Vidyarohi Learning

CBSE Class 12

Relations and Functions Previous Year Questions

Solution PDF

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Relations and Functions

Solution 1

$f(x)=\dfrac{|x-1|}{(x-1)}$

This function $f(x)$ is not defined at x = 1.
For $x \geq 1$, $|x-1|=x-1$

$\therefore f(x)=\dfrac{|x-1|}{(x-1)}=\dfrac{(x-1)}{(x-1)}=1$

For $x<1, |x-1|=-(x-1)=1-x$

$\therefore f(x)=\dfrac{|x-1|}{(x-1)}=\dfrac{1-x}{(x-1)}=-1$

Thus, the range of the function is −1 or 1 at all points excluding x = 1.

Solution 2

Given that * is a binary operation on Q defined by $a*b=\dfrac{3ab}{5}$, where a, b ∈ Q

Now, $b*a=\dfrac{3ba}{5}$

As ab = ba, $\dfrac{3ab}{5}=\dfrac{3ba}{5}$

$\therefore a*b=b*a$

Thus, the binary operation * is commutative.
Let a, b, c ∈ Q

$a*(b*c)=a*\dfrac{3bc}{5}$

$\Rightarrow a*(b*c)=\dfrac{3\times a\times\dfrac{3bc}{5}}{5}$

$\Rightarrow a*(b*c)=\dfrac{9abc}{25}$ ...(1)

$(a*b)*c=\dfrac{3ab}{5}*c$

$\Rightarrow (a*b)*c=\dfrac{3\times\dfrac{3ab}{5}\times c}{5}$

$\Rightarrow (a*b)*c=\dfrac{9abc}{25}$ ...(2)

From equations (1) and (2):
a * (b * c) = (a * b) * c
Thus, the binary operation * is associative.
For a binary operation *: $A \times A \to A$, if an element $e \in A$ such that $a * e = e * a = a$ &mnForE $a \in A$, then e is called the identity element.

Now, $a*\dfrac{5}{3}=\dfrac{3a\times\dfrac{5}{3}}{5}=a$

Also, $\dfrac{5}{3}*a=\dfrac{3\times\dfrac{5}{3}\times a}{5}=a$

Now, $a*\dfrac{5}{3}=\dfrac{5}{3}*a=a$ and $\dfrac{5}{3}\in Q$.

Thus, $\dfrac{5}{3}$ is the identity element for the binary operation *.

Solution 3

It is known that a relation R in a set A is transitive if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R$; $a_1, a_2, a_3 \in A$.
It can be observed that (1, 2), (2, 1) ∈ R, but (1, 1) ∉ R.
Thus, the relation R in the set {1, 2, 3} is not transitive.

Solution 4

The binary operation * on the set {1, 2, 3, 4, 5} is defined by $a * b$ = min {a, b}
1 * 1 = min {1, 1} = 1
1 * 2 = min {1, 2} = 1
2 * 3 = min {2, 3} = 2
4 * 5 = min {4, 5} = 4
The operation table for the given operation * on the given set can be written as:

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Solution 5

The given function is $f(x)=\frac{4x+3}{6x-4}$.

Let $f(x_1) = f(x_2)$ $\Rightarrow \frac{4x_1+3}{6x_1-4}=\frac{4x_2+3}{6x_2-4}$

$\Rightarrow 24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 + 18x_1 - 16x_2 - 12$
$\Rightarrow 18x_2 + 16x_2 = 18x_1 + 16x_1$
$\Rightarrow 34x_2 = 34x_1 \Rightarrow x_1 = x_2$
Therefore $f(x)$ is one − one.

Since, $\frac{4x+3}{6x-4}$ is a real number, therefore, for every y in the co–domain (f), there exists a number x in $\mathrm{IR}-\left\{\frac{2}{3}\right\}$ such that $f(x)=y=\frac{4x+3}{6x-4}$

Therefore, $f(x)$ is onto

Now, let $y=\frac{4x+3}{6x-4}$

$\Rightarrow 6xy-4y=4x+3 \Rightarrow x=\frac{4y+3}{6y-4} \Rightarrow f^{-1}(x)=\frac{4x+3}{6x-4}$

Solution 6

Since $a * b$ = LCM (a,b)
Given numbers are 5 and 7.
Now, 5 * 7 = LCM (5,7)
= 35
Hence, 5 * 7 = 35

Solution 7

Given, A = **R** − {3} and **B** = R − {1}.

f : A → B is a function defined by $f(x)=\left(\frac{x-2}{x-3}\right)$.

Let $x_1, x_2 \in$ A.
Now,

$$f(x_1)=f(x_2)$$
$$\Rightarrow \frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$$
$$\Rightarrow (x_1-2)(x_2-3)=(x_1-3)(x_2-2)$$
$$\Rightarrow x_1x_2-2x_2-3x_1+6=x_1x_2-3x_2-2x_1+6$$
$$\Rightarrow x_1=x_2$$

∴f is one-one.
Again, let y be any arbitrary element of B.

$$f(x)=y$$
$$\Rightarrow \frac{x-2}{x-3}=y$$
$$\Rightarrow (x-2)=y(x-3)$$
$$\Rightarrow x-2=xy-3y$$
$$\Rightarrow x-xy=2-3y$$
$$\Rightarrow x(1-y)=2-3y$$

$\Rightarrow x=\frac{2-3y}{1-y}$, which is a real number for all y ≠ 1.

Also, $\frac{2-3y}{1-y} \neq 3$ for any y, because if we take $\frac{2-3y}{1-y} = 3$

$\Rightarrow 2-3y = 3-3y$

⇒ 2 = 3, which is not possible.

∴ $x \in \mathbf{R} - \{3\}$ for all $y \in \mathbf{R} - \{1\}$.

Thus, for all $y \in \mathbf{R} - \{1\}$ there exists $x = \frac{2-3y}{1-y} \in \mathbf{R} - \{3\}$ such that

$$f(x) = f\left(\frac{2-3y}{1-y}\right) = \left(\frac{\frac{2-3y}{1-y} - 2}{\frac{2-3y}{1-y} - 3}\right) = \left(\frac{2-3y-2+2y}{2-3y-3+3y}\right) = y$$

∴ Every element y in B has a pre-image x in A which is given by $x = \frac{2-3y}{1-y}$.

∴ f is onto.

Hence, f is one-one and onto.

To find f^{-1}:

Let $f(x) = y$ where $x \in \mathbf{R} - \{3\}$ and $y \in \mathbf{R} - \{1\}$.

$$\therefore y = \frac{x-2}{x-3}$$

$$\Rightarrow x = \frac{2-3y}{1-y}$$

$$\Rightarrow f^{-1}(y) = \frac{2-3y}{1-y}$$

∴ $f^{-1} : \mathbf{R} - \{1\} \to \mathbf{R} - \{3\}$ is defined by $f^{-1}(x) = \frac{2-3x}{1-x}$ for all $x \in \mathbf{R} - \{1\}$.

Solution 8

It is given that $a * b = \frac{ab}{5}$, where $a,\ b \in R - \{0\}$ and $2 * (x * 5) = 10$.

Using $a * b = \frac{ab}{5}$ in the equation $2 * (x * 5) = 10$, we get:

$$2 * \left(\frac{5x}{5}\right) = 10 \qquad \left(\because\ x * 5 = \frac{5x}{5}\right)$$

$$\Rightarrow 2 * x = 10$$

$$\Rightarrow \frac{2x}{5} = 10 \qquad \left(\because 2 * x = \frac{2x}{5}\right)$$

$$\Rightarrow x = 25$$

∴ The value of x is 25.

Solution 9

A = {1, 2, 3, ..., 9} ⊂ ℕ, the set of natural numbers

Let R be the relation in A × A defined by (a, b) R (c, d) if $a + d = b + c$ for (a, b), (c, d) in A × A.

We have to show that R is an equivalence relation.

Reflexivity:

Let (a, b) be an arbitrary element of A × A. Then, we have:

(a, b) ∈ A × A

⇒ a, b ∈ A

⇒ $a + b = b + a$ (by commutativity of addition on A ⊂ ℕ)

⇒ (a, b) R (a, b)

Thus, (a, b) R (a, b) for all (a, b) ∈ A × A.

So, R is reflexive.

Symmetry:

Let (a, b), (c, d) ∈ A × A such that (a, b) R (c, d).

$a + d = b + c$

⇒ $b + c = a + d$

⇒ $c + b = d + a$ (by commutativity of addition on A ⊂ ℕ)

⇒ (c, d) R (a, b)

Thus, (a, b) R $(c, d) \Rightarrow (c, d)$ R (a, b) for all $(a, b), (c, d) \in$ A × A.
So, R is symmetric.

Transitivity:
Let $(a, b), (c, d), (e, f) \in$ A × A such that (a, b) R (c, d) and (c, d) R (e, f). Then, we have:

(a, b) R (c, d)
$\Rightarrow a + d = b + c$ ... (1)

(c, d) R (e, f)
$\Rightarrow c + f = d + e$ ... (2)

Adding equations (1) and (2), we get:

$(a + d) + (c + f) = (b + c) + (d + e)$
$\Rightarrow a + f = b + e$
$\Rightarrow (a, b)$ R (e, f)

Thus, (a, b) R (c, d) and (c, d) R $(e, f) \Rightarrow (a, b)$ R (e, f) for all $(a, b), (c, d), (e, f) \in$ A × A.
So, R is transitive on A × A.
Thus, R is reflexive, symmetric and transitive.
∴ R is an equivalence relation.

To write the equivalence class of [(2, 5)], we need to search all the elements of the type (a, b) such that $2 + b = 5 + a$.

∴ Equivalence class of [(2, 5)] = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

Solution 10

To prove a relation R is an equivalence relation, it will be sufficient to prove it as a reflexive, symmetric and transitive relation.

i) Reflexivity:
Let (a, b) be an arbitrary element of N × N.
Now,
$a, b \in$ N

$$\Rightarrow ab(a+b) = ba(a+b)$$
$$\Rightarrow (a,b)\mathrm{R}(a,b)$$

∴ (a, b)R(a, b) for all $(a, b) \in$ N × N
Hence, R is reflexive.

ii) Symmetry:
Let $(a, b), (c, d)$ be an arbitrary element of N × N such that (a, b)R(c, d).

$$\therefore\ ad(b+c) = bc(a+d)$$
$$\Rightarrow cb\,(d+a) = da\,(c+b)$$
$$\Rightarrow (c,d)\mathrm{R}\,(a,b)$$

∴ (a, b)R$(c, d) \Rightarrow (c, d)R(a, b)$ for all $(a, b), (c, d) \in$ N × N
Hence, R is symmetric.

iii) Transitivity:
Let $(a, b), (c, d), (e, f)$ be an arbitrary element of N × N such that (a, b)R(c, d) and (c, d)R(e, f).

$ad\left(b+c\right)=bc\left(a+d\right)$
$\Rightarrow adb+adc=abc+bcd$
$\Rightarrow cd\left(a-b\right)=ab\left(c-d\right) \quad .....(1)$
Also,
$cf\left(d+e\right)=de\left(c+f\right)$
$\Rightarrow cfd+cfe=dec+def$
$\Rightarrow cd\left(f-e\right)=ef\left(d-c\right) \quad .....(2)$
From (1) and (2), we have
$\frac{a-b}{f-e}=-\frac{ab}{ef}$
$\Rightarrow aef-bef=-abf+aeb$
$\Rightarrow aef+abf=aeb+bef$
$\Rightarrow af\left(b+e\right)=be\left(a+f\right)$
$\Rightarrow (a,\ b)\mathrm{R}\left(e,\ f\right)$

∴(a, b)R(c, d) and (c, d)R(e, f) ⇒ (a, b)R(e, f) for all (a, b), (c, d), (e, f) ∈ N × N
Hence, R is transitive.
Thus, R being reflexive, symmetric and transitive, is an equivalence relation on N × N.

Solution 11

Given: $f\left(x\right)=9x^2+6x-5$
Let y = $9x^2+6x-5$
$\Rightarrow y=\left(3x+1\right)^2-1-5=\left(3x+1\right)^2-6$
$\Rightarrow \left(3x+1\right)^2 = y + 6$
$\Rightarrow 3x+1 = \sqrt{y + 6}$
$\Rightarrow x = \frac{\sqrt{y+6}-1}{3}$ as $x\in \mathrm{N}$
$\Rightarrow \sqrt{y + 6}-1>0$
$\Rightarrow y + 6>1$
$\Rightarrow y > -5$ and $y\in \mathrm{N}$

So, the function is invertible if the range of the function $f(x)$ is {1, 2, 3, ...}.
Therefore, the inverse of the function $f(x)$ is $f^{-1}\left(y\right)$, i.e. x.
Now,
$$f^{-1}\left(y\right)=\frac{\sqrt{y+6}-1}{3}$$
$$f^{-1}\left(43\right)=\frac{\sqrt{43+6}-1}{3}=2$$
$$f^{-1}\left(163\right)=\frac{\sqrt{163+6}-1}{3}=4$$

Solution 12

f : $\mathrm{R}_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$
To show: f is one-one and onto.
Let us assume that f is not one-one.

Therefore there exist two or more numbers for which images are same.
For $x_1,\ x_2\in \mathrm{R}_+$ and $x_1\neq\ x_2$
Let $f\left(x_1\right)=f\left(x_2\right)$
$\Rightarrow 9{x_1}^2+6x_1-5=9{x_2}^2+6x_2-5$
$\Rightarrow 9{x_1}^2+6x_1=9{x_2}^2+6x_2$
$\Rightarrow 9{x_1}^2-9{x_2}^2+6x_1-6x_2=0$
$\Rightarrow 9\left({x_1}^2-{x_2}^2\right)+6\left(x_1-x_2\right)=0$
$\Rightarrow \left(x_1-x_2\right)\left[9\left(x_1+x_2\right)+6\right]=0$
Since x_1 and x_2 are positive,

$9\left(x_1+x_2\right)+6>0$
$\therefore x_1-x_2=0 \Rightarrow x_1=x_2$
Therefore, it contradicts our assumption.

Hence the function *f* is one-one.

Now, let is prove that *f* is onto.
A function *f* : *X* → *Y* is onto if for every *y* ∈ *Y,* there exist a pre-image in *X*.

$$\begin{aligned} f(x) &= 9x^2+6x-5 \\ &= 9x^2+6x+1-6 \\ &= (3x+1)^2-6 \end{aligned}$$

Now, for all $x \in R^+ \text{ or } [0, \infty),\ f(x) \in [-5,\ \infty)$.

$\therefore$ Range = co-domain.

Hence, *f* is onto.

Therefore, function *f* is invertible.

Now, let $y = 9x^2 + 6x - 5$
$9x^2+6x-5-y=0$ where $x \in R^+$
$\Rightarrow x=\frac{-6 \pm \sqrt{36+36(5+y)}}{2 \times 9}$
$\Rightarrow x=\frac{-6+6 \sqrt{6+y}}{18}$ $\quad [As\ x\ \in R^+, \text{ we neglecting the negative part}]$
$\Rightarrow x=\frac{-1+\sqrt{6+y}}{3}$
$\Rightarrow f^{-1}(y)=\frac{\sqrt{y+6}-1}{3}$

$(i)\ f^{-1}(10)=\frac{\sqrt{10+6}-1}{3}=\frac{4-1}{3}=1$

$\text{(ii) } f^{-1}(y)=\frac{4}{3}$

$\therefore \frac{\sqrt{y+6}-1}{3}=\frac{4}{3}$
$\Rightarrow \sqrt{y+6}-1=4$
$\Rightarrow \sqrt{y+6}=5$
$\Rightarrow y+6=25$
$\Rightarrow y=19$

OR

Given, * is a binary operation on Q − {1} defined by $a * b=a-b+ab$
Commutativity:
For $a$ny $a,\ b \in A$, we have
$a * b=a-b+ab$ and $b * a=b-a+ba$
Since, $a-b+ab \neq b-a+ab$
$\therefore a * b \neq b * a$
So, * is not commutative on A.
Associativity:

Let $a,\ b,\ c \in A$

$(a * b) * c = (a - b + ab) * c$

$\Rightarrow (a * b) * c = (a - b + ab) - c + (a - b + ab)c$

$\Rightarrow (a * b) * c = a - b + ab - c + ac - bc + abc$

$a * (b * c) = a * (b - c + bc)$

$\Rightarrow a * (b * c) = a - (b - c + bc) + a\,(b - c + bc)$

$\Rightarrow a * (b * c) = a - b + c - bc + ab - ac + abc$

$\Rightarrow (a * b) * c \neq a * (b * c)$

So, * is not associative on A.

Identity Element

Let *e* be the identity element in A, then

$a * e = a = e * a \qquad \forall a \in Q - \{1\}$

$\Rightarrow a - e + ae = a$

$\Rightarrow (a - 1)e = 0$

$\Rightarrow e = 0 \qquad (As\ a \neq 1)$

So, 0 is the identity element in A.

Inverse of an Element

Let *a* be an arbitrary element of A and *b* be the inverse of *a*. Then,

$a * b = e = b * a$

$\Rightarrow a * b = e$

$\Rightarrow a - b + ab = 0 \qquad [\because e = 0]$

$\Rightarrow a = b\,(1 - a)$

$\Rightarrow b = \frac{a}{1-a}$

Since, $b \in Q - 1$

So, every element of A is invertible.

Solution 13

Given: *a* * *b* denotes the larger of '*a*' and '*b*'.

Also, $a \circ b = (a * b) + 3$

For *a* = 5 and *b* = 10

a * *b* = 5 * 10 = 10

$a \circ b = 5 \circ 10 = (5 * 10) + 3 = 10 + 3 = 13.$

Solution 14

$A = \{0,1,2,3,4,5,6,7,8,9,10,11,12\}$

$R = \{(a,b) : a,b \in Z,\ |a-b| \text{ is divisible by } 4\}$

For reflexive,

for every $a \in A$

$|a-a| = 0$ which is divisible by 4

then $(a,a) \in R$

Hence, it is reflexive.

For symmetric,

If $(a,b) \in R$ then $(b,a) \in R$

$|a-b| = |b-a|$

Hence, it is symmetric.

For transitive,

If $(a,b) \in R \Rightarrow |a-b|$ is divisible by 4 (Say $|a-b| = 4k_1 \Rightarrow a-b = \pm 4k_1$)

and $(b,c) \in R \Rightarrow |b-c|$ is divisible by 4 (Say $|b-c| = 4k_2 \Rightarrow b-c = \pm 4k_2$)

$\therefore |a-c| = |\pm 4k_1 \pm 4k_2|$ which is divisible by 4

then $(a,c) \in R$

Hence, it is transitive.

Also, the relation is equivalence.

Set of elements related to 1 is $\{(1,1),(1,5),(1,9),(5,1),(9,1)\}$

Let $(x,2) \in R;\ (x \in A)$

$|x-2| = 4k$ (k is whole number, $k \leq 3$)

$\therefore x = 2, 6, 10$

Equivalence class $[2]$ is $\{2,6,10\}$

OR

Given, $y = \frac{x}{x^2+1}$

$\Rightarrow yx^2 - x + y = 0$

Here, $a = y,\ b = -1$ and $c = y$

$\therefore x = \frac{-\left(-1\right) \pm \sqrt{1-4y^2}}{2y}$

Clearly for every value of y, x will have two different values so the function is many − one not one − one.

Since $1-4y^2 \geq 0 \Rightarrow \left(1+2y\right)\left(1-2y\right) \geq 0 \Rightarrow \frac{-1}{2} \leq y \leq \frac{1}{2}$

That means no matter what is x, y always belongs to the interval $\left[\frac{-1}{2}, \frac{1}{2}\right]$

So, the function is not onto.

Now, $fog\left(x\right) = \frac{2x-1}{(2x-1)^2+1} = \frac{2x+1}{4x^2-4x+1+1} = \frac{2x+1}{2(2x^2-2x+1)}$.

Solution 15

R = {(a, b); a ≤ b}

Clearly (a, a) ∈ R as a = a.

∴ R is reflexive.

Now,

(2, 4) ∈ R (as 2 < 4)

But, (4, 2) ∉ R as 4 is greater than 2.

∴ R is not symmetric.

Now, let (a, b), (b, c) ∈ R.

Then,

a ≤ b and b ≤ c
⇒ a ≤ c
⇒ (a, c) ∈ R
∴ R is transitive.
Hence, R is reflexive and transitive but not symmetric.

OR

The given function is
f : N → N
$f(x) = x^2 + x + 1$
Let x_1, x_2 6N

$$\text{So let } f(x_1) = f(x_2)$$
$$x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$$
$$x_1^2 - x_2^2 + x_1 - x_2 = 0$$
$$(x_1 - x_2)(x_1 + x_2 + 1) = 0$$
$$\because x_2 = x_1$$
$$\text{or } x_2 = -x_1 - 1$$
$$\because x_1 \in \mathrm{N}$$
$$\therefore -x_1 - 1 \in \mathrm{N}$$
$$\text{So } x_2 \neq -x_1 - 1$$
$$\because f(x_2) = f(x_1) \text{ only for } x_1 = x_2$$

So $f(x)$ is one-one function.

$$\because f(x) = x^2 + x + 1$$
$$f(x) = \left(x + \tfrac{1}{2}\right)^2 + \tfrac{3}{4}$$

Which is an increasing function.
$f(1) = 3$
∵ Range of $f(x)$ will be {3, 7, ...}
Which is a subset of N.
So it is an into function.
i.e., $f(x)$ is not an onto function.

$$\text{let } y = x^2 + x + 1$$
$$x^2 + x + 1 - y = 0$$
$$x = \frac{-1 \pm \sqrt{1 - 4(1-y)}}{2}$$
$$x = \frac{-1 \pm \sqrt{4y-3}}{2}$$

So two possibilities are their for $f^{-1}(x)$

$f^{-1}(x) = \frac{-1+\sqrt{4x-3}}{2}, \frac{-1-\sqrt{4x-3}}{2}$ and we know $f^{-1}(3) = 1$ because $f(1) = 3$

$$\text{so } f^{-1}(x) = \frac{-1+\sqrt{4x-3}}{2}$$

Solution 16

The given operation is $a * b = ab + 1$
If any operation is a binary operation then it must follow the closure property.
Let $a \in R$, $b \in R$
then $a * b \in R$
also $ab + 1 \in R$
i.e. $a * b \in R$
so * on R satisfies the closure property
Now if this binary operation satisfies associative law then
$(a * b) * c = a * (b * c)$
$(a * b) * c = (ab + 1) * c$
$= (ab + 1) c + 1$
$= abc + c + 1$

$a * (b * c) = a * (bc + 1)$
$= a(bc + 1) + 1$
$= abc + a + 1$
∴ $(a * b) * c \neq a * (b * c)$
i.e., * operation does not follow associative law.

Solution 17

Given: $f(x) = ax + \frac{b}{x}\ (a > 0,\ b > 0,\ x > 0)$
As $a,\ b$ and $x > 0$
We can use $\text{AM} \geq \text{GM}$
$\frac{ax+\frac{b}{x}}{2} \geq \sqrt{ax \times \frac{b}{x}}$
$\frac{ax+\frac{b}{x}}{2} \geq \sqrt{ab}$
$ax + \frac{b}{x} \geq 2\sqrt{ab}$
Hence, the minimum value of $f(x)$ is $2\sqrt{ab}$.

Solution 18

Every relation in a set *A* is called reflexive relation, if each element of *A* is related to itself.
For example, if *A* = {*p, q, r*} then *R* = $\{(p,p),(q,q),(r,r),(p,r)\}$ is a reflexive relation.

Solution 19

We have,
$R = \{(a,\ b)\ :\ a < b\}, \quad \text{where } a,\ b \in \mathbb{R}$
(i) Symmetry
We observe that $(2,\ 3) \in R$ but $(3,\ 2) \notin R$.
So, *R* is not symmetric.

(ii) Transitivity
Let $(a,\ b) \in R$ and $(b,\ c) \in R$. Then,
$\Rightarrow a < b$ and $b < c$
$\Rightarrow a < c$
$\Rightarrow (a,c) \in R$

So, *R* is transitive.

Solution 20

If $(a_1,\ a_2) \in \text{R}$ and $(a_2,\ a_3) \in \text{R}$ then $(a_1,\ a_3) \in \text{R}$
This category of relation is called transitive.
Hence, set A is called a transitive set.

Solution 21

Given: *X* = {1, 2, 3} and *R* = {(1, 3), (2, 2), (3, 2)}
To make R reflexive (1, 1) and (3, 3) need to be added and to make *R* symmetric (3, 1) and (2, 3) need to be added.
Thus, a minimum of 4 order pairs, i.e., {(1, 1), (3, 3), (3, 1), (2, 3)} need to be added to make *R* both reflexive and symmetric.

Hence, the correct answer is option (c).

Solution 22

$x^2 + y^2 \leq 4$
$\Rightarrow y^2 \leq 4 - x^2$

The value of y^2 will always be a positive value so, the value of $4 - x^2$ must be greater than or equal to zero.

$\therefore 4 - x^2 \geq 0$
$\Rightarrow x^2 \leq 4$
$\Rightarrow -2 \leq x \leq 2$

Since the given relation in set **Z**, the domain of R is {−2, −1, 0, 1, 2}.

Hence, the correct answer is option (b).

Solution 23

Let X = {x^2 : *x* ∈ **N**} and *f* : **N** → X such that *f*(*x*) = x^2, *x* ∈ **N**.
Let *x*, *y* ∈ **N** such that *f*(*x*) = *f*(*y*).

⇒ $x^2 = y^2$
⇒ $x = y$
So, *f* is a one-one function.
Let $y \in \mathbf{X}$.
$\Rightarrow f(x) = y$

$\Rightarrow x^2 = y$

$\Rightarrow x = \sqrt{y} \in \mathbf{N} \quad [\because y \in \mathrm{X} = \{x^2 : x \in \mathbf{N}\}]$

So, *f* is onto function
Therefore, *f* is a bijective function.

Hence, the correct answer is option (d).

Solution 24

Given: *f*: **R** → **R** defined by $f(x) = 2 + x^2$

For One-One:
Let *x* and *y* be two arbitrary elements of **R** such that *f*(*x*) = *f*(*y*).
Then, *f*(*x*) = *f*(*y*)
⇒ $2 + x^2 = 2 + y^2$
⇒ $x^2 = y^2$
⇒ $x = \pm y$
Here, *f*(*x*) = *f*(*y*) does not provide the unique solution *x* = *y* but it provides $x = \pm y$.
Thus, *f* is not a one-one function.

For Onto:
Clearly $f(x) = 2 + x^2 \geq 2$ for all $x \in \mathbf{R}$.
So, negative real numbers in **R**(co-domain) do not have their pre-images in **R**(domain).
Thus, *f* is not an onto function.

Therefore, *f* is neither one-one nor onto.
Hence, the correct answer is option (d).

Solution 25

Given: A = {3, 5}
The number of reflexive relations on a set with the '*n*' number of elements is given by $2^{(n^2-n)}$.

Here, *n* = 2
∴ The number of reflexive relations on a set A = $2^{(2^2-2)} = 2^{(4-2)} = 2^2 = 4$

Hence, the correct answer is option (b).

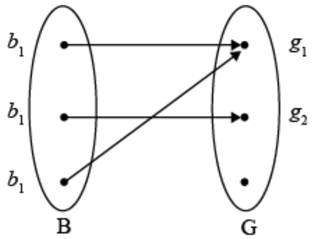
Solution 26

(I) Total number of relations from B to G = $2^{3\times 2}$
= 2^6
= 64
(II) Total number of functions from B to G = 2^3
= 8
(III) *R* = {(*x, y*) ; *x* and *y* are students of same}, R: B → B
<u>For Reflexive</u>, (*x, x*) ∈ R → *x* and *x* are students of same sex which is true
So, it is reflective relation
<u>For Symmetric,</u> If (*x, y*) ∈R → *x* and *y* are of same sex then (*y, x*) ∈R → *y* and *x* are of same sex which is true.
So, it is symmetric relation.

<u>For transitive</u>, if (*x, y*) ∈R → *x* and *y* are of same sex
(*y, z*)∈R → *y* and *z* are of same sex then (*x, z*) ∈R → *x* and *z* are of same sex which is true
So, it is transitive relation.
∴ It is an equivalence Relation.

OR

(III) $f = \{(b_1, g_1)\ (b_2, g_2)\ (b_3, g_1)\}$



It is many-one and onto function.
So, it is a surjective function.

Solution 27

R = {a, b): $a = b - 2$, $b > 6$}
If $b = 7$, $a = 5 \rightarrow (5, 7)$
$b = 8$, $a = 6 \rightarrow (6, 8)$

Hence, the correct answer is option (b).

Solution 28

f $f(x) = |\cos x|$

$$f\left(\frac{3\pi}{4}\right) = \left|\cos\left(\frac{3\pi}{4}\right)\right|$$
$$= \left|\cos\left(\pi - \frac{\pi}{4}\right)\right|$$
$$= \left|-\cos\frac{\pi}{4}\right|$$
$$= \left|-\frac{1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$$

Hence, the correct answer is option (d).