Board Paper of Class 12-Science 2023 Math Delhi(Set 1)

Total Time: 180

Total Marks: 80.0

Section A

Q.No.1: Let A = {3, 5}. Then number of reflexive relations on A is
(a) 2
(b) 4
(c) 0
(d) 8 **Marks:[1.00]**

Q.No.2: $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to
(a) 1
(b) $\frac{1}{2}$
(c) $\frac{1}{3}$
(d) $\frac{1}{4}$ **Marks:[1.00]**

Q.No.3: If for a square matrix, A, $A^2 - A + I = O$, then A^{-1} equals
(a) A
(b) A + I
(c) I − A
(d) A − I **Marks:[1.00]**

Q.No.4: If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and A = B^2, then *x* equals
(a) ± 1

(b) −1
(c) 1
(d) 2

Marks:[1.00]

Q.No.5: If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is
(a) 1
(b) 2
(c) 3
(d) 4

Marks:[1.00]

Q.No.6: The derivative of x^{2x} w.r.t. x is
(a) x^{2x-1}
(b) $2x^{2x} \log x$
(c) $2x^{2x}(1 + \log x)$
(d) $2x^{2x}(1 - \log x)$

Marks:[1.00]

Q.No.7: The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at
(a) $x = 1$
(b) $x = 1.5$
(c) $x = -2$
(d) $x = 4$

Marks:[1.00]

Q.No.8: If $x = \text{A} \cos 4t + \text{B} \sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to
(a) x
(b) $-x$
(c) $16x$
(d) $-16x$

Marks:[1.00]

Q.No.9: The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is
(a) $(-1,\ \infty)$
(b) $(-2,\ -1)$

(c) $(-\infty,\ -2)$
(d) $[-1,\ 1]$ **Marks:[1.00]**

Q.No.10: $\int \frac{\sec\ x}{\sec\ x - \tan\ x}\ dx$ equals
(a) sec *x* − tan *x* + *c*
(b) sec *x* + tan *x* + *c*
(c) tan *x* − sec *x* + *c*
(d) −(sec *x* + tan *x*) + *c* **Marks:[1.00]**

Q.No.11: $\int\limits_{-1}^{1} \frac{|x-2|}{x-2} dx,\ x \neq 2$ is equal to
(a) 1
(b) −1
(c) 2
(d) −2 **Marks:[1.00]**

Q.No.12: The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is
(a) 2
(b) 3
(c) 5
(d) 0 **Marks:[1.00]**

Q.No.13: Two vectors $\overrightarrow{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\overrightarrow{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if
(a) $a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$
(b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$
(c) $a_1 = b_1,\ a_2 = b_2,\ a_3 = b_3$
(d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$ **Marks:[1.00]**

Q.No.14: The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is
(a) 1
(b) 5
(c) 7

(d) 12 **Marks:[1.00]**

Q.No.15: If a line makes angles of 90°, 135° and 45° with the *x*, *y* and *z* axes respectively, then its direction cosines are
(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
(b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$
(d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ **Marks:[1.00]**

Q.No.16: The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is
(a) 0°
(b) 30°
(c) 45°
(d) 90° **Marks:[1.00]**

Q.No.17: If for any two events A and B, P(A)$= \frac{4}{5}$ and P(A ∩ B)$= \frac{7}{10}$, then P(B/A) is equal to
(a) $\frac{1}{10}$
(b) $\frac{1}{8}$
(c) $\frac{7}{8}$
(d) $\frac{17}{20}$ **Marks:[1.00]**

Q.No.18: Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is
(a) $\frac{27}{32}$
(b) $\frac{5}{32}$
(c) $\frac{31}{32}$
(d) $\frac{1}{32}$ **Marks:[1.00]**

Q.No.19: Assertion (A): Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R): Let E and F be two events with a random experiment, then $\mathrm{P}\left(\mathrm{F}/\mathrm{E}\right) = \frac{\mathrm{P}(\mathrm{E} \cap \mathrm{F})}{\mathrm{P}(\mathrm{E})}$.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
(c) (A) is true and (R) is false.
(d) (A) is false, but (R) is true. **Marks:[1.00]**

Q.No.20: Assertion (A): $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx = 3$

Reason (R): $\int_a^b f\left(x\right) dx = \int_a^b f\left(a+b-x\right) dx$

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
(c) (A) is true and (R) is false.
(d) (A) is false, but (R) is true. **Marks:[1.00]**

Section B

Q.No.21: Write the domain and range (principle value branch) of the following functions:
$f(x) = \tan^{-1} x$ **Marks:[2.00]**

Q.No.22: If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.

OR

Find the value (s) of 'λ', if the function

$f\left(x\right) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$. **Marks:[2.00]**

Q.No.23: Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration. **Marks:[2.00]**

Q.No.24: If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3,\ |\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

Marks:[2.00]

Q.No.25: Find the vector and the cartesian equations of a line that passes through the point A(1, 2, −1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

Marks:[2.00]

Section C

Q.No.26: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = 0$.

Marks:[3.00]

Q.No.27: Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$.

OR

If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

Marks:[3.00]

Q.No.28: Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$

OR

Find : $\int \frac{x^4}{(x-1)(x^2+1)} dx$

Marks:[3.00]

Q.No.29: Find the area of the following region using integration :
$\{(x, y) : y^2 \le 2x \text{ and } y \ge x - 4\}$

Marks:[3.00]

Q.No.30: Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$.

OR

Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity

$\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $\left|\vec{a}\right|=3$, $\left|\vec{b}\right|=4$ and $\left|\vec{c}\right|=2$.

Marks:[3.00]

Q.No.31: Find the distance between the lines :

$\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$;

$\vec{r}=\left(3\hat{i}+3\hat{j}-5\hat{k}\right)+\mu\left(4\hat{i}+6\hat{j}+12\hat{k}\right)$

Marks:[3.00]

Section D

Q.No.32: The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers. **Marks:[5.00]**

Marks:[5.00]

Q.No.33: Evaluate: $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)dx$

Q.No.34: Solve the following Linear Programming Problem graphically:
Maximize : P = 70x + 40y
subject to: 3x + 2y $\leq$ 9,
3x + y $\leq$ 9,
x $\geq$ 0, y $\geq$0 **Marks:[5.00]**

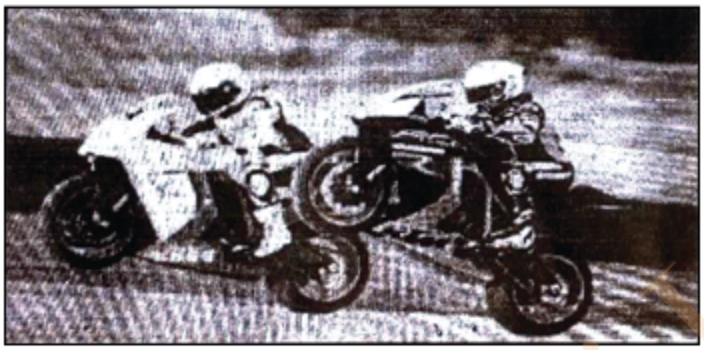
Q.No.35: In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?

OR

A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize. **Marks:[5.00]**

Q.No.36: Case Study

An organization conducted bike race under two different categories - Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.
Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions
(I) How many relations are possible from B to G ?
(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?
(III) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

(III) A function $f : B \to G$ be defined by $f = \{b_1, g_1). (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer. **Marks:[4.00]**

Q.No.37: Case Study

Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250.
Based on the above information, answer the following questions:
(I) Convert the given above situation into a matrix equation of the form AX = B.
(II) Find $|A|$.
(III) Find A^{-1}.

OR

(III) Determine $P = A^2 - 5A$. **Marks:[4.00]**

Q.No.38: Case Study

An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x,\ y)$ is said to be homogeneous if *F*(*x*, *y*) is a homogeneous function of degree zero, whereas a function F(*x*, *y*) is a homogenous function of degree *n* if $F(\lambda x,\ \lambda y) = \lambda^n\ F(x,\ y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x,\ y) =\ g\left(\frac{y}{x}\right)$, we make the substitution *y* = *vx* and then separate the variables.

Based on the above, answer the following questions:

(I) Show that $(x^2 - y^2)\ dx + 2xy\ dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution. **Marks:[4.00]**