JEE Main 25 July 2022(First Shift)

Total Time: 180

Total Marks: 300.0

Solution 1

[η] = $[ML^{-1}T^{-1}]$
Now if [η] = $[P]^a\ [A]^b\ [T]^c$
⇒ $[ML^{-1}T^{-1}] = [MLT^{-1}]^a[L^2]^b[T]^c$
⇒ $a = 1,\ a + 2b = -1,\ -a + c = -1$
⇒ $a = 1,\ b = -1,\ c = 0$
⇒ [η] = $[P]\ [A]^{-1}\ [T]^0$
= $[PA^{-1}T^0]$

Hence, the correct answer is option A.

Solution 2

Electric displacement $\left(\vec{D}\right) = \varepsilon_0\vec{E}$

$$\Rightarrow \left[\vec{D}\right] = \left[\varepsilon_0\right]\left[\overline{E}\right] = \left[M^{-1}L^{-3}T^4A^2\right]\left[M^1L^1A^{-1}T^{-3}\right]$$

$$\Rightarrow \left[\vec{D}\right] = \left[L^{-2}T^1A^1\right]$$

[Surface charge density] $= \frac{[Q]}{[A]}$

[σ] = $[ATL^{-2}]$

$\Rightarrow \vec{D}$ and σ

have same dimensions

Hence, the correct answer is option A.

Solution 3

Distance travelled = 60 m
⇒ Angle covered = 135°
Displacement $= 2R\sin\left(\frac{135^\circ}{2}\right)$
$= 2\left(\frac{60}{135} \times \frac{180}{\pi}\right)\left[\frac{1-\cos(135^\circ)}{2}\right]^{\frac{1}{2}}$
$= 2\left(\frac{180}{\pi}\right)(0.85)^{\frac{1}{2}}$
$\approx 47 \text{ m}$

Hence, the correct answer is option B.

Solution 4

$v = 3x^2 + 4$
at $x = 0$, $v_1 = 4$ m/s
$x = 2$, $v_2 = 16$ m/s
⇒ Work done = Δ kinetic energy
$= \frac{1}{2} \times m\left(v_2^2 - v_1^2\right)$
$= \frac{1}{4}(256 - 16)$
$= 60 \text{ J}$

Hence, the correct answer is option B.

Solution 5

$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$

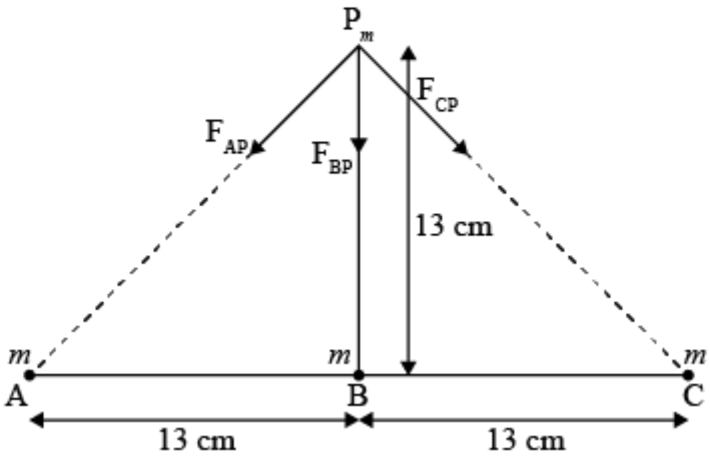
$v = \sqrt{\frac{2Sg\sin\theta}{1+\frac{K^2}{R^2}}}$

$\Rightarrow \frac{v_c}{v_{ss}} = \sqrt{\frac{1+\frac{K_{ss}^2}{R^2}}{1+\frac{K_c^2}{R^2}}} = \sqrt{\frac{1+\frac{2}{5}}{1+\frac{1}{2}}}$

$\Rightarrow \sqrt{\frac{\frac{7}{5}}{\frac{3}{2}}} = \sqrt{\frac{14}{15}}$

Hence, the correct answer is option D.

Solution 6



$$m=100\ \text{kg}$$

$$F_{AP}=\frac{Gm^2}{\left(13\sqrt{2}\right)^2}$$

$$F_{BP}=\frac{Gm^2}{13^2}$$

$$F_{CP}=\frac{Gm^2}{\left(13\sqrt{2}\right)^2}$$

$$F_{\text{net}}\ =\ F_{BP}\ +\ F_{AP}\ \cos 45^\circ\ +\ F_{CP}\ \cos 45^\circ$$

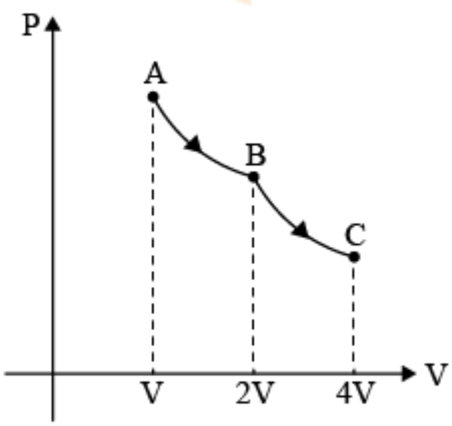
$$\frac{Gm^2}{13^2}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$=\frac{G100^2}{169}(1+0.707)$$

$$\simeq\ 100\,G$$

Hence, the correct answer is option B.

Solution 7



Let *AB* is isothermal process and *BC* is adiabatic process then for *AB* process
$P_A V_A = P_B V_B$
$\Rightarrow P_B = 10^7 \text{ Nm}^{-2}$
For process *BC*
$$P_B V_B^r = P_C V_C^r$$
$P_C = 3.536 \times 10^6$ Pa

Hence, the correct answer is option B.

Solution 8

Because KE ∝ T so A is correct, B is incorrect, statement C can not be said, statement D is contradicting it self, statement E is incorrect (Isothermal process) So No answer correct (Bonus) If the statement of D would have been. "Pressure of gas increases with increase in temperature at constant volume, "then statement D would have been correct, so in that case answer would have been 'A'

Disclaimer: No option is correct.

Solution 9

Both the springs are in parallel combination in both the diagrams so
$$T_1 = 2\pi\sqrt{\frac{3m}{2k}}$$
and $T_2 = 2\pi\sqrt{\frac{m}{3k}}$

So, $\frac{T_1}{T_2} = \frac{3}{\sqrt{2}}$

Hence, the correct answer is option A.

Solution 10

$Q = CV$
As capacitance is constant $Q \propto V$
and $V_f = \frac{Q_f}{C} = \frac{5}{2\times10^{-6}} = 2.5 \times 10^6 \text{ V}$

Hence, the correct answer is option A.

Solution 11

We know that
$$R = \frac{mv}{Bq} = \sqrt{\frac{2mK}{Bq}}$$

$$\Rightarrow \text{Ratio of radii} = \frac{R_1}{R_2} = \sqrt{\frac{m_1}{m_2}\frac{q_2}{q_1}}$$

$$\Rightarrow \frac{6}{5} = \sqrt{\frac{9}{4}\frac{q_2}{q_1}}$$

$$\Rightarrow \frac{q_1}{q_2} = \frac{3}{2} \times \frac{5}{6} = \frac{5}{4}$$

Hence, the correct answer is option B.

Solution 12

Resonant frequency $= \frac{1}{\sqrt{LC}} = \omega_0$

⇒ If we decrease *C*, ω_0 would increase

⇒ Another capacitor should be added in series.

Hence, the correct answer is option C.

Solution 13

We know ϕ = *Mi*

Let *i* current be flowing in the larger loop

$$\Rightarrow \phi = \left[4 \times \frac{\mu_0 i}{4\pi\left(\frac{L}{2}\right)}\left[\sin 45^\circ + \sin 45^\circ\right]\right] \times \text{Area}$$

$$= \frac{2\sqrt{2}\mu_0 i}{\pi L} \times l^2$$

$$\Rightarrow M = \frac{\phi}{i} = \frac{2\sqrt{2}\mu_0 l^2}{\pi L}$$

Hence, the correct answer is option C.

Solution 14

$$Z_C = \frac{V}{I}$$

$$\Rightarrow \frac{1}{\omega C} = \frac{230}{6.9}\ \text{M}\Omega$$

$$\Rightarrow C = \frac{6.9}{230\omega}\ \mu\text{F}$$

$$= \frac{6.9}{230 \times 600}\ \mu\text{F}$$

$$C = 50\ \text{pF}$$

Hence, the correct answer is option B.

Solution 15

In primary rainbow, observer sees red colour on the top and violet on the bottom.

Hence, the correct answer is option A.

Solution 16

$t_2 - t_1 = 5 \times 10^{-10}$

$\Rightarrow \frac{d}{v_B} - \frac{d}{v_A} = 5 \times 10^{-10}$

and, $\frac{v_B}{v_A} = \frac{\mu_A}{\mu_B} = \frac{1}{2}$

$$\Rightarrow d\left(1 - \frac{v_B}{v_A}\right) = 5 \times 10^{-10} \times v_B$$

$$\Rightarrow d\left(1 - \frac{1}{2}\right) = 5 \times 10^{-10} \times v_B$$

$$\Rightarrow d = 10 \times 10^{-10} \times v_B \mathrm{m}$$

$$\Rightarrow d = 5 \times 10^{-10} \times v_A \mathrm{m}$$

Hence, the correct answer is option A.

Solution 17

$$\because\ K_m = \frac{hc}{\lambda} - \phi$$

$$\Rightarrow K = \frac{1230}{800} - \phi$$

$$\text{and, } 2K = \frac{1230}{500} - \phi$$

$$\Rightarrow 2 \times \frac{1230}{800} - 2\phi = \frac{1230}{500} - \phi$$

$$\Rightarrow \phi = 0.615 \text{ eV}$$

Hence, the correct answer is option C.

Solution 18

$$\because\ mvr = \frac{nh}{2\pi}$$

$$\Rightarrow mv = \frac{nh}{2\pi r}$$

Hence, the correct answer is option A.

Solution 19

$$\because\ \vec{\mu} = \frac{q\vec{L}}{2m}$$

$$\Rightarrow \vec{\mu} = \frac{-e\vec{L}}{2m}$$

Hence, the correct answer is option B.

Solution 20

Given circuit is equivalent to an AND gate.

$\therefore$ A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Hence, the correct answer is option A.

Solution 21

$F_R d = \frac{1}{2}\mathrm{mv}^2$

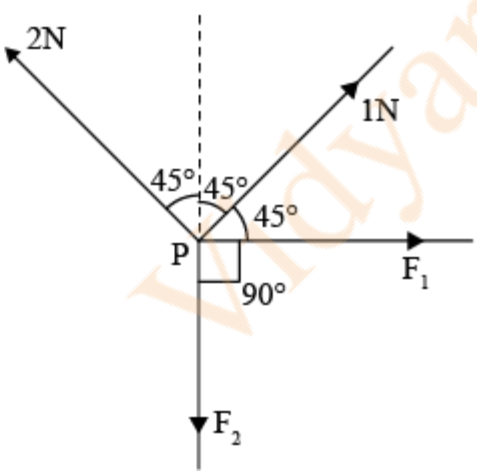
$\frac{d_2}{d_1} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{1}{3}\right)^2$

$d_2 = d_1 \times \frac{1}{9} = 3\ \mathrm{m}$

Solution 22

$F_1 = +2 \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$F_2 = 2 \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$



$\frac{F_1}{F_2} = \frac{1}{3} = \frac{1}{x}$

$\Rightarrow x = 3$

Solution 23

$$\frac{\frac{F}{A}}{\frac{\Delta L}{L}} = Y$$

$$\Rightarrow \Delta L = \frac{FL}{AY}$$

$$\frac{\Delta L_2}{\Delta L_1} = \left(\frac{F_2}{F_1}\right) \times \left(\frac{L_2}{L_1}\right) \times \left(\frac{A_1}{A_2}\right)$$

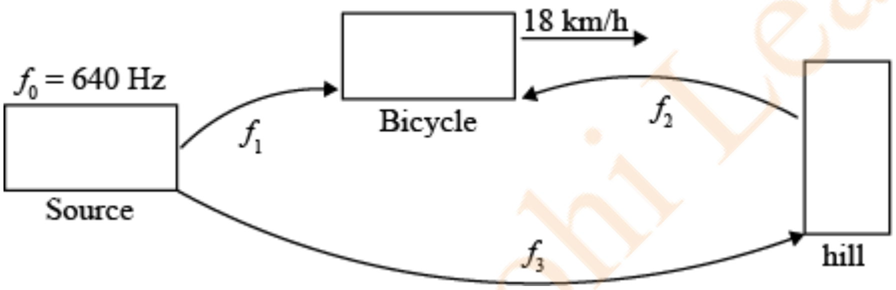
$$= 4 \times 4 \times \frac{1}{16} = 1$$

$$\Delta\ L_2 = \Delta L_1 = 5\ \text{cm.}$$

Solution 24

$\Delta L_1 = \alpha_1 L_1 \Delta T$
$\Delta L_2 = \alpha_2 L_2 \Delta T$
$\alpha_1 L_1 = \alpha_2 L_2$
$1.2 \times 10^{-5} \times L_1 = 1.8 \times 10^{-5} L_2$
$L_1 = \frac{1.8}{1.2} \times 40 = 60\ \text{cm}$

Solution 25



$$f_1 = f_0\left(\frac{320-5}{320}\right) = 640\left(\frac{315}{320}\right)$$
$$= 630\ \text{Hz}$$

$f_3 = f_0$ [No relative motion]

$$f_1 = f_0\left(\frac{320+5}{320}\right) = 640\left(\frac{325}{320}\right)$$
$$= 650$$

Beat frequency = $f_2 - f_1$
= 650 – 630 = 20 Hz

Solution 26

ρ = 2 μc/cm^3
R = 6 m
Number of lines of force per unit area = Electric field at surface.

$= \frac{KQ}{R^2}$

$= \frac{1}{4\pi\varepsilon_0} \frac{\rho \frac{4}{3}\pi R^3}{R^2}$

$= \frac{\rho R}{3\in_0}$

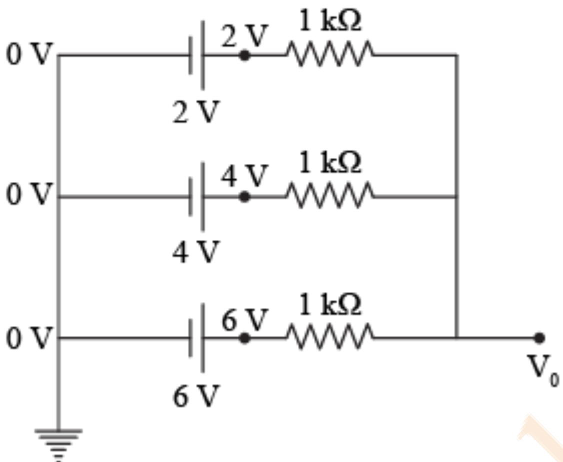
$= \frac{2\times10^{-6}\times10^{6}\times6}{3\times8.85\times10^{-12}}$

$= 0.45197 \times 10^{12}$

$= 45.19 \times 10^{10}$ N/C

$\simeq 45 \times 10^{10}$

Solution 27



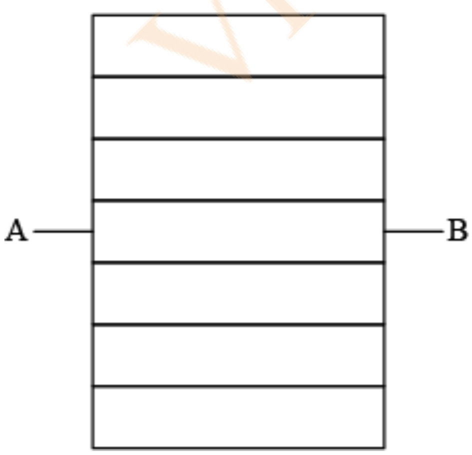
Using Kirchhoff's junction rule.

$\frac{2-V_0}{1} + \frac{4-V_0}{1} + \frac{6-V_0}{1} = 0$

$12 - 3V_0 = 0$

$V_0 = 4\text{V}$

Solution 28



RAB = R

$$R = \frac{1}{8}\left(\text{Resistance of one wire}\right)$$

$$= \frac{1}{8}\rho\frac{l}{\pi\frac{d^2}{4}} = \frac{\rho l}{2\pi d^2}$$

Resistance of copper wire of length 2*l* and diameter *x* = *R*.

$$\rho\frac{2l}{\pi\frac{x^2}{4}} = R$$

$$\frac{8\rho l}{\pi x^2} = \frac{\rho l}{2\pi d^2}$$

$$16d^2 = x^2$$

$$x = 4d$$

Solution 29

Energy corresponding to wavelength 4000 Å

$$E = \frac{hc}{\lambda}$$

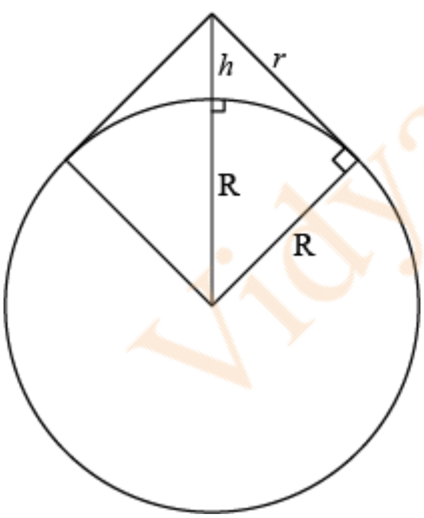
$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}\times1.6\times10^{-19}}\text{eV}$$

$$= \frac{12400}{4000}$$

$$= 3.1\text{ eV}$$

$$\approx 3\text{ eV}$$

Solution 30



$r=\sqrt{(h+R)^2-R^2}\cong\sqrt{2hR}$

$A=\frac{6.03\times10^5}{100}$

$\pi r^2=6.03\times10^3$

$\pi 2Rh=6.03\times10^3$

$h=\frac{6.03\times10^3}{2\times\pi\times R}=0.015\times10\times10^3\ \mathrm{m}$

$=150\ \mathrm{m}$

Solution 31

$SO_2Cl_2 + 2H_2O \rightarrow H_2SO_4 + 2HCl$
Moles of NaOH required for complete neutralisation of resultant acidic mixture = 16 moles
And 1 mole of SO_2Cl_2 produced 4 moles of H^+.
∴ Moles of SO_2Cl_2 used will be = $\frac{16}{4}$ = 4 moles

Hence, the correct answer is option C.

Solution 32

If *n* = 3, then possible values of *l* = 0, 1, 2
But in option (C), the value of *l* is given '3', this is not possible.

Hence, the correct answer is option C.

Solution 33

ΔT_f of formic acid = 0.0405°C
Concentration = 0.5 mL/L
and density = 1.05 g/mL
∴ Mass of formic acid in solution = 1.05 × 0.5 g = 0.525 g
∴ According to Van't Hoff equation,
$\Delta T_f = iK_f \cdot m$

$0.0405=i\times1.86\times\frac{0.525}{46\times1}$

(Assuming mass of 1 L water = kg)

$i=\frac{0.0405\times46}{1.86\times0.525}=1.89\approx1.9$

Hence, the correct answer is option C.

Solution 34

$$\underset{\substack{20\ \mathrm{mL},\ 0.1\ \mathrm{M}\\ 2\ \mathrm{mmole}\\ ---}}{\mathrm{NH_4Cl}} + \underset{\substack{40\ \mathrm{mL},\ 0.05\ \mathrm{M}\\ 2\ \mathrm{mmole}\\ ---}}{\mathrm{HCl}} \longrightarrow \underset{2\ \mathrm{mmoles}}{\mathrm{NH_2Cl} + \mathrm{H_2O}}$$

∴ In final solution 2 millimoles of NH_4Cl is present.

$$\therefore \; [NH_4\,Cl] = \frac{1}{30}\text{molar}$$

$$\begin{aligned}\rho H &= \frac{1}{2}\left[\rho k_w - \rho k_b - \log\ C\right] \\ &= \frac{1}{2}\left[14 - 5 - (-1.48)\right] \\ &= 5.24\end{aligned}$$

Hence, the correct answer is option C.

Solution 35

Here, we have to match the reactions with their correct catalyst:

(A) $N_2(g) + 3H_2(g) \xrightarrow{Fe_x\,O_y + K_2O + \ Al_2\,O_3} 2NH_3(g)$

(B) $CO(g) + 3H_2(g) \xrightarrow{Ni} CH_4(g) + H_2O(g)$

(C) $CO(g) + H_2(g) \xrightarrow{Cu} HCHO(g)$

(D) $CO(g) + 2H_2(g) \xrightarrow{Cu\,/\,ZnO\text{–}Cr_2\,O_3} CH_3OH(g)$

Hence, the correct answer is option C.

Solution 36

The element with electronic configuration [Rn] $5f^{14}6d^{1}7s^{2}$ has atomic number →103

∴ Its IUPAC name is : Unniltrium

Hence, the correct answer is option D.

Solution 37

The compound(s) that are removed as a slag during the extraction of copper is:

$$FeS \xrightarrow{O_2/\,SiO_2} \underset{\text{slag}}{FeSiO_3} + SO_2$$

∴ Only iron oxide (FeO) formed slag during extraction of copper.

Hence, the correct answer is option D.

Solution 38

The reaction of $KMnO_4$ with H_2O_2 in acidic medium is as

$2KMnO_4 + 3H_2SO_4 + 5H_2O_2 \rightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 5O_2$

∴ Mn^{2+} will be formed as the product.

Hence, the correct answer is option A.

Solution 39

The increasing order of density of alkali metals as

$$\underset{0.53}{\text{Li}} < \underset{0.86}{\text{K}} < \underset{0.97}{\text{Na}} < \underset{1.53}{\text{Rb}} < \underset{1.87}{\text{Cs}} \ \left(\text{in g/dm}^3\right)$$

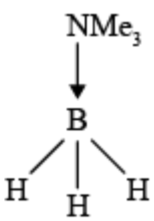
'K' metal has less density as compare to 'Na' metal.

Hence, the correct answer is option A.

Solution 40

$$2\,BF_3 + 6\,NaH \xrightarrow{450\text{ K}} \underset{(A)}{B_2H_6} + 6\,NaF$$

$$\underset{(A)}{B_2F_6} + 2\,NMe_3 \longrightarrow \underset{(B)}{2BH_3 \cdot NMe_3}$$



$\therefore$ Geometry of boron will be tetrahedral.

Hence, the correct answer is option B.

Solution 41

$$Br_2 + \underset{(\text{Excess})}{5F_2} \longrightarrow 2\,BrF_5$$

If BrF_5 undergoes hydrolysis it will produce halide.

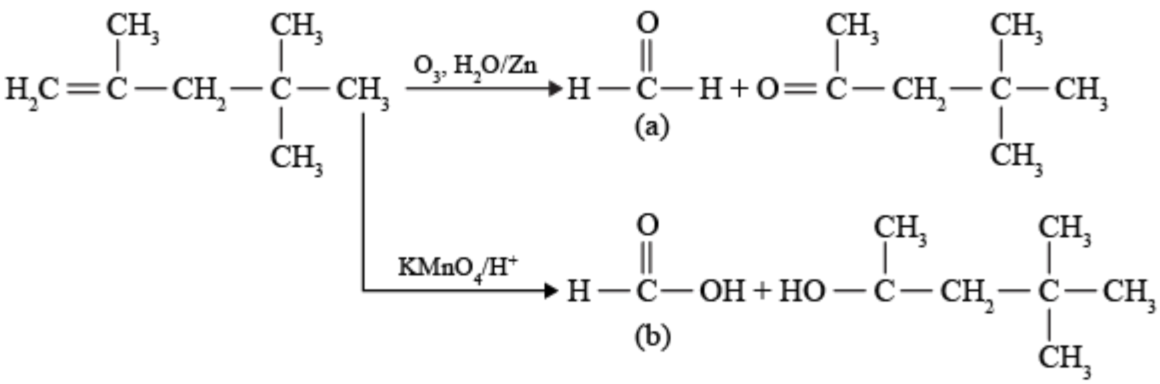
Hence, the correct answer is option B.

Solution 42

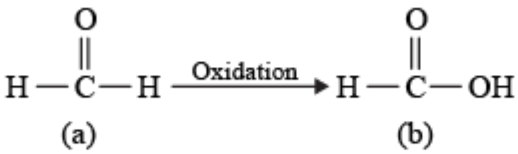
Photochemical smog contain:
Ozone, nitric oxide, organic compounds, nitrogen dioxide, formaldehyde.
$\therefore$ SO_2 is not the part of photochemical smog.

Hence, the correct answer is option C.

Solution 43

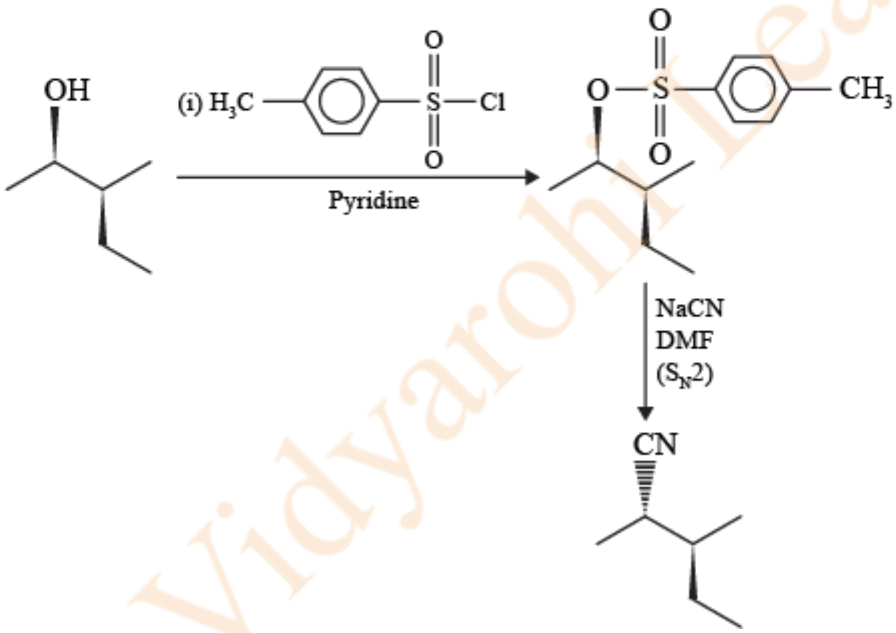


*Ants produces formic acid in their venom gland.



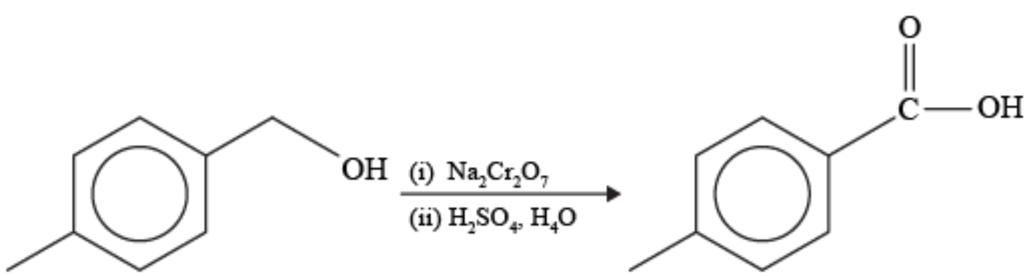
Hence, the correct answer is option D.

Solution 44



Hence, the correct answer is option B.

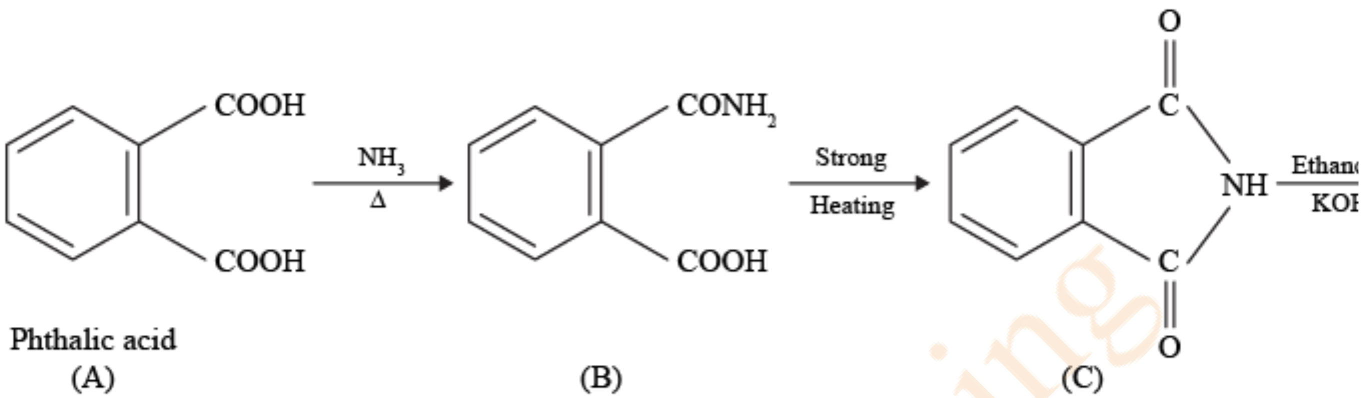
Solution 45

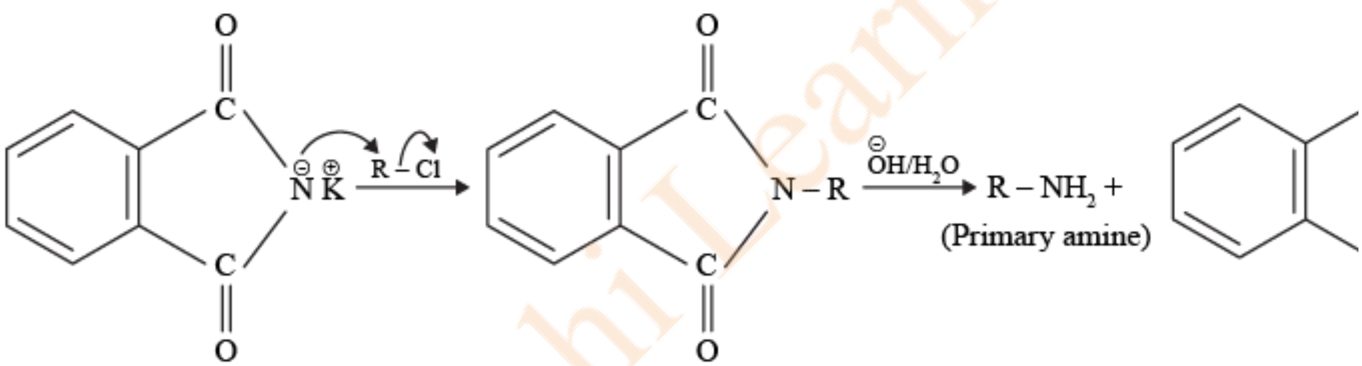


$Na_2Cr_2O_7$, H_2SO_4/H_2O is the strongest oxidising agent and it will oxidise 1° alcohol into acids.

Hence, the correct answer is option D.

Solution 46



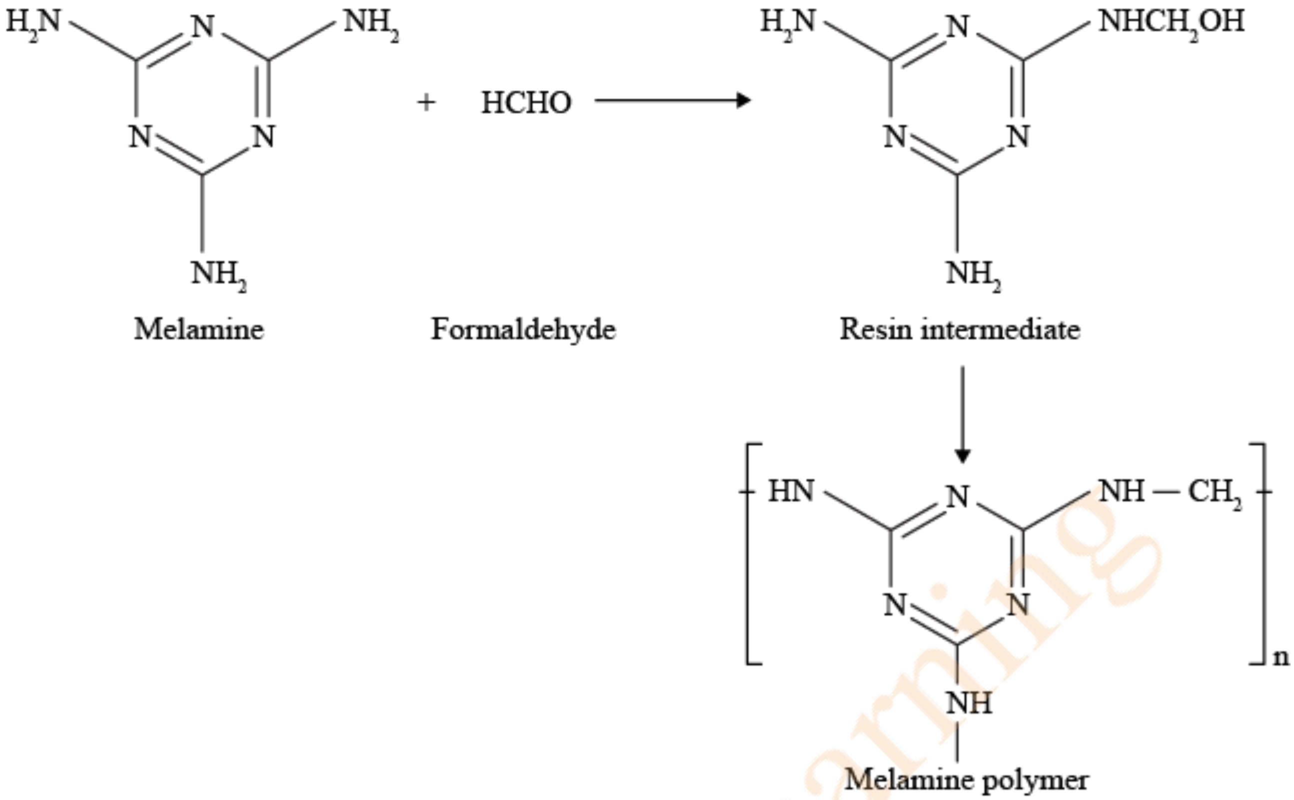


All the given reactions can be explained if organic compound (A) is phthalic acid.

Hence, the correct answer is option C.

Solution 47

Melamine polymer is formed by the condensation polymerisation of melamine and formaldehyde.



Hence, the correct answer is option A.

Solution 48

During the denaturation of proteins hydrogen bonds are disturbed. As a result, the secondary and tertiary structures are destroyed but the primary structures remain intact.

Hence, the correct answer is option A.

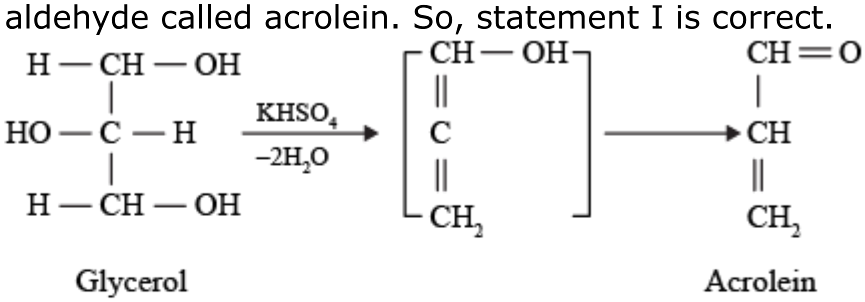
Solution 49

Drugs that bind to the receptor site and inhibit its natural function are called Antagonists.

Hence, the correct answer is option B.

Solution 50

Glycerol, on heating with $KHSO_4$, undergoes dehydration to give unsaturated aldehyde called acrolein. So, statement I is correct.



Acrolein has piercing unpleasant smell. So, statement-II is incorrect.

Hence, the correct answer is option C.

Solution 51

According to molecules orbital theory. The electronic configurations of the given species are

$N_2: \ \sigma 1s^2 \sigma * 1s^2 \sigma 2s^2 \sigma * 2s^2 \pi 2p_x^2 = \pi 2p_y^2 \sigma 2p_z^2$

$N_2^+: \ \sigma 1s^2 \sigma * 1s^2 \sigma 2s^2 \sigma * 2s^2 \pi 2p_x^2 = \pi 2p_y^2 \sigma 2p_z^1$

$N_2^-: \ \sigma 1s^2 \sigma * 1s^2 \sigma 2s^2 \sigma * 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi * 2p_x^1$

$N_2^{2-}: \ \sigma 1s^2 \sigma * 1s^2 \sigma 2s^2 \sigma * 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi * 2p_x^1 = \pi * 2p_y^1$

$O_2: \ \sigma 1s^2 \sigma * 1s^2 \sigma 2s^2 \sigma * 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi * 2p_x^1 = \pi * 2p_y^1$

$O_2^+: \ \sigma 1s^2 \sigma * 1s^2 \sigma 2s^2 \sigma * 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi * 2p_1^x$

$O_2^-: \ \sigma 1s^2 \sigma * 1s^2 \sigma 2s^2 \sigma * 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi * 2p_x^2 = \pi * 2p_y^1$

$O_2^{2-}: \ \sigma 1s^2 \sigma * 1s^2 \sigma 2s^2 \sigma * 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi * 2p_x^2 = \pi * 2p_y^2$

Diamagnetic species are N_2 and O_2^{2-}

∴ Number of species showing diamagnetism = 2

Solution 52

Enthalpy of combustion of propane, graphite and H_2 at 298K are

$$C_3H_8(g) + 5O_2(g) \to 3\,CO_2(g) + 4H_2O(l), \ \Delta H_1 = -2220 \text{ kJ mol}^{-1}$$

$$C(\text{graphite}) + O_2(g) \to CO_2(g), \ \Delta H_2 = -393.5 \text{ kJ mol}^{-1}$$

$$H_2(g) + \tfrac{1}{2}O_2(g) \to H_2O(l), \ \Delta H_3 = -285.8 \text{ kJ mol}^{-1}$$

The desired reaction is $3C(\text{graphite}) + 4H_2(g) \to C_3H_8(g)$

$$\Delta H_f = 3\,\Delta H_2 + 4\,\Delta H_3 - \Delta H_1$$

$$= 3(-393.5) + 4(-285.8) - (-2220)$$

$$= -103.7 \text{ kJ mol}^{-1}$$

$$|\Delta H_f| \simeq 104 \text{ kJ mol}^{-1}$$

Solution 53

From ideal gas equation,

$P \propto \frac{1}{V}$

$P_1V_1 = P_2V_2$

Pressure of the gas = 4 – 0.4 = 3.6 atm

$3.6\ V_1 = P_2\ (2V_1)$

$P_2 = 1.8$ atm

Hence, new pressure of moist gas is 1.8 + 0.4 = 2.2 atm = 22×10^{-1} atm

Solution 54

$$A:\ Zn \rightarrow Zn^{2+} + 2e^-$$
$$C:\ Sn^{+x} + xe^- \rightarrow Sn$$
$$E^o_{Cell} = E^o_{Zn|Zn^{2+}} + E^o_{Sn^{+x}|Sn}$$
$$\Rightarrow 0.763 + 0.008 = 0.771\ V$$

From Nernst equation,

$$E_{Cell} = E^o_{Cell} \frac{-2.303\ RT}{nF} \log Q$$
$$0.801 = 0.771 - \frac{0.06}{n} \log\ 10^{-2}$$
$$0.03 = \frac{0.06}{n} \times 2$$
$$n = 4$$

Solution 55

$(t_{1/2})_A$ = 240 s when P = 500 torr

$(t_{1/2})_A$ = 4 min = 4 × 60 = 240 sec when P = 250 torr

If means half-life is independent of concentration of reactant present.

∴ Order of reaction = 1

Solution 56

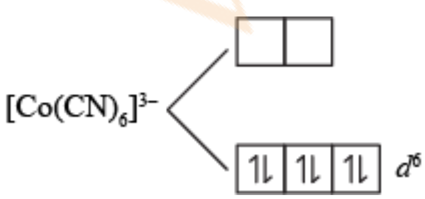
In all complexes, Co is present in +3 oxidation state and all complexes are low spin or inner orbital complex.

Stronger the ligand, higher the crystal field splitting.

So, order of crystal field splitting is

$[Co(CN)_6]^{3-} > [Co(NH_3)_6]^{3+} > [Co(NH_3)_5(H_2O)]^{3+} > [CoCl(NH_3)_5]^{2+}$.

Shortest wavelength is shown by complex having maximum crystal field splitting.



Spin only magnetic moment = $\sqrt{0(0+2)} = 0\ B.M$

Solution 57

Co^{3+} will not liberate H_2 gas an reaction with dilute acid

$E^{o}_{Co^{3+}/Co^{2+}} = +1.97$

And Co^{3+} has electronic configuration = [Ar] $3d^6$

∴ 4 unpaired e^- are present in it

∴ Spin-only magnetic moment $= \sqrt{4(4+2)}\ \mathrm{B.M.} = 4.92\ \mathrm{B.M.} \approx 5\ \mathrm{B.M.}$

Solution 58

NH_3 gas is neutralized by 2·5 mL of 2 M H_2SO_4

∴ Moles of NH_3 neutralized = 2·5 × 2 × 2 millimoles = 10 × 10^{-3} moles

∴ Weight of N present in compound will be = 10 × 10^{-3} × 14 = 0.14 g

∴ % of 'N' in compound

$= \frac{0.14}{0.25} \times 100$

$= 56\%$

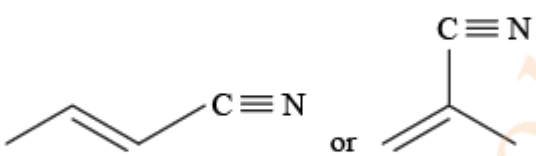
Solution 59

C_4H_5N

$$\mathrm{DBE} = (C+1) - \left(\frac{H+X-N}{2}\right)$$

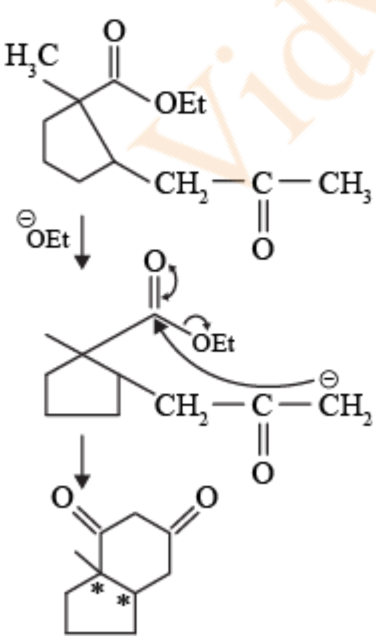
$$= 4+1-\left(\frac{5-1}{2}\right) = 5-2 = 3$$

3 double bond equivalent are present in compound



Only1 sp^3 hybridised carbon is there (Keeping compound as acyclic)

Solution 60



2 chiral carbons are there in product A.

Solution 61

Case 1: If $f(3) = 3$ then $f(1)$ and $f(2)$ take 1 OR 2
No. of ways = 2·6 = 12
Case 2: If $f(3) = 5$ then $f(1)$ and $f(2)$ take 2 OR 3 OR 1 and 4
No. of ways = 2·6·2 = 24
Case 3: If $f(3) = 2$ then $f(1) = f(2) = 1$
No. of ways = 6
Case 4: If $f(3) = 4$ then $f(1) = f(2) = 2$
No. of ways = 6
OR $f(1)$ and $f(2)$ take 1 and 3
No. of ways = 12
Case 5: If $f(3) = 6$ then $f(1) = f(2) = 3 \Rightarrow 6$ ways
OR $f(1)$ and $f(2)$ take 1 and 5 ⇒ 12 ways
OR $f(2)$ and $f(1)$ take 2 and 4 ⇒ 12 ways

Hence, the correct answer is option B.

Solution 62

$x^4 + x^3 + x^2 + x + x + 1 = 0 \text{ OR } \frac{x^5-1}{x-1} = 0 \left(x \neq 1\right)$

$\text{So roots are } e^{\frac{i2\pi}{5}}, e^{\frac{i4\pi}{5}}, e^{\frac{i6\pi}{5}}, e^{\frac{i8\pi}{5}}$

$\text{i.e. } \alpha, \beta, \gamma \text{ and } \delta$

$\text{From properties of } n^{\text{th}} \text{ root of unity}$

$1^{2021} + \alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} = 0$

$\Rightarrow \alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} = -1$

Hence, the correct answer is option B.

Solution 63

$S_n = \left\{z \in C : \left|z - 3 + 2i\right| = \frac{n}{4}\right\}$ represents a circle with centre $C_1(3, -2)$ and radius $r_1 = \frac{n}{4}$
Similarly T_n represents circle with centre $C_2(2, -3)$ and radius $r_2 = \frac{1}{n}$
As $S_n \cap T_n = \phi$
$C_1C_2 > r_1 + r_2$ OR $C_1C_2 < |r_1 - r_2|$
$\sqrt{2} > \frac{n}{4} + \frac{1}{2}$ OR $\sqrt{2} < \left|\frac{n}{4} - \frac{1}{n}\right|$
$n = 1, 2, 3, 4$ n may take infinite values

***Disclaimer*:** *No option is correct.*

Solution 64

$$\Delta = \begin{vmatrix} 3\sin 3\theta & -1 & 1 \\ 3\cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix}$$

= 3sin3θ(7) + 1(21cos2θ – 18) + 1(21cos2θ – 24)

Δ = 21sin3θ + 42cos2θ – 42

For no solution

sin3θ + 2cos2θ = 2

⇒ sin3θ = 2·2sin^{2}θ

⇒ 3sinθ – 4sin^{3}θ = 4sin^{2}θ

⇒ sinθ(3 – 4sinθ – 4sin^{2}θ) = 0

$\sin\theta = 0 \text{ OR } \sin\theta = \frac{1}{2}$

$\theta = \pi,\ 2\pi,\ 3\pi,\ \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{13\pi}{6},\ \frac{17\pi}{6}$

Hence, the correct answer is option B.

Solution 65

$$\lim_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$$

$$=\lim_{n\to\infty}\left[\sqrt{1-\frac{1}{n}-\frac{1}{n^2}}+\alpha+\frac{\beta}{n}\right]=0$$

$\therefore\ \alpha\ =-1$

Now,

$$\lim_{n\to\infty} n\left[\left(1-\frac{1}{n}-\frac{1}{n^2}\right)^{\frac{1}{2}}+\frac{\beta}{n}-1\right]=0$$

$$=\lim_{n\to\infty}\frac{\left(1-\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n^2}\right)+\ldots\right)+\frac{\beta}{n}-1}{\frac{1}{n}}=0$$

$\Rightarrow \beta-\frac{1}{2}=0$

$\therefore\ \beta=\frac{1}{2}$

Now, $8\left(\alpha\ +\ \beta\right)=8\left(-\frac{1}{2}\right)=-4$

Hence, the correct answer is option C.

Solution 66

Given, $f(x)=\underbrace{\left(x^2-2x+7\right)}_{f_1(x)}\underbrace{e^{\left(4x^3-12x^2-180x+31\right)}}_{f_2(x)}$

$f_1(x) = x^2 - 2x + 7$
$f'_1(x) = 2x - 2$, so $f(x)$ is decreasing in $[-3, 0]$ and positive also

$$f_2\left(x\right) = e^{4x^3 - 12x^2 - 180x + 31}$$

$$f_2'\left(x\right) = e^{4x^3 - 12x^2 - 180x + 31}.\ 12x^2 - 24x - 180$$

$$= 12\left(x - 5\right)\left(x + 3\right)\ e^{4x^3 - 12x^2 - 180x + 31}$$

So, $f_2(x)$ is also decreasing and positive in $\{-3, 0\}$
∴ absolute maximum value of $f(x)$ occurs at $x = -3$
∴ $\alpha = -3$

Hence, the correct answer is option B.

Solution 67

$f(x) = y = ax^3 + bx^2 + cx + 5$ ...(i)

$$\frac{dy}{dx} = 3ax^2 + 2bx + c \qquad \ldots\left(\text{ii}\right)$$

Touches x-axis at $P(-2, 0)$
⇒ $y|_{x = -2} = 0$
⇒ $-8a + 4b - 2c + 5 = 0$ ...(iii)
Touches x-axis at $P(-2, 0)$ also implies

$$\frac{dy}{dx}\Big|_{x=-2} = 0$$

$$\Rightarrow 12a\ - 4b + c = 0 \qquad \ldots\left(\text{iv}\right)$$

$y = f(x)$ cuts y-axis at $(0, 5)$

$$\text{Given, } \frac{dy}{dx}\Big|_{x=0} = c = 3 \qquad \ldots\left(\text{v}\right)$$

From (iii), (iv) and (v)

$$a = -\frac{1}{2},\ b = -\frac{3}{4},\ c = 3$$

$$\Rightarrow f\left(x\right) = \frac{-x^2}{2} - \frac{3}{4}x^2 + 3x + 5$$

$$f'\left(x\right) = \frac{-3}{2}x^2 - \frac{3}{2}x + 3$$

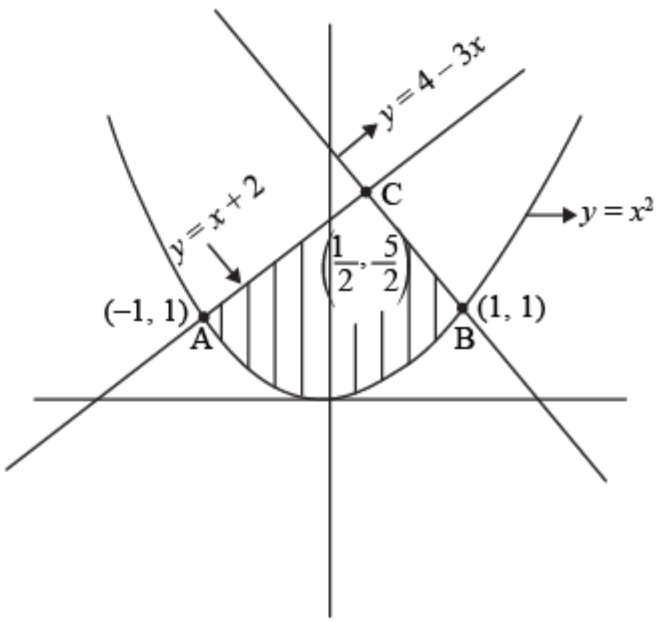
$$= \frac{-3}{2}\left(x + 2\right)\left(x - 1\right)$$

$f'(x) = 0$ at $x = -2$ and $x = 1$
By first derivative test $x = 1$ in point of local maximum
Hence local maximum value of $f(x)$ is $f(1)$
i.e., $\frac{27}{4}$

Hence, the correct answer is option A.

Solution 68

$A = \{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}$



So area of required region

$$A = \int_{-1}^{\frac{1}{2}} \left(x + 2 - x^2\right) dx + \int_{\frac{1}{2}}^{1} \left(4 - 3x - x^2\right) dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{\frac{1}{2}} + \left[4x - \frac{3x^2}{2} - \frac{x^3}{3}\right]_{\frac{1}{2}}^{1}$$

$$= \left(\frac{1}{8} + 1 - \frac{1}{24}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(4 - \frac{3}{2} - \frac{1}{3}\right) - \left(2 - \frac{3}{8} - \frac{1}{24}\right)$$

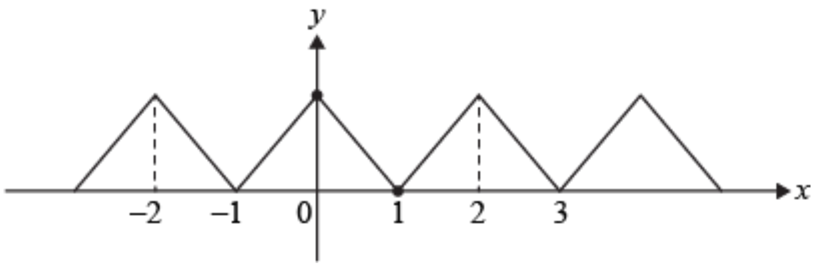
$$= \frac{17}{6}$$

Hence, the correct answer is option B.

Solution 69

$$f(x) = \begin{cases} x - [x], & \text{if } [x] \text{ is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even.} \end{cases}$$

Graph of $f(x)$



$f(x)$ is an even and periodic function

So, $\frac{\pi^2}{10} \int_{-10}^{10} f(x)\cos\pi x \, dx = \frac{\pi^2}{10} \cdot 20 \int_0^1 f(x)\cos\pi x \, dx$

$= 2\pi^2 \int_0^1 (1-x)\cos\pi x \, dx$

$= 2\pi^2 \left\{ (1-x)\frac{\sin\pi x}{\pi}\Big|_0^1 - \frac{\cos\pi x}{\pi^2}\Big|_0^1 \right\} = 4$

Hence, the correct answer is option A.

Solution 70

$\frac{dy}{dx} = \frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}} = e^{2x} - \frac{6e^{-x}}{2+9e^{-2x}}$

$\int dy = \int e^{2x}dx - 3\int \underbrace{\frac{e^{-x}}{1+\left(\frac{3e^{-x}}{\sqrt{2}}\right)^2}}_{\text{put } e^{-x}=1} dx$

$= \frac{e^{2x}}{2} + 3\int \frac{dt}{1+\left(\frac{3t}{\sqrt{2}}\right)^2}$

$= \frac{e^{2x}}{2} + \sqrt{2}\tan^{-1}\frac{3t}{\sqrt{2}} + C$

$y = \frac{e^{2x}}{2} + \sqrt{2}\tan^{-1}\left(\frac{3e^{-x}}{\sqrt{2}}\right) + C$

It is given that the curve passes through

$\left(0,\ \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$

$\frac{1}{2} + \frac{\pi}{2\sqrt{2}} = \frac{1}{2} + \sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right) + C$

$\Rightarrow C = \frac{\pi}{2\sqrt{2}} - \sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$

Now if $\left(\alpha,\ \frac{1}{2}e^{2\alpha}\right)$ satisfies the curve, then

$$\frac{1}{2}e^{2\alpha} = \frac{e^{2\alpha}}{2} + \sqrt{2}\tan^{-1}\left(\frac{3e^{-\alpha}}{\sqrt{2}}\right) + \frac{\pi}{2\sqrt{2}} - \sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$$

$$\tan^{-1}\left(\frac{3}{\sqrt{2}}\right) - \tan^{-1}\left(\frac{3e^{-\alpha}}{\sqrt{2}}\right) = \frac{\pi}{2\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

$$\frac{\frac{3}{\sqrt{2}} - \frac{3e^{-\alpha}}{\sqrt{2}}}{1 + \frac{9}{2}e^{-\alpha}} = 1$$

$$\frac{3}{\sqrt{2}}e^{\alpha} - \frac{3}{\sqrt{2}} = e^{\alpha} + \frac{9}{2}$$

$$e^{\alpha} = \frac{\frac{9}{2} + \frac{3}{\sqrt{2}}}{\frac{3}{\sqrt{2}} - 1} = \frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$$

Hence, the correct answer is option B.

Solution 71

$$\left(x - y^2\right)dx + y\left(5x + y^2\right)dy = 0$$

$$y\frac{dy}{dx} = \frac{y^2 - x}{5x + y^2}$$

Let $y^2 = t$

$$\frac{1}{2} \cdot \frac{dt}{dx} = \frac{t - x}{5x + t}$$

Now substitute, $t = vx$

$$\frac{dt}{dx} = v + x\frac{dv}{dx}$$

$$\frac{1}{2}\left\{v+x\frac{dv}{dx}\right\}=\frac{v-1}{5+v}$$

$$x\frac{dv}{dx}=\frac{2v-2}{5+v}-v=\frac{-3v-v^2-2}{5+v}$$

$$\int\frac{5+v}{v^2+3v+2}dv=\int-\frac{dx}{x}$$

$$\int\frac{4}{v+1}dv-\int\frac{3}{v+2}dv=-\int\frac{dx}{x}$$

$$4\,\text{In}\left|v+1\right|-3\,\text{In}\left|v+2\right|=-\,\text{In}\,x+\text{InC}$$

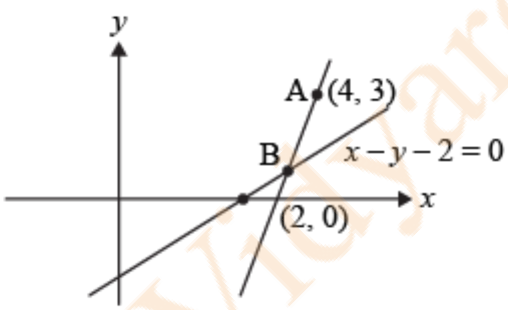
$$\left|\frac{(v+1)^4}{(v+2)^3}\right|=\frac{C}{x}$$

$$\left|\frac{\left(\frac{y^2}{x}+1\right)^4}{\left(\frac{y^2}{x}+2\right)^3}\right|=\frac{C}{x}$$

$$\left|\left(y^2+x\right)^4\right|=C\left|\left(y^2+2x\right)^3\right|$$

Hence, the correct answer is option A.

Solution 72



Let inclination of required line is θ,
So the coordinates of point B can be assumed as
$\left(4-\frac{\sqrt{29}}{3}\cos\theta,\ 3-\frac{\sqrt{29}}{3}\sin\theta\right)$
Which satisfices $x - y - 2 = 0$

$$4-\frac{\sqrt{29}}{3}\cos\theta-\ 3+\frac{\sqrt{29}}{3}\sin\theta\ -2=0$$

$$\sin\theta-\cos\theta=\frac{3}{\sqrt{29}}$$

By squaring

$\sin 2\theta = \frac{20}{29} = \frac{2\tan\theta}{1+\tan^2\theta}$

$\tan\theta = \frac{5}{2}$ only(because slope is greater than 1)

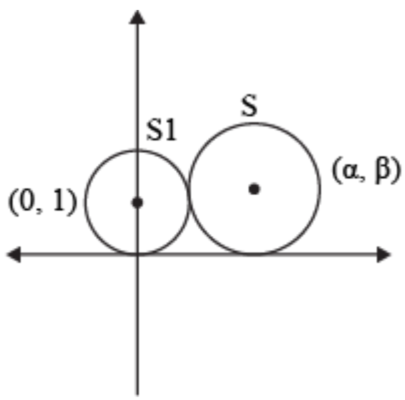
$\sin\theta = \frac{5}{\sqrt{29}}, \cos\theta = \frac{2}{\sqrt{29}}$

$\text{Point B}: \left(\frac{10}{3}, \frac{4}{3}\right)$

Which also satisfies $x + 2y = 6$

Hence, the correct answer is option C.

Solution 73



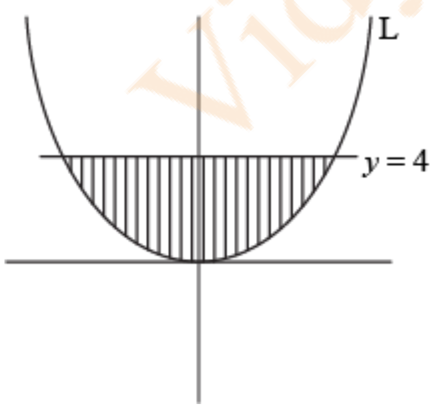
Radius of circle S touching x-axis and centre (α, β) is $|\beta|$. According to given conditions

$\alpha^2 + (\beta - 1)^2 = (|\beta| + 1)^2$

$\alpha^2 + \beta^2 - 2\beta + 1 = \beta^2 + 1 + 2|\beta|$

$\alpha^2 = 4\beta$ as $\beta > 0$

$\therefore$ Required louse is L : $x^2 = 4y$



The area of shaded region $=2\int_0^4 2\sqrt{y}\,dy$

$$=4\cdot\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

$$=\frac{64}{3} \text{ square units.}$$

Hence, the correct answer is option C.

Solution 74

Let <*a*, *b*, *c*> be direction ratios of plane containing lines $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and $\frac{x}{3}=\frac{y}{7}=\frac{z}{8}$.

$\therefore$ $2a + 3b + 5c = 0$ ...(i)

and $3a + 7b + 8c = 0$ ...(ii)

from eq. (i) and (ii): $\frac{a}{24-35}=\frac{b}{15-16}=\frac{c}{14-9}$

$\therefore$ D.R^S. of plane are < 11, 1, −5>

Let D.R^S of plane P be $<a_1, b_1, c_1>$ then.

$11a_1 + b_1 - 5c_1 = 0$ ...(iii)

and $9a_1 - b_1 - 5c_1 = 0$ ...(iv)

From eq. (iii) and (iv):

$\frac{a_1}{-5-5}=\frac{b_1}{-45+55}=\frac{c_1}{-11-9}$

$\therefore$ D.A^5 . of plane P are <1, −1, 2>

Equation plane P is: $1(x - 3) - 1(y + 4) + 2(z - 7) = 0$

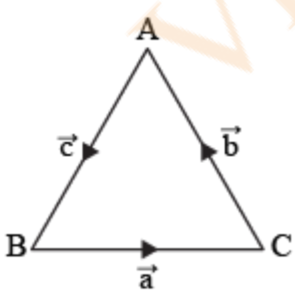
$\Rightarrow x - y + 2z - 21 = 0$

Distance from point (2, −5, 11) is $d=\frac{|2+5+22-21|}{\sqrt{6}}$

$\therefore d^2=\frac{32}{3}$

Hence, the correct answer is option C*.

Solution 75



$$\because\ \vec{a}+\vec{b}+\vec{c}=0 \qquad \ldots(\mathrm{i})$$

$$\text{then } \overline{a}+\vec{c}=-\vec{b}$$

$$\text{then } \left(\vec{a}+\vec{c}\right)\times\vec{b}=-\vec{b}\times\overline{b}$$

$$\therefore\ \vec{a}\times\vec{b}+\vec{c}\times\vec{b}=\overline{0} \qquad \ldots(\mathrm{i})$$

$$\text{For } (\mathrm{S1}):$$

$$\left|\vec{a}\times\vec{b}+\vec{c}\times\vec{b}\right|-\left|\vec{c}\right|=6\left(2\sqrt{2}-1\right)$$

$$\left|\left(\vec{a}+\vec{c}\right)\times\vec{b}\right|-\left|\vec{c}\right|=6\left(2\sqrt{2}-1\right)$$

$$\left|\vec{\mathrm{c}}\right|=6-12\sqrt{2}\left(\text{not possible}\right)$$

$$\text{Hence } (\mathrm{S1}) \text{ is not correct}$$

$$\text{For } (\mathrm{S2}): \text{ from } (\mathrm{i})\ \vec{b}+\vec{c}=-\vec{a}$$

$$\Rightarrow \vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{b}=-\vec{a}\cdot\vec{b}$$

$$\Rightarrow 12+12=-6\sqrt{2}\cdot 2\sqrt{3}\cos\left(\pi-\angle\mathrm{ACB}\right)$$

$$\therefore\ \cos\left(\angle\mathrm{ACB}\right)=\sqrt{\frac{2}{3}}$$

$$\therefore\ \angle\mathrm{ACB}=\cos^{-1}\sqrt{\frac{2}{3}}$$

$$\therefore\ \mathrm{S}(2) \text{ is correct.}$$

Hence, the correct answer is option C.

Solution 76

If *n* is number of trails, *p* is probability of success and *q* is probability of unsuccess then,

Mean = *np* and variance = *npq*.
Here $np + npq = 24$ ...(i)
$np \cdot npq = 128$ ...(ii)
and $q = 1 - p$ ...(iii)
From eq. (i), (ii) and (iii): $p = q = \frac{1}{2}$ and $n = 32$.
∴ Required probability = $p(X = 1) + p(X = 2)$

$$= {}^{32}C_1 \cdot \left(\frac{1}{2}\right)^{32} + {}^{32}C_2 \cdot \left(\frac{1}{2}\right)^{32}$$

$$= \left(32 + \frac{32 \times 31}{2}\right) \cdot \frac{1}{2^{32}}$$

$$= \frac{33}{2^{28}}$$

Hence, the correct answer is option C.

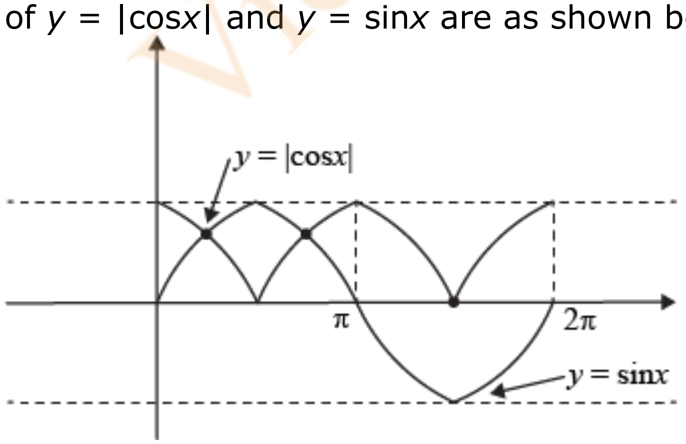
Solution 77

For $x^2 + \alpha x + \beta > 0\ \forall\ x \in R$ to hold, we should have $\alpha^2 - 4\beta < 0$
If α = 1, β can be 1, 2, 3, 4, 5, 6 i.e., 6 choices
If α = 2, β can be 2, 3, 4, 5, 6 i.e., 5 choices
If α = 3, β can be 3, 4, 5, 6 i.e., 4 choices
If α = 4, β can be 5 or 6 i.e., 2 choices
If α = 6, No possible value for β i.e., 0 choices
Hence total favourable outcomes
= 6 + 5 + 4 + 2 + 0 + 0
= 17
Total possible choices for α and β = 6 × 6 = 36
Required probability $= \frac{17}{36}$

Hence, the correct answer is option A.

Solution 78

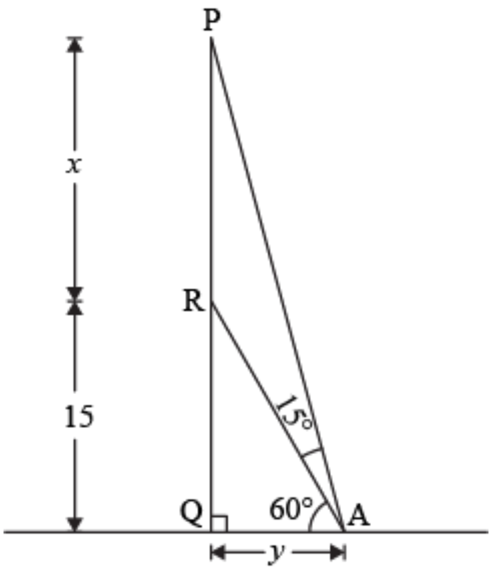
Number of solutions of the equation $|\cos x| = \sin x$ for $x \in [-4\pi, 4\pi]$ will be equal to 4 times the number of solutions of the same equation for $x \in [0, 2\pi]$. Graphs of $y = |\cos x|$ and $y = \sin x$ are as shown below.



Hence, two solutions of given equation in [0, 2π]
⇒ Total of 8 solutions in [−4π, 4π]

Hence, the correct answer is option C.

Solution 79



From ΔAPQ

$\frac{x+15}{y} = \tan 75^\circ \qquad \ldots\left(\text{i}\right)$

From ΔRQA,

$\frac{15}{y} = \tan 60^\circ \qquad \ldots\left(\text{ii}\right)$

From (i) and (ii)

$\frac{x+15}{15} = \frac{\tan 75^\circ}{\tan 60^\circ} = \frac{\tan(45^\circ+30^\circ)}{\tan 60^\circ} = \frac{\sqrt{3}+1}{\left(\sqrt{3}+1\right).\sqrt{3}}$

On simplification, $x = 10\sqrt{3}\text{m}$

Hence height of the tower $= \left(15 + 10\sqrt{3}\right)\text{m}$

$$= 5\left(2\sqrt{3} + 3\right)\text{m}$$

Hence, the correct answer is option A.

Solution 80

Truth Table

					A	B	C	D
p	*q*	~*p*	~*q*	(~*p*) ∨ *q*	((~*p*)∨ *q*) → *p*	*p*→((~*p*) ∨ *q*)	(~*p*) ∨ *q* → *q*	*q*→((~*p*) ∨ *q*)
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	F	T	T
F	T	T	F	T	F	T	T	T
F	F	T	T	T	F	T	F	T

Hence, the correct answer is option D.

Solution 81

Here $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$

We get $A^2 = A$ and similarly for

$B = A - l = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$

We get $B^2 = -B$

$\Rightarrow B^3 = B$

$\therefore A^n + (\omega B)^n = A + (\omega B)^n$ for $n \in N$

For ω^n to be unity n shall be multiple of 3 and for B^n to be B. n shall be 3, 5, 7, ... 99

$\therefore n$ = {3, 9, 15, ... 99}

Number of elements = 17.

Solution 82

Arranging letter in alphabetical order A D I K M N N for finding rank of MANKIND making arrangements of dictionary we get

$A \ldots\ldots\ldots\ldots\ldots\ldots \rightarrow \frac{6!}{2!} = 360$

$D \ldots\ldots\ldots\ldots\ldots\ldots \rightarrow 360$

$I \ldots\ldots\ldots\ldots\ldots\ldots \rightarrow 360$

$K \ldots\ldots\ldots\ldots\ldots\ldots \rightarrow 360$

$M\ A\ D \ldots\ldots\ldots\ldots\ldots \rightarrow \frac{4!}{2!} = 12$

$M\ A\ I \ldots\ldots\ldots\ldots\ldots \rightarrow 12$

$M\ A\ K \ldots\ldots\ldots\ldots\ldots \rightarrow 12$

$M\ A\ N\ D \ldots\ldots\ldots\ldots\ldots \rightarrow 3! = 6$

$M\ A\ N\ I \ldots\ldots\ldots\ldots\ldots \rightarrow 6$

$M\ A\ N\ K\ D \ldots\ldots\ldots\ldots \rightarrow 2$

$M\ A\ N\ K\ I\ D \ldots\ldots\ldots\ldots\ldots \rightarrow 1$

$M\ A\ N\ K\ I\ N\ D \ldots\ldots\ldots\ldots\ldots\ldots \rightarrow 1$

$\therefore$ Rank of MANKIND = 1440 + 36 + 12 + 2 + 2 = 1492

Solution 83

General Term = $15C_r \left(t^2 x^{\frac{1}{5}}\right)^{15-r} \left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$ for term independent on t

$2(15 - r) - r = 0$

$\Rightarrow r = 10$
$\therefore T_{11} = ^{15}C_{10} x(1-x)$

Maximum value of $x(1 - x)$ occur at $x = \frac{1}{2}$
i.e., $(x(1-x))_{\max} = \frac{1}{4}$
$\Rightarrow K = ^{15}C_{10} \times \frac{1}{4}$
$\Rightarrow 8K = 2\left(^{15}C_{10}\right) = 6006$

Solution 84

$\because$ Roots of $2ax^2 - 8ax + 1 = 0$ are $\frac{1}{p}$ and $\frac{1}{r}$ and roots of $6bx^2 + 12bx + 1 = 0$ are $\frac{1}{q}$ and $\frac{1}{s}$.
Let $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ as $\alpha - 3\beta,\ \alpha - \beta,\ \alpha + \beta,\ \alpha + 3\beta$
So sum of roots $2\alpha - 2\beta = 4$ and $2\alpha + 2\beta = -2$
Clearly $\alpha = \frac{1}{2}$ and $\beta = -\frac{3}{2}$
Now products of roots, $\frac{1}{p} \cdot \frac{1}{r} = \frac{1}{2a} = -5 \Rightarrow \frac{1}{a} = -10$ and
$\frac{1}{q} \cdot \frac{1}{x} = \frac{1}{6b} = -8 \Rightarrow \frac{1}{b} = -48$
So, $\frac{1}{a} - \frac{1}{b} = 38$

Solution 85

$a_1 = b_1 = 1$
$a_n = a_{n-1} + 2 \text{ (for } n \geq 2)$ $\quad b_n = a_n + b_{n-1}$
$a_2 = a_1 + 2 = 1 + 2 = 3$ $\quad b_2 = a_2 + b_1 = 3 + 1 = 4$
$a_3 = a_2 + 2 = 3 + 2 = 5$ $\quad b_3 = a_3 + b_2 = 5 + 4 = 9$
$a_4 = a_3 + 2 = 5 + 2 = 7$ $\quad b_4 = a_4 + b_3 = 7 + 9 = 16$
$a_{15} = a_{14} + 2 = 29$ $\quad b_{15} = 225$

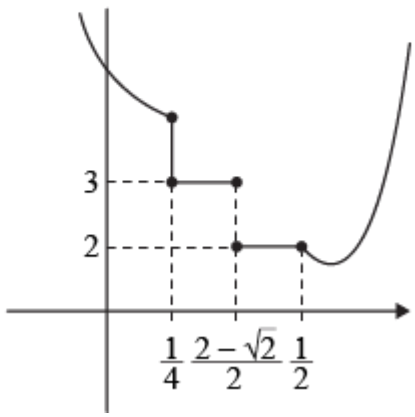
$\sum_{n=1}^{15} a_n b_n = 1 \times 1 + 3 \times 4 + 5 \times 9 + \ldots\ldots 29 \times 225$
$\therefore \sum_{n=1}^{11} a_n b_n = \sum_{n=1}^{15} (2n-1)n^2 = \sum_{n=1}^{15} 2n^3 - \sum_{n=1}^{15} n^2$
$= 2\left[\frac{15 \times 16}{2}\right]^2 - \left[\frac{15 \times 16 \times 31}{6}\right] = 27560.$

Solution 86

$$f(x) = \begin{cases} \left|4x^2 - 8x + 5\right|, & \text{if } 8x^2 - 6x + 1 \geq 0 \\ \left[4x^2 - 8x + 5\right], & \text{if } 8x^2 - 6x + 1 < 0 \end{cases}$$

$$= \begin{cases} 4x^2 - 8x + 5, & \text{if } x \in \left[-\infty, \frac{1}{4}\right] \cup \left[\frac{1}{2}, \infty\right) \\ \left[4x^2 - 8x + 5\right] & \text{if } x \in \left(\frac{1}{4}, \frac{1}{2}\right) \end{cases}$$

$$f(x) = \begin{cases} 4x^2 - 8x + 5 & if\ x \in \left(-\infty, \frac{1}{4}\right] \cup \left[\frac{1}{2}, \infty\right) \\ 3 & x \in \left(\frac{1}{4}, \frac{2-\sqrt{2}}{2}\right) \\ 2 & x \in \left(\frac{2-\sqrt{2}}{2}, \frac{1}{2}\right) \end{cases}$$



$\therefore$ Non – diff at $x = \frac{1}{4}, \frac{2-\sqrt{2}}{2}, \frac{1}{2}$

Solution 87

$$\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{k-1} \frac{1}{n} \sum_{r=1}^{n} \left(k + \frac{r}{n}\right) = 33 \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \left(\frac{r}{n}\right)^k$$

$\Rightarrow \int_0^1 (k + x)dx = 33 \int_0^1 x^k dx$

$\Rightarrow \frac{2k+1}{2} = \frac{33}{k+1}$

$\Rightarrow k = 5$

Solution 88

$x^2 + y^2 - 2x + 2fy + 1 = 0$ [entre = (1, −f]

Diameter $2px - y = 1$ ...(i)

$2x + py = 4p$ ...(ii)

$x = \frac{5P}{2P^2+2}$ $y = \frac{4P^2-1}{1+P^2}$

$\because x = 1$ $f = 0$ $\left[\text{for } P = \frac{1}{2}\right]$

$\frac{5P}{2P^2+2} = 1$ $f = 3$ [for $P = 2$]

$\therefore P = \frac{1}{2},\ 2$

Centre can be $\left(\frac{1}{2},\ 0\right)$ or (1, 3)

$\left(\frac{1}{2},\ 0\right)$ will not satisfy

$\therefore$ Tangent should pass through

(2, 3) for $3x^2 - y^2 = 3$

$\frac{x^2}{1} - \frac{y^2}{3} = 1$

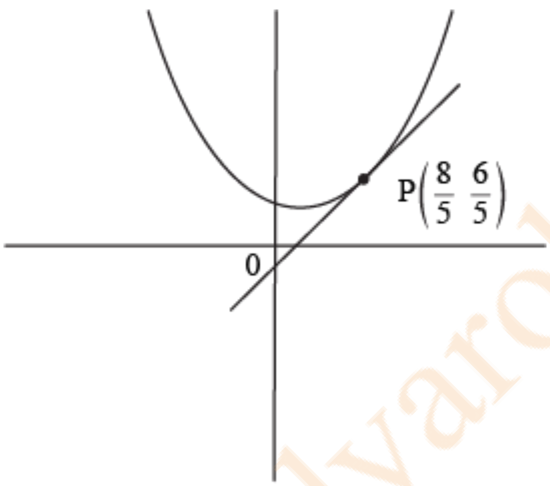
$y = mx\ \pm\ \sqrt{m^2 - 3}$

substitute (2, 3)

$3 = m \pm \sqrt{m^2 - 3}$

$\therefore\ \boxed{m = 2}$

Solution 89



Equation of tangent to the parabola at $P\left(\frac{8}{5},\ \frac{6}{5}\right)$

$75x.\frac{8}{5} = 160\left(y + \frac{6}{5}\right) - 192$

$\Rightarrow 120x\ =\ 160y$

$\Rightarrow 3x = 4y$

Equation of circle touching the given parabola at *P* can be taken as

$\left(x - \frac{8}{5}\right)^2 + \left(y - \frac{6}{5}\right)^2 + \lambda\left(3x - 4y\right) = 0$

If this circle touches *y*-axis then

$\frac{64}{25} + \left(y - \frac{6}{5}\right)^2 + \lambda\left(-4y\right) = 0$

$$\Rightarrow y^2 - 2y\left(2\lambda + \frac{6}{5}\right) + 4 = 0$$

$$\Rightarrow D = 0$$

$$\Rightarrow \left(2\lambda + \frac{6}{6}\right)^2 = 4$$

$$\Rightarrow \lambda = \frac{2}{5} \text{ or } -\frac{8}{5}$$

Radius = 1 or 4
Sum of diameter = 10

Solution 90

Line of shortest distance will be along $\overrightarrow{b_1} \times \overrightarrow{b_2}$
Where, $\overrightarrow{b_1} = \hat{j} + \hat{k}$ and $\overrightarrow{b}_2 = 2\hat{i} + 2\hat{j} + \hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 2 & 2 & 1 \end{vmatrix} = -\hat{i} + 2\hat{j} - 2\hat{k}$$

Angle between $\overline{b_1} \times \overline{b_2}$ and plane P,

$$\sin\theta = \left|\frac{-a-2+2}{3.\sqrt{a^2+2}}\right| = \frac{5}{\sqrt{27}}$$

$$\Rightarrow \frac{|a|}{\sqrt{a^2+2}} = \frac{5}{\sqrt{3}}$$

$$\Rightarrow a^2 = -\frac{25}{11}\left(\text{not possible}\right)$$

Disclaimer: No answer is correct.