 7*x* + 4*y* + 2 = 0 is

A. 21*x* + 46*y* – 180 = 0
B. 21*x* – 46*y* + 96 = 0
C. 46*x* + 21*y* – 155 = 0
D. 46*x* – 21*y* – 29 = 0

Marks:[2.50]

Q.No.57: The equation of the ellipse whose centre is at the origin, major axis is along *x*-axis with eccentricity $\frac{3}{4}$ and latus rectum 4 units is

A. $\frac{x^2}{1024} + \frac{7y^2}{64} = 1$
B. $\frac{49x^2}{1024} + \frac{7y^2}{64} = 1$
C. $\frac{7x^2}{1024} + \frac{49y^2}{64} = 1$
D. $\frac{x^2}{1024} + \frac{y^2}{64} = 1$

Marks:[2.50]

Q.No.58: The equation of the circle which passes through the points (1, 0), (0, –6) and (3, 4) is

A. $4x^2 + 4y^2 + 142x + 47y + 140 = 0$
B. $24x^2 + 4y^2 - 142x - 47y + 138 = 0$
C. $4x^2 + 4y^2 - 142x + 47y + 138 = 0$
D. $4x^2 + 4y^2 + 150x - 49y + 138 = 0$

Marks:[2.50]

Q.No.59: A variable plane passes through a fixed point (*a*, *b*, *c*) and cuts the axes in A, B and C respectively. The locus of the centre of the sphere OABC, O being the origin, is

A. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
B. $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$
C. $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$
D. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 2$

Marks:[2.50]

Q.No.60: The equation of the plane passing through the line of intersection of the planes $x + y + z = 1$, $2x + 3y + 4z = 7$, and perpendicular to the plane $x - 5y + 3z = 5$ is given by

A. $x + 2y + 3z - 6 = 0$
B. $x + 2y + 3z + 6 = 0$
C. $3x + 4y + 5z - 8 = 0$
D. $3x + 4y + 5z + 8 = 0$

Marks:[2.50]

Q.No.61: The inverse of the function $y = 5^{\ln x}$ is

A. $x = y^{\frac{1}{\ln 5}},\ y > 0$
B. $x = y^{\ln 5},\ y > 0$
C. $x = y^{\frac{1}{\ln 5}},\ y < 0$
D. $x = 5\ \ln\ y,\ y > 0$

Marks:[2.50]

Q.No.62: A function is defined as follows:

$$f(x) = \begin{cases} -\frac{x}{\sqrt{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Which one of the following is correct in respect of the above function?

A. $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$

Typesetting math: 100%

B. $f(x)$ is continuous as well as differentiable at $x = 0$
C. $f(x)$ is discontinuous at $x = 0$
D. None of the above

Marks:[2.50]

Q.No.63: If $y = (\cos\ x)^{(\cos\ x)^{(\cos\ x)^{\cdot^{\cdot^{\cdot \infty}}}}}$, then $\frac{dy}{dx}$ is equal to

A. $-\frac{y^2\ \tan\ x}{1-y\ \ln\ (\cos\ x)}$
B. $\frac{y^2\ \tan\ x}{1+y\ \ln\ (\cos\ x)}$
C. $\frac{y^2\ \tan\ x}{1-y\ \ln\ (\sin\ x)}$
D. $\frac{y^2\ \sin\ x}{1+y\ \ln\ (\sin\ x)}$

Marks:[2.50]

Q.No.64: Consider the following:
1. $x + x^2$ is continuous at $x = 0$
2. $x + \cos \frac{1}{x}$ is discontinuous at $x = 0$
3. $x^2 + \cos \frac{1}{x}$ is continuous at $x = 0$
Which of the above are correct?

A. 1 and 2 only
B. 2 and 3 only
C. 1 and 3 only
D. 1, 2 and 3

Marks:[2.50]

Q.No.65: Consider the following statements:
1. dy/dx at a point on the curve gives slope of the tangent at that point.
2. If $a(t)$ denotes acceleration of a particle, then $\int a(t)dt + c$ gives velocity of the particle.
3. If $s(t)$ gives displacement of a particle at time t, then ds/dt gives its acceleration at that instant.
Which of the above statements is/are correct?

A. 1 and 2 only
B. 2 only
C. 1 only
D. 1, 2 and 3

Marks:[2.50]

Typesetting math: 100%

Q.No.66: If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx}$ is equal to

A. 0
B. 1
C. $\frac{x-1}{x+1}$
D. $\frac{x+1}{x-1}$

Marks:[2.50]

Q.No.67: What is $\int \tan^{-1}\left(\sec x + \tan x\right) dx$ equal to?

A. $\frac{\pi x}{4} + \frac{x^2}{4} + c$
B. $\frac{\pi x}{2} + \frac{x^2}{4} + c$
C. $\frac{\pi x}{4} + \frac{\pi x^2}{4} + c$
D. $\frac{\pi x}{4} - \frac{x^2}{4} + c$

Marks:[2.50]

Q.No.68: A function is defined in (0, ∞) by

$$f\left(x\right) = \begin{Bmatrix} 1 - x^2 \text{ for } 0 < x \leq 1 \\ \ln\ x \text{ for } 1 < x \leq 2 \\ \ln\ 2 - 1 + 0\ .5\ x \text{ for } 2 < x < \infty \end{Bmatrix}$$

Which of the following is correct in respect of the derivation of the function, i.e., $f'(x)$?

A. $f'(x) = 2x$ for $0 < x \leq 1$
B. $f'(x) = -2x$ for $0 < x \leq 1$
C. $f'(x) = -2x$ for $0 < x < 1$
D. $f'(x) = 0$ *for* $0 < x < \infty$

Marks:[2.50]

Q.No.69: Which of the following is correct in respect of the function $f(x) = x(x - 1)(x+1)$?

A. The local maximum value is larger than local minimum value
B. The local maximum value is smaller than local minimum value
C. The function has no local maximum
D. The function has no local minimum

Marks:[2.50]

Q.No.70: Consider the following statements:
1. Derivative of *f*(*x*) may not exist at some point.
2. Derivative of *f*(*x*) may exist finitely at some point.
3. Derivative of *f*(*x*) may be infinite (geometrically) at some point.
Which of the above statements are correct?

A. 1 and 2 only
B. 2 and 3 only
C. 1 and 3 only
D. 1, 2 and 3

Marks:[2.50]

Q.No.71: The maximum value of $\frac{\ln x}{x}$ is

A. *e*
B. $\frac{1}{e}$
C. $\frac{2}{e}$
D. 1

Marks:[2.50]

Q.No.72: The function $f(x) = |x| - x^3$ is

A. odd
B. even
C. both even and odd
D. neither even nor odd

Marks:[2.50]

Typesetting math: 100%

Q.No.73: If

$l_1 = \frac{d}{dx}\left(e^{\sin\,x}\right)$

$l_2 = \lim\limits_{h\to 0}\frac{e^{\sin\,(x+h)}-e^{\sin\,x}}{h}$

$l_3 = \int e^{\sin\,x}\cos x\,dx$

then which one of the following is correct?

A. $l_1 \neq l_2$

B. $\frac{d}{dx}\left(l_3\right) = l_2$

C. $\int l_3 dx = l_2$

D. $l_2 = l_3$

Marks:[2.50]

Q.No.74: The general solution of

$\frac{dy}{dx} = \frac{ax+h}{by+k}$

represents a circle only when

A. $a = b = 0$

B. $a = -b \neq 0$

C. $a = b \neq 0$, $h = k$

D. $a = b \neq 0$

Marks:[2.50]

Q.No.75: If $\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin x}{x} = l$ and $\lim\limits_{x\to\infty}\frac{\cos\,x}{x} = m$, then which of the following is correct?

A. $l = 1$, $m = 1$

B. $l = \frac{2}{\pi},\ m = \infty$

C. $l = \frac{2}{\pi},\ m = 0$

D. $l = 1$, $m = \infty$

Marks:[2.50]

Q.No.76: What is $\int\limits_0^{2\pi}\sqrt{1+\sin\frac{x}{2}}dx$ equal to?

A. 8

B. 4

Typesetting math: 100%

C. 2
D. 0

Marks:[2.50]

Q.No.77: The area bounded by the curve $|x| + |y| = 1$ is

A. 1 square unit
B. $2\sqrt{2}$ square units
C. 2 square units
D. $2\sqrt{3}$ square units

Marks:[2.50]

Q.No.78: If *x* is any real number, then $\frac{x^2}{1+x^4}$ belongs to which of the following intervals?

A. (0,1)
B. $\left(0, \frac{1}{2}\right]$
C. $\left[0, \frac{1}{2}\right]$
D. [0,1]

Marks:[2.50]

Q.No.79: The left-hand derivative of
f(*x*) = [*x*] sin (π*x*) at *x* = *k*
where *k* is an integer and [*x*] is the greatest integer function, is

A. $(-1)^k \left(k - 1\right)\pi$
B. $(-1)^{k-1} \left(k - 1\right)\pi$
C. $(-1)^k k\pi$
D. $(-1)^{k-1} k\pi$

Marks:[2.50]

Q.No.80: If $f\left(x\right) = \frac{x}{2} - 1$, then on the interval [0, π] which of the following is correct?

A. tan [*f*(*x*)], where [·] is the greatest integer function, and $\frac{1}{f(x)}$ are both continuous
B. tan [*f*(*x*)], where [·] is the greatest integer function, and $f^{-1}(x)$ are both continuous

Typesetting math: 100%

C. tan $[f(x)]$, where $[\cdot]$ is the greatest integer function, and $\frac{1}{f(x)}$ are both discontinuous

D. tan $[f(x)]$, where $[\cdot]$ is the greatest integer function, is discontinuous but $\frac{1}{f(x)}$ is continuous

Marks:[2.50]

Q.No.81: The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \rho^2\left[\frac{d^2y}{dx^2}\right]^2$ are respectively

A. 3 and 2
B. 2 and 2
C. 2 and 3
D. 1 and 3

Marks:[2.50]

Q.No.82: If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to

A. $-\frac{2}{1+x^2}$ for all $|x| < 1$
B. $-\frac{2}{1+x^2}$ for all $|x| > 1$
C. $\frac{2}{1+x^2}$ for all $|x| < 1$
D. None of the above

Marks:[2.50]

Q.No.83: The set of all points, where the function $f(x) = \sqrt{1-e^{-x^2}}$ is differentiable, is

A. (0, ∞)
B. (−∞, ∞)
C. (−∞, 0) ∪ (0, ∞)
D. (−1, ∞)

Marks:[2.50]

Q.No.84: Match List-I with List-II and select the correct answer using the code given below the lists:

List–I (Function)	List–II (Maximum value)	
A. sin x + cos x	1.	$\sqrt{10}$

B.	$3 \sin x + 4 \cos x$	2.	$\sqrt{2}$
C.	$2 \sin x + \cos x$	3.	5
D.	$\sin x + 3 \cos x$	4.	$\sqrt{5}$

Code:

A.

A	B	C	D
2	3	1	4

B.

A	B	C	D
2	3	4	1

C.

A	B	C	D
3	2	1	4

D.

A	B	C	D
3	2	4	1

Marks:[2.50]

Q.No.85: If $f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$, then *f*(*x*) is

A. continuous but not differentiable at $x = 0$

B. differentiable at $x = 0$

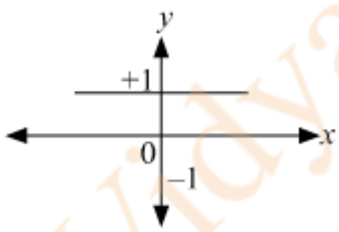
C. not continuous at $x = 0$

D. None of the above

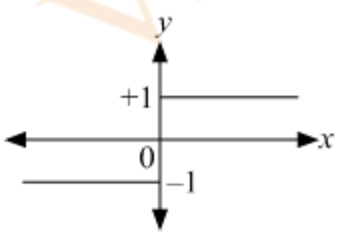
Marks:[2.50]

Q.No.86: Which of the following graphs represents the function $f(x) = \frac{x}{x}$, $x \neq 0$?

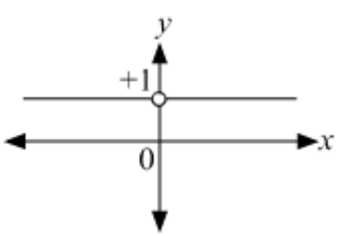
A.



B.



C.



D. None of the above

Marks:[2.50]

Q.No.87: Let $f\left(n\right)=\left[\frac{1}{4}+\frac{n}{1000}\right]$, where [*x*] denotes the integral part of *x*. Then the value of $\sum_{n=1}^{1000} f(n)$ is

A. 251
B. 250
C. 1
D. 0

Marks:[2.50]

Q.No.88: $\int(\ln\ x)^{-1}dx-\int(\ln\ x)^{-2}dx$ is equal to

A. $x(\ln x)^{-1}+c$
B. $x(\ln x)^{-2}+c$
C. $x(\ln x)+c$
D. $x(\ln x)^{2}+c$

Marks:[2.50]

Q.No.89: A cylindrical jar without a lid has to be constructed using a given surface area of a metal sheet. If the capacity of the jar is to be maximum, then the diameter of the jar must be *k* times the height of the jar. The value of *k* is

A. 1
B. 2
C. 3
D. 4

Marks:[2.50]

Q.No.90: The value of $\int\limits_0^{\frac{\pi}{4}}\sqrt{\tan\ x}dx+\int\limits_0^{\frac{\pi}{4}}\sqrt{\cot\ x}dx$ is equal to

A. $\frac{\pi}{4}$
B. $\frac{\pi}{2}$
C. $\frac{\pi}{2\sqrt{2}}$
D. $\frac{\pi}{\sqrt{2}}$

Marks:[2.50]

Typesetting math: 100%

Q.No.91: Let g be the greatest integer function. Then the function $f(x) = (g(x))^2 - g(x)$ is discontinuous at

A. all integers

B. all integers except 0 and 1

C. all integers except 0

D. all integers except 1

Marks:[2.50]

Q.No.92: The differential equation of the minimum order by eliminating the arbitrary constants A and C in the equation $y = A[\sin(x + C) + \cos(x + C)]$ is

A. $y'' + (\sin x + \cos x)y' = 1$

B. $y'' = (\sin x + \cos x)y'$

C. $y'' = (y')^2 + \sin x \cos x$

D. $y'' + y = 0$

Marks:[2.50]

Q.No.93: Consider the following statements:

Statements I:

$x > \sin x$ for all $x > 0$

Statement II:

$f(x) = x - \sin x$ is an increasing function for all $x > 0$

Which one of the following is correct in respect of the above statements?

A. Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I.

B. Both Statement I and Statement II are true and Statement II is not the correct explanation of Statement I

C. Statement I is true but Statement II is false

D. Statement I is false but Statement II is true

Marks:[2.50]

Q.No.94: The solution of the differential equation $\frac{dy}{dx} = \frac{y\phi'(x) - y^2}{\phi(x)}$ is

A. $y = \frac{x}{\phi(x) + c}$

B. $y = \frac{\phi(x)}{x} + c$

C. $y = \frac{\phi(x) + c}{x}$

D. $y = \frac{\phi(x)}{x + c}$

Marks:[2.50]

Q.No.95: If $f\left(x\right) = \frac{4x+x^4}{1+4x^3}$ and $g\left(x\right) = \ln\left(\frac{1+x}{1-x}\right)$, then what is the value of $f\circ g\left(\frac{e-1}{e+1}\right)$ equal to?

A. 2
B. 1
C. 0
D. $\frac{1}{2}$

Marks:[2.50]

Q.No.96: The value of the determinant $\begin{vmatrix} 1-\alpha & \alpha-\alpha^2 & \alpha^2 \\ 1-\beta & \beta-\beta^2 & \beta^2 \\ 1-\gamma & \gamma-\gamma^2 & \gamma^2 \end{vmatrix}$ is equal to

A. $(\alpha-\beta)(\beta-\gamma)(\alpha-\gamma)$
B. $(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$
C. $(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$
D. 0

Marks:[2.50]

Q.No.97: The adjoint of matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ is

A. $\begin{bmatrix} -1 & 6 & 2 \\ -2 & 1 & -4 \\ 6 & 3 & 1 \end{bmatrix}$
B. $\begin{bmatrix} 1 & 6 & -2 \\ -2 & 1 & 4 \\ 6 & -3 & 1 \end{bmatrix}$
C. $\begin{bmatrix} 6 & 1 & 2 \\ 4 & -1 & 2 \\ 6 & 3 & -1 \end{bmatrix}$
D. $\begin{bmatrix} -6 & 2 & 1 \\ 4 & -2 & 1 \\ 3 & 1 & -6 \end{bmatrix}$

Marks:[2.50]

Q.No.98: If $A = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$, then which of following is correct?

A. $A^2 = -2A$

B. $A^2 = -4A$

C. $A^2 = -3A$

D. $A^2 = 4A$

Marks:[2.50]

Q.No.99: Geometrically, Re $(z^2 - i) = 2$, where $i = \sqrt{-1}$ and Re is the real part, represents

A. circle

B. ellipse

C. rectangular hyperbola

D. parabola

Marks:[2.50]

Q.No.100: If $p + q + r = a + b + c = 0$, then the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ equals

A. 0

B. 1

C. $pa + qb + rc$

D. $pa + qb + rc + a + b + c$

Marks:[2.50]

Q.No.101: A committee of two persons is selected from two men and two women. The probability that the committee will have exactly one woman is

A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Marks:[2.50]

Q.No.102: Let a die be loaded in such a way that even faces are twice likely to

Typesetting math: 100%

occur as the odd faces. What is the probability that a prime number will show up when the die is tossed?

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{4}{9}$

D. $\frac{5}{9}$

Marks:[2.50]

Q.No.103: Let the sample space consist of non-negative integers up to 50. X denotes the numbers which are multiples of 3 and Y denotes the odd numbers. Which of the following is/are correct?

1. $P\left(X\right) = \frac{8}{25}$
2. $P\left(Y\right) = \frac{1}{2}$

Select the correct answer using the code given below.

A. 1 only

B. 2 only

C. Both 1 and 2

D. Neither 1 nor 2

Marks:[2.50]

Q.No.104: For two events A and B, let $P\left(A\right) = \frac{1}{2}$, $P\left(A \cup B\right) = \frac{2}{3}$ and $P\left(A \cap B\right) = \frac{1}{6}$. What is $P\left(\overline{A} \cap B\right)$ equal to?

A. $\frac{1}{6}$

B. $\frac{1}{4}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Marks:[2.50]

Q.No.105: Consider the following statements:

1. Coefficient of variation depends on the unit of measurement of the variable.
2. Range is a measure of dispersion.
3. Mean deviation is the least when measured about median.

Which of the above statements are correct?

A. 1 and 2 only

B. 2 and 3 only

 y

D. 1, 2 and 3

Marks:[2.50]

Q.No.106: Given that the arithmetic mean and standard deviation of a sample of 15 observations are 24 and 0, respectively. Then which one of the following is the arithmetic mean of the smallest five observations in the data?

A. 0

B. 8

C. 16

D. 24

Marks:[2.50]

Q.No.107: Which of the following can be considered as the appropriate pair of values of regression coefficient of *y* on *x* and regression coefficient of *x* on *y*?

A. (1, 1)

B. (−1, 1)

C. $\left(-\frac{1}{2},\ 2\right)$

D. $\left(\frac{1}{3},\ \frac{10}{3}\right)$

Marks:[2.50]

Q.No.108: Let *A* and *B* be two events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{6}$ and $P\left(A \cap B\right) = \frac{1}{12}$. What is $\mathrm{P}\left(B \middle| \overline{A}\right)$ equal to?

A. $\frac{1}{5}$

B. $\frac{1}{7}$

C. $\frac{1}{8}$

D. $\frac{1}{10}$

Marks:[2.50]

Typesetting math: 100%

Q.No.109: In a binomial distribution, the mean is $\frac{2}{3}$ and the variance is $\frac{5}{9}$. What is the probability that $X = 2$?

A. $\frac{5}{36}$

B. $\frac{25}{36}$

C. $\frac{25}{216}$

D. $\frac{25}{54}$

Marks:[2.50]

Q.No.110: The probability that a ship safely reaches a port is $\frac{1}{3}$. The probability that out of 5 ships, at least 4 ships would arrive safely is

A. $\frac{1}{243}$

B. $\frac{10}{243}$

C. $\frac{11}{243}$

D. $\frac{13}{243}$

Marks:[2.50]

Q.No.111: What is the probability that at least two persons out of a group of three persons were born in the same month (disregard the year)?

A. $\frac{33}{144}$

B. $\frac{17}{72}$

C. $\frac{1}{144}$

D. $\frac{2}{9}$

Marks:[2.50]

Q.No.112: It is given that $\overline{X} = 10$, $\overline{Y} = 90$, $\sigma_X = 3$, $\sigma_Y = 12$ and $r_{XY} = 0.8$. The regression equation of X and Y is

A. $Y = 3.2X + 58$

B. $X = 3.2Y + 58$

C. $X = -8 + 0.2Y$

D. $Y = -8 + 0.2X$

Marks:[2.50]

Typesetting math: 100%

Q.No.113: If $P(B) = \frac{3}{4}$, $P\left(A \cap B \cap \overline{C}\right) = \frac{1}{3}$ and $P\left(\overline{A} \cap B \cap \overline{C}\right) = \frac{1}{3}$, then what is $P(B \cap C)$ equal to?

A. $\frac{1}{12}$

B. $\frac{3}{4}$

C. $\frac{1}{15}$

D. $\frac{1}{9}$

Marks:[2.50]

Q.No.114: The following table gives the monthly expenditure of two families:

	Expenditure (in Rs)	
Items	Family A	Family B
Food	3,500	2,700
Clothing	500	800
Rent	1,500	1,000
Education	2,000	1,800
Miscellaneous	2,500	1,800

In constructing a pie diagram to the above data, the radii of the circles are to be chosen by which of the following ratios?

A. 1 : 1

B. 10 : 9

C. 100 : 91

D. 5 : 4

Marks:[2.50]

Q.No.115: If a variable takes values 0, 1, 2, 3, ... , n with frequencies 1, $C(n, 1)$, $C(n, 2)$, $C(n, 3)$, ... , $C(n, n)$ respectively, then the arithmetic mean is

A. $2n$

B. $n + 1$

C. n

D. $\frac{n}{2}$

Marks:[2.50]

Q.No.116: In a multiple choice test, an examinee either knows the correct answer with probability p, or guesses with probability $1 - p$. The probability of answering a question correctly is $\frac{1}{m}$, if he or she merely guesses. If the examinee answers a question correctly, the probability that he or she really

 er is

A. $\frac{mp}{1+mp}$
B. $\frac{mp}{1+(m-1)p}$
C. $\frac{(m-1)p}{1+(m-1)p}$
D. $\frac{(m-1)p}{1+mp}$

Marks:[2.50]

Q.No.117: If x_1 and x_2 are positive quantities, then the condition for the difference between arithmetic mean and the geometric mean to be greater than 1 is

A. $x_1 + x_2 > 2\sqrt{x_1 x_2}$
B. $\sqrt{x_1} + \sqrt{x_2} > \sqrt{2}$
C. $\left|\sqrt{x_1} - \sqrt{x_2}\right| > \sqrt{2}$
D. $x_1 + x_2 < 2\left(\sqrt{x_1 x_2} + 1\right)$

Marks:[2.50]

Q.No.118: Consider the following statements:
1. Variance is unaffected by the change of origin and change of scale.
2. Coefficient of variance is independent of the unit of observations.
Which of the statements given above is/are correct?

A. 1 only
B. 2 only
C. Both 1 and 2
D. Neither 1 nor 2

Marks:[2.50]

Q.No.119: Five sticks of lengths 1, 3, 5, 7 and 9 feet are given. Three of these sticks are selected at random. What is the probability that the selected sticks can form a triangle?

A. 0.5
B. 0.4
C. 0.3
D. 0

Marks:[2.50]

Q.No.120: The coefficient of correlation when coefficients of regression are 0.2 and 1.8 is

Typesetting math: 100%

B. 0.2
C. 0.6
D. 0.9

Marks:[2.50]

Typesetting math: 100%