NDA II 2017_Mathematics

Total Time: 150

Total Marks: 300.0

Section A

Solution 1

$x + \log_{10}(1 + 2^x) = x\log_{10}5 + \log_{10}6$

$\Rightarrow \log_{10}(10^x) + \log_{10}(1 + 2^x) = \log_{10}(5^x) + \log_{10}(6)$

$\Rightarrow \log_{10}(10^x(1 + 2^x)) = \log_{10}(6 \times 5^x)$

$\Rightarrow 10^x(1 + 2^x) = 6 \times 5^x$

$\Rightarrow 2^x(1 + 2^x) = 6$

Let $2^x = t$

$\Rightarrow t(1 + t) = 6$

$\Rightarrow t^2 + t - 6 = 0$

$\Rightarrow (t - 2)(t + 3) = 0$

$\Rightarrow t = 2, t = -3$

$\therefore 2^x = 2$

$\Rightarrow x = 1$

Also, $2^x \neq -3$ as it is not true for any real value of x.

Hence, the correct answer is option C

Solution 2

We have
$(101110)_2 = (46)_{10}$ and $(110)_2 = (6)_{10}$
Remainder = $(4)_{10} = (100)_2$
Quotient = $(7)_{10} = (111)_2$
Hence, the correct answer is option B.

Solution 3

If AB exists, then number of columns of A is equal number of rows of B.
$x + 5 = y$ .....(1)

If BA exists, then number of columns of B is equal number of rows of A.
$11 - y = x$ .....(2)

Solving (1) and (2), we get
$x = 3, y = 8$

Hence, the correct answer is option C.

Solution 4

$S_n = nP + \frac{n(n-1)Q}{2}$,

$S_n = nP + \frac{n(n-1)Q}{2}$

Putting $n = 1,\ S_1 = P$

Putting $n = 2,\ S_2 = 2P + Q$

Second term $(T_2) = S_2 - S_1$

$\Rightarrow T_2 = 2P + Q - P = P + Q$

First Term $\left(T_1\right) = S_1 = P$

Common Difference, $d = T_2 - T_1 = Q$

Hence, the correct answer is option D.

Solution 5

Putting x = 1 in the given equation, we get

(q – r)(1)2 + (r – p)(1) + (p – q)
= q – r + r – p + p – q
= 0

For ax2 + bx + c = 0, if a + b + c = 0, it implies x = 1 is root of the equation.
Hence, 1 is a root.

Now, product of roots = $\frac{p-q}{q-r}$

As one of the roots is 1,
$\therefore$ Other root = $\frac{p-q}{q-r} \div 1 = \frac{p-q}{q-r}$

Hence, the correct answer is option B.

Solution 6

E – (E –(E – (E – (E – A))))
= E – (E – (E – (E – A′)))
= E – (E – (E – A))
= E – (E – A′) = E – A = A ′
= $(B \cup C)' = B' \cap C'$
Hence, the correct answer is option C.

Solution 7

A = { 2, 4, 6, 8, 10...}

B = { 5, 10, 15, 20, 25...}

C = {10, 20, 30, 40, 50...}

Clearly,
$A \cap (B \cap C) = A \cap C = \{10,\ 20,\ 30, \ldots\} = C$

Hence, the correct answer is option C.

Solution 8

Given: α and β are the roots of equation $1 + x + x^2 = 0$.

We know
$1 + \omega + \omega^2 = 0$, where ω is the cube root of unity.
Hence, $\alpha = \omega,\ \beta = \omega^2$

$$\begin{bmatrix} 1 & \beta \\ \alpha & \alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ 1 & \beta \end{bmatrix} = \begin{bmatrix} \alpha+\beta & \beta(1+\beta) \\ \alpha(1+\alpha) & 2\alpha\beta \end{bmatrix}$$
$$= \begin{bmatrix} \omega+\omega^2 & \omega^2\left(1+\omega^2\right) \\ \omega(1+\omega) & 2\omega^3 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & \omega^2(-\omega) \\ \omega\left(-\omega^2\right) & 2\omega^3 \end{bmatrix} \quad (\because 1+\omega+\omega^2=1)$$
$$= \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix} \quad (\because \omega^3=1)$$

Hence, the correct answer is option B.

Solution 9

We know that |*ab*| = |*a*||*b*|, this is true for all real values of *a*, *b*.

Also, by triangular inequality, we have

$|a\ +\ b|\ \leq\ \ |a|\ +\ |b|$ and

$|a-b|\ \geq\ ||a|\ -\ |b||$

Hence, the correct answer is option D.

Solution 10

There are 11 letters in the word 'PERMUTATION', of which 2 are T's and remaining all are each of its own kind.
So, total number of arrangements = $\frac{11!}{2!}=19958400$

Hence, the correct answer is option A.

Solution 11

$$kA=\frac{1}{2i}\begin{bmatrix} 4i\ -\ 6 & 10i \\ 14i & 6+4i \end{bmatrix}$$
$$=\frac{-i}{2}\begin{bmatrix} 4i\ -\ 6 & 10i \\ 14i & 6+4i \end{bmatrix}$$
$$=\begin{bmatrix} 2+3i & 5 \\ 7 & 2\ -\ 3i \end{bmatrix} \quad (\because i^2=-1)$$

Hence, the correct answer is option A.

Solution 12

The given equation is $|x-3|^2+|x-3|-2=0$.
Let |*x* − 3| = *y*
So, the equation becomes
$y^2\ +\ y\ -\ 2\ =\ 0$
$\Rightarrow (y-1)(y+2)\ =\ 0$
$\Rightarrow y\ =\ 1,\ -2$

If *y* = 1
$\Rightarrow |x-3|=1$
$\Rightarrow x-3=\pm 1$
$\Rightarrow x=2,\ 4$

If *y* = −2
$\Rightarrow |x-3|=-2$
which is not true for any value of *x*.

Therefore, the sum of all real roots is 2 + 4 = 6.

Hence, the correct answer is option D.

Solution 13

It is given that the roots of equation $x^2 - 4x - \log_3 P = 0$ are real.

$\therefore D \geq 0$

$\Rightarrow 16 + 4\log_3(P) \geq 0$

$\Rightarrow 4\log_3(P) \geq -16$

$\Rightarrow \log_3(P)^4 \geq -16$

$\Rightarrow P^4 \geq 3^{-16}$

$\Rightarrow P \geq 3^{-4}$

$\Rightarrow P \geq \frac{1}{81}$

So, minimum value of P is $\frac{1}{81}$.

Hence, the correct answer is option C.

Solution 14

We know

adj A^T = $(\text{adj}A)^T$

⇒ adj A^T – $(\text{adj } A)^T$ = 0

Hence, the correct answer is option C.

Solution 15

$6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times 6^{\frac{1}{16}} \times \ldots$

$= 6^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\right)}$

$= 6^{\left(\frac{1}{2\cdot 2}+\frac{1}{2\cdot 2^2}+\frac{1}{2\cdot 2^3}+\ldots\right)}$ $\quad \left(\text{Sum of infinite GP } = \frac{a}{1-r}\right)$

$= 6^{\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)}$

$= 6$

Hence, the correct answer is option A.

Solution 16

$$\begin{vmatrix} \cos^2\frac{\theta}{2} & \sin^2\frac{\theta}{2} \\ \sin^2\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{vmatrix} = \cos^4\left(\frac{\theta}{2}\right) - \sin^4\left(\frac{\theta}{2}\right)$$

$$= \left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)$$

$$= 1 \times \left(\cos 2\left(\frac{\theta}{2}\right)\right)$$

$$= \cos\theta$$

Hence, the correct answer is option B.

Solution 17

Number of terms in expansion of $(x + a)^{100}$

$= \left({}^{100}C_0\, x^{100} + {}^{100}C_1 x^{99} a^1 + {}^{100}C_2\, x^{98} a^2 + \ldots + {}^{100}C_{100} a^{100}\right)$

$= 101$

Number of terms in expansion of $(x - a)^{100}$

$= \left({}^{100}C_0\, x^{100} - {}^{100}C_1 x^{99} a^1 + {}^{100}C_2\, x^{98} a^2 - \ldots + {}^{100}C_{100} a^{100}\right)$

$= 101$

So, number of terms in expansion of $(x + a)^{100} + (x - a)^{100}$

$= 2\left({}^{100}C_0\, x^{100} + {}^{100}C_2 x^{98} a^2 + {}^{100}C_4\, x^{96} a^4 + \ldots + {}^{100}C_{100} a^{100}\right)$

$= 51$

Hence, the correct answer is option C.

Solution 18

In the expansion of $(1 + x)^{50}$,

Sum of coefficients of odd powers

$= C_1 + C_3 + C_5 + ..$

$= \frac{1}{2}(C_0 + C_1 + C_2 + C_3 + \ \ldots)$

$= \frac{1}{2} \times 2^{50} \quad (\because C_0 + C_1 + C_2 + \ \ldots \ + C_n = 2^n)$

$= 2^{49}$

Hence, the correct answer is option B.

Solution 19

$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = diag \begin{bmatrix} a, \ b, \ c \end{bmatrix}$

$A^{-1} = diag^{-1} \left[a, \ b, \ c\right]$

$= diag \left[a^{-1}, \ b^{-1}, \ c^{-1}\right]$

$= \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$

Hence, the correct answer is option A.

Solution 20

Let us assume that the total minutes taken by the person to count 4500 notes = x
Number of notes counted in the first nine minutes = 150 × 9 = 1350

Let the remaining minutes = $y = x - 9$
Let a_y denote the number of notes that he counts in the y^{th} minute.
Therefore, the number of notes counted in y minutes = $a_{10} + a_{11} + a_{12} + \ldots + a_y$

Total number of notes
= Number of notes counted in the first nine minutes + Number of notes counted in y minutes

$\Rightarrow (150 \times 9) + \frac{y}{2}[2 \times 150 + (y-1)(-2)] = 4500$

$\Rightarrow 1350 + y(151 - y) = 4500$

$\Rightarrow y(151 - y) = 3150$

$\Rightarrow y^2 - 151y + 3150 = 0$

$\Rightarrow (y - 126)(y - 25) = 0$

$\Rightarrow y = 25 \text{ or } 126$

Rejecting *y* = 126, as $a_{126} < 0$, which is not possible.
We get *y* = 25.
So, *x* = *y* + 9 = 25 + 9 = 34 minutes.
Hence, the correct answer is option B.

Solution 21

$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n = \left(\frac{(1+i)^2}{2}\right)^n = (i)^n$

$\Rightarrow i^n = 1$

$n = 4, 8, 12, 16...$

$\text{Minimum value of } n = 4$

Hence, the correct answer is option B.

Solution 22

(*a*, *b*) R (*c*, *d*) ⇒ *a* + *d* = *b* + *c*
a + *a* = *a* + *a*
(*a*, *a*) R (*a*, *a*) ⇒ R is reflexive.

Next, (*a*, *b*) R (*c*, *d*) ⇒ *a* + *d* = *b* + *c*
⇒*c* + *b* = *d* + *a* ⇒ (*c*, *d*) R (*a*, *b*)
⇒ R is symmetric.
Next, Let (*a*, *b*) R (*c*, *d*) and (*c*, *d*) R (*e*, *f*)
⇒ *a* + *d* = *b* + *c* and *c* + *f* = *d* + *e*
⇒ *a* + *d* + *c* + *f*= *b* + *c* + *d* + *e*
⇒ *a* + *f*= *b* + *e* ⇒ (*a*, *b*) R (*e*, *f*)
⇒ R is transitive
So R is an equivalence relation.
Hence, correct answer is option C.

Solution 23

$y = x + x^2 + x^3 + ...$

$y \text{ is an infinite GP}$

$y = \frac{x}{1-x} \quad \left[\because S_\infty = \frac{a}{1-r}\right]$

$x = \frac{y}{1+y}$

Hence, the correct answer is option A.

Solution 24

$3x^2 + 2x + 1 = 0$

$\alpha + \beta = -\frac{2}{3}, \alpha\beta = \frac{1}{3}$

For the equation having $(\alpha + \beta^{-1})$ and $(\beta + \alpha^{-1})$ as roots

$$S = \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta}$$
$$= \alpha + \beta + \frac{\alpha+\beta}{\alpha\beta}$$
$$= -\frac{2}{3} - 2 = -\frac{8}{3}$$
$$P = (\alpha + \beta^{-1})(\beta + \alpha^{-1})$$
$$= \alpha\beta + 2 + \frac{1}{\alpha\beta}$$
$$= \frac{16}{3}$$

Required equation is

$$x^2 - Sx + P = 0$$
$$x^2 + \frac{8}{3}x + \frac{16}{3} = 0$$
$$3x^2 + 8x + 16 = 0$$

Hence, the correct answer is option A.

Solution 25

$$\frac{1}{\log_3(e)} + \frac{1}{\log_3(e^2)} + \frac{1}{\log_3(e^4)} + \dots$$
$$= \log_e(3) + \frac{1}{2}\log_e(3) + \frac{1}{4}\log_e(3) + \dots$$
$$= \log_e 3\left(1 + \frac{1}{2} + \frac{1}{4} + \dots\right)$$
$$= \log_e 3\left(\frac{1}{1-\frac{1}{2}}\right)$$
$$= 2\log_e 3$$
$$= \log_e 9$$

Hence, the correct answer is option A.

Solution 26

Four on one side $= {}^8P_4$

Two on one side $= {}^8P_2$

Remainning 10 $= 10!$

Total possible arrangements $= \frac{8!}{4!} \times \frac{8!}{6!} \times 10! = 210 \times (8!)^2$

Hence, the correct answer is option C.

Solution 27

For given system of equation to have no solution.

$$\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$$
$$k(k^2-1) - (k-1) + (1-k) = 0$$
$$k(k+1)-1-1 = 0$$
$$k^2 + k - 2 = 0$$
$$k = 1, -2$$

For $k = 1$, equations become same.

So $k = -2$

Hence, the correct answer is option D.

Solution 28

$S = 1.3 + 2.3^2 + 3.3^3 + \ldots + n.3^n \quad \ldots(1)$

$3S = \quad 1.3^2 + 2.3^3 + \ldots + (n-1).3^n + n.3^{n+1} \quad \ldots(2)$

Subtractring (2) from (1)

$-2S = \left(3 + 3^2 + 3^3 + \ldots\ldots.+3^n\right) - n.3^{n+1}$

$-2S = \frac{3(3^n-1)}{2} - n.3^{n+1}$

$-2S = \frac{3^{n+1}-3-2n.3^{n+1}}{2}$

$S = \frac{-3^{n+1}+3+2n.3^{n+1}}{4}$

$S = \frac{3^{n+1}(2n-1)+3}{4} = \frac{(2n-1)3^a+b}{4}$

On comparing

$a = n+1,\ b = 3$

Hence, the correct answer is option D.

Solution 29

$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = -\frac{b}{a}$

$\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right) = \frac{c}{a}$

$\angle P + \angle Q = 90^\circ$

$\frac{\angle P}{2} + \frac{\angle Q}{2} = 45^\circ$

$\Rightarrow \tan\left(\frac{P+Q}{2}\right) = \frac{\tan\left(\frac{P}{2}\right)+\tan\left(\frac{Q}{2}\right)}{1-\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)}$

$\Rightarrow \tan 45^\circ = \frac{\tan\left(\frac{P}{2}\right)+\tan\left(\frac{Q}{2}\right)}{1-\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)}$

$\Rightarrow 1 = \frac{\tan\left(\frac{P}{2}\right)+\tan\left(\frac{Q}{2}\right)}{1-\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)}$

$\Rightarrow \tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = 1 - \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)$

$\Rightarrow -\frac{b}{a} = 1 - \frac{c}{a}$

$\Rightarrow -b = a - c$

$\Rightarrow c = a + b$

Hence, the correct answer is option C.

Solution 30

$\left|z - \frac{4}{z}\right| \geq |z| - \left|\frac{4}{z}\right|$

$\left|x - \frac{a}{x}\right| \geq 2$ (for any x)

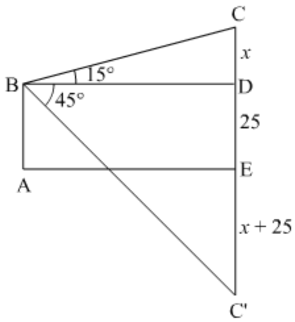
$|z| - \left|\frac{4}{z}\right| \leq 2$

$\left|z\right|^2 - 2\left|z\right| - 4 \leq 0$

$\left|z\right|_{max} = \frac{2+2\sqrt{5}}{2} = 1+\sqrt{5}$

Hence, the correct answer is option B.

Solution 31



In $\triangle BDC'$

$\tan 45^\circ = \frac{x+50}{BD}$

$\Rightarrow BD = x + 50$

In $\triangle BCD$

$\tan 15^\circ = \frac{x}{BD}$

$2 - \sqrt{3} = \frac{x}{x+50}$

$x = \frac{50\left(2-\sqrt{3}\right)}{\sqrt{3}-1} = 25\left(\sqrt{3} - 1\right)$

Height of cloud above lake $= x + 25$

$= 25\left(\sqrt{3} - 1\right) + 25 = 25\sqrt{3}$

Hence, the correct answer is option B.

Solution 32

$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$= \left(\tan 9^\circ + \cot 9^\circ\right) - \left(\tan 27^\circ + \cot 27^\circ\right)$

$= \left(\frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\cos 9^\circ}{\sin 9^\circ}\right) - \left(\frac{\sin 27^\circ}{\cos 27^\circ} + \frac{\cos 27^\circ}{\sin 27^\circ}\right)$

$= \left(\frac{1}{\sin 9^\circ \cos 9^\circ}\right) - \left(\frac{1}{\sin 27^\circ \cos 27^\circ}\right)$

$= \frac{2.2 \cos 36 \sin 18}{\sin 18^\circ \sin 54^\circ} = 4$

Hence, the correct answer is option D.

Solution 33

$\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$

$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$

$= 2\left(\frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{2 \sin 20^\circ \cos 20^\circ}\right)$

$= 2 \times 2\left(\frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\sin 40^\circ}\right)$

$= 2 \times 2 \frac{\sin (60^\circ - 20^\circ)}{\sin 40^\circ} = 4$

Hence, the correct answer is option A.

Solution 34

$\alpha = A + B$ and $x = A - B$

$A = \frac{x+\alpha}{2}, B = \frac{\alpha - x}{2}$

$\frac{\tan A}{\tan B} = \frac{\tan\left(\frac{x+\alpha}{2}\right)}{\tan\left(\frac{\alpha-x}{2}\right)} = \frac{p}{q}$

$\frac{2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{\alpha-x}{2}\right)}{2\cos\left(\frac{\alpha+x}{2}\right)\sin\left(\frac{\alpha-x}{2}\right)} = \frac{p}{q}$

$\frac{\sin\alpha + \sin x}{\sin\alpha - \sin x} = \frac{p}{q}$

$\frac{\sin\alpha + \sin x + \sin\alpha - \sin x}{\sin\alpha + \sin x - \sin\alpha + \sin x} = \frac{p+q}{p-q}$

$\sin x = \frac{(p-q)\sin\alpha}{p+q}$

Hence, the correct answer is option D.

Solution 35

$\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7}$ $\left[\because \sin^{-1}x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right]$

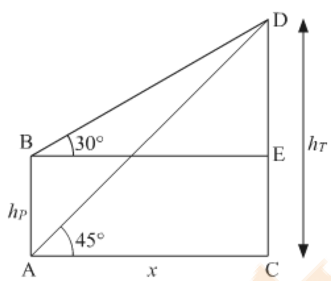
$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$ $\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= \tan^{-1}\left(\frac{21+4}{28-3}\right)$

$= \tan^{-1}\left(\frac{25}{25}\right) = \tan^{-1}1 = \frac{\pi}{4}$

Hence, the correct answer is option B.

Solution 36



Let distance between pole and tower is x

In $\Delta\, ACD$

$\frac{h_T}{x} = \tan 45^\circ = 1$

$\Rightarrow h_T = x$

In $\Delta\, BED$

$\frac{h_T - h_P}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{h_T - h_P}{h_t} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{h_T - h_P}{h_T} = \frac{1}{\sqrt{3}}$

$\Rightarrow 1 - \frac{h_P}{h_T} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{h_P}{h_T} = \frac{\sqrt{3}-1}{\sqrt{3}}$

$\Rightarrow \frac{h_T}{h_P} = \frac{\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow \frac{h_T}{h_P} - 1 = \frac{\sqrt{3}}{\sqrt{3}-1} - 1$

$\Rightarrow \frac{h_T - h_P}{h_P} = \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2} \left(\text{Statement 2 is correct}\right)$

$\Rightarrow \frac{h_T}{h_P} - 1 = \frac{\sqrt{3}+1}{2}$

$\Rightarrow \frac{h_T}{h_P} = \frac{\sqrt{3}+1}{2} + 1$

$\Rightarrow \frac{h_T}{h_P} = \frac{\sqrt{3}+3}{2}$

$\Rightarrow \frac{2h_T h_P}{\sqrt{3}+3} = h_P^2 \qquad \left(\text{Statement 1 is correct}\right)$

$\frac{h_T - h_P}{h_P} + 2 = \frac{\sqrt{3}+1}{2} + 2$

$\frac{h_T + h_P}{h_P} = \frac{5+\sqrt{3}}{2} \left(\text{Statement 3 is incorrect}\right)$

Hence, the correct answer is option C.

Solution 37

$a + c = 2b$

$\cot\left(\frac{A}{2}\right)\cot\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-a)}{(s-b)(s-c)} \times \frac{s(s-c)}{(s-a)(s-b)}}$

$= \frac{s}{s-b} = \frac{2s}{2s-2b} = \frac{a+b+c}{a+b+c-2b} = \frac{3b}{b} = 3$

Hence, the correct answer is option B.

Solution 38

$$\sqrt{1+\sin A}=\sqrt{\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)^2}=\left|\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)\right|$$

$$\left|\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)\right|=\left\{\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)\qquad \frac{3\pi}{4}<\frac{A}{2}<\frac{5\pi}{4}\right.$$

$$\left.-\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)\qquad \text{Otherwise}\right\}$$

$$\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)=\sqrt{2}\left(\cos\left(\frac{A}{2}-\frac{\pi}{4}\right)\right)$$

$$\cos\left(\frac{A}{2}-\frac{\pi}{4}\right)<0,\ for\ \frac{\pi}{2}<\frac{A}{2}-\frac{\pi}{4}<\pi$$

$$\frac{\pi}{2}<\frac{\text{A}}{2}-\frac{\pi}{4}<\pi$$

$$\frac{3\pi}{4}<\frac{\text{A}}{2}<\frac{5\pi}{4}$$

$$\frac{3\pi}{2}<\text{A}<\frac{5\pi}{2}$$

Hence, the correct answer is option A.

Solution 39

$$\frac{\sin^2 A+\sin^2 B+\sin^2 C}{\cos^2 A+\cos^2 B+\cos^2 C}=2$$

$$\sin^2 A+\sin^2 B+\sin^2 C = 2\left(\cos^2 A+\cos^2 B+\cos^2 C\right)$$

$$\left(1-\cos^2 A+1-\cos^2 B+1-\cos^2 C\right) = 2\left(\cos^2 A+\cos^2 B+\cos^2 C\right)$$

$$\left(\cos^2 A+\cos^2 B+\cos^2 C\right) = 1$$

$$\frac{3}{2}+\frac{1}{2}\left(\cos 2A + \cos 2B + \cos 2C\right) = 1$$

$$\cos 2A + \cos 2B + \cos 2C = -1$$

$$2\cos\left(A+B\right)\cos\left(A-B\right) = -\left(1+\cos 2C\right)$$

$$-2\cos C\cos\left(A-B\right) = -2\cos^2 C$$

$$\cos\left(A-B\right) = \cos C$$

$$A-B = C$$

$$\text{Again, } A+B+C = \pi$$

$$A+B+A-B = \pi$$

$$A = \frac{\pi}{2}$$

$$\text{So } \Delta \text{ is right angled.}$$

Hence, the correct answer is option A.

Solution 40

Principle value of $\sin^{-1}(x)$ lies between $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hence, the correct answer is option B.

Solution 41

Using hit and trial, we can observe all the given points lie in the line *ay* = *bx*. So all the points are colinear.
Hence, the correct answer is option D.

Solution 42

Length of normal from origin to plane is

$$\frac{|D|}{\sqrt{A^2+B^2+C^2}} = \frac{9}{3}=3$$

Hence, the correct answer is option C.

Solution 43

Here $\cos\alpha,\ \cos\beta,\ \cos\gamma$ are direction cosines.
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\cos^2\alpha + \cos^2\beta = 1 - \cos^2\gamma$
$\cos^2\alpha + \cos^2\beta = \sin^2\gamma \qquad (1 \text{ is correct})$
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$(1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$
$3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$
$(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 2 \quad (3 \text{ is correct})$

Hence, correct answer is option C.

Solution 44

Slope of line $x + y - 3 = 0 \Rightarrow m_1 = -1$
Slope of line $x - y + 3 = 0 \Rightarrow m_2 = 1$
$m_1 \times m_2 = -1$
$\alpha = 90^\circ$
It is given that β is acute so $\alpha > \beta$

Hence, the correct answer is option B.

Solution 45

Let $\delta = a\,\hat{i} + b\,\hat{j} + c\,\hat{k}$
$\vec{\alpha}.\vec{\delta} = 0 \Rightarrow a + 2b - c = 0 \ldots (1)$
$\vec{\beta}.\vec{\delta} = 0 \Rightarrow 2a - b + 3c = 0 \ldots (2)$
From (1) and (2)
$\frac{a}{5} = \frac{b}{-5} = \frac{c}{-5}$
$\frac{a}{1} = \frac{b}{-1} = \frac{c}{-1} = \lambda$
$a = \lambda,\ b = -\lambda,\ c = -\lambda$
$\vec{\delta}.\vec{\gamma} = 10$
$2a + b + 6c = 2\lambda - \lambda - 6\lambda = 10$
$\lambda = -2$
$\vec{\delta} = -2\hat{i} + 2\,\hat{j} + 2\,\hat{k}$
$\left|\vec{\delta}\right| = \sqrt{12} = 2\sqrt{3}$

Hence, the correct answer is option B.

Solution 46

$\left(\hat{a} + \hat{b}\right)\left(\hat{a} \times \hat{b}\right) = \hat{a} \times \left(\hat{a} \times \hat{b}\right) + b \times \left(\hat{a} \times \hat{b}\right)$
$= \left(\hat{a}.\hat{b}\right)\hat{a} - \left(\hat{a}.\hat{a}\right)\hat{b} + \left(\hat{b}.\hat{b}\right)\hat{a} - \left(\hat{b}.\hat{a}\right)\hat{b}$
$= k\hat{a} - \hat{b} + \hat{a} - k\hat{b}$
$= \left(k+1\right)\left(\hat{a} - \hat{b}\right)$

This is parallel to (a − b) vector
Hence, the correct answer is option A.

Solution 47

$\overrightarrow{F} = \hat{i} + 3\hat{j} + 2\hat{k}$, $\overrightarrow{OA} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\overrightarrow{OB} = 3\hat{i} - \hat{j} + 5\hat{k}$

Displacement $\left(\overrightarrow{AB}\right) = \overrightarrow{OB} - \overrightarrow{OA} = 2\hat{i} - 3\hat{j} + 8\hat{k}$

$W = \overrightarrow{F} . \overrightarrow{AB} = \left(\hat{i} + 3\hat{j} + 2\hat{k}\right).\left(2\hat{i} - 3\hat{j} + 8\hat{k}\right) = 9$ units

Hence, the correct answer is option C.

Solution 48

Let $\overrightarrow{a} = x\,\hat{i} + y\,\hat{j} + z\hat{k}$

$\left|\overrightarrow{a} \times \hat{i}\right|^2 = z^2 + y^2$

$\left|\overrightarrow{a} \times \hat{j}\right|^2 = x^2 + z^2$

$\left|\overrightarrow{a} \times \hat{k}\right|^2 = x^2 + y^2$

$\left|\overrightarrow{a} \times \hat{i}\right|^2 + \left|\overrightarrow{a} \times \hat{j}\right|^2 + \left|\overrightarrow{a} \times \hat{k}\right|^2 = 2\left(x^2 + y^2 + z^2\right) = 2\left|\overrightarrow{a}\right|^2$

Hence, the correct answer is option B.

Solution 49

Man is moving in a path such that at every point sum of the distance of two fixed points from him is always constant, so the man is moving in elliptical path.

Let F_1 and F_2 be two fixed points at any point P.

$PF_1 + PF_2 = 2a = 10$ m

$\Rightarrow a = 5$

Distance between two fixed points $(2c) = 8$ m

$\Rightarrow c = 4$

Now, $a^2 + b^2 = c^2$

$\Rightarrow b = 3$

Area of ellipse $= \pi ab = 15\pi$ sq metres

Hence, the correct answer is option B.

Solution 50

Equation of line parallel to $4x + y = 4$ and passing through (1, 3)

$y - 3 = -4(x - 1)$

$y + 4x = 7$ .....(1)

Given line: $2x + 3y = 6$ .....(2)

Solving (1) and (2)

$x = \frac{3}{2}, y = 1$

The distance of point (1,3) from the line $2x + 3y = 6$, measured parallel to the line $4x + y = 4$ will be distance between (1, 3) and $\left(\frac{3}{2}, 1\right)$

$d = \sqrt{\left(\frac{3}{2} - 1\right)^2 + (1 - 3)^2}$

$d = \sqrt{\frac{17}{4}}$

$d = \frac{\sqrt{17}}{2}$

Hence, the correct answer is option D.

Solution 51

If given vectors are coplanar then

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1, \ C_3 \to C_3 - C_1$

$$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$$

$a(b-1)(c-1) - (1-a)(c-1) - (1-a)(b-1) = 0$

Dividing by $(1-a)(1-b)(1-c)$

$\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$

$\frac{1}{1-b} + \frac{1}{1-c} = -\frac{a}{1-a}$

$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = -\frac{a}{1-a} + \frac{1}{1-a} = 1$

$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

Hence, the correct answer is option B.

Solution 52

Equation of line is

$\frac{x+3}{8} = \frac{y-4}{-10} = \frac{z+8}{12} = \lambda$

$x = 8\lambda - 3, \ y = -10\lambda + 4, \ z = 12\lambda - 8,$

since line intersects XY plane, so, $z = 0$

$\lambda = \frac{2}{3},$

$x = \frac{7}{3}, \ y = -\frac{8}{3}$

Point $\left(\frac{7}{3}, \frac{-8}{3}, 0\right)$

Hence, the correct answer is option A.

Solution 53

$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1+b_1+c_1}\sqrt{a_2+b_2+c_2}}$

$\Rightarrow \cos\frac{\pi}{4} = \frac{2x-3+10}{\sqrt{9}\sqrt{x^2+34}}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{2x-3+10}{3\sqrt{x^2+34}}$

$\Rightarrow 2x-3+10 = 3\sqrt{\frac{x^2+34}{2}}$

$\Rightarrow 4x^2 + 49 + 28x = \frac{9\left(x^2+34\right)}{2}$

$\Rightarrow 8x^2 + 98 + 56x = 306 + 9x^2$

$\Rightarrow x^2 - 56x + 208 = 0$

$\Rightarrow x = \frac{56 \pm \sqrt{3136-832}}{2} = 28 \pm 24 = 4, \ 52$

Smaller value $= 4$

Hence, the correct answer is option B.

Solution 54

$2(1)^2 + 7(2)^2 - 20 = 2 + 28 - 20 > 0$

Thus, point lies outside the elipse.

Hence, the correct answer is option A.

Solution 55

Slope of line $\left(m\right) = \tan 120^{\circ} = -\sqrt{3}$

Line is making an intercept of 5 unit in negative y direction

hence it passes through $\left(0, -5\right)$

Equation of line

$y + 5 = -\sqrt{3}\left(x - 0\right)$

$y + \sqrt{3}x + 5 = 0$

Hence, the correct answer is option A.

Solution 56

Equation of line passing through intersection of two given lines

$\left(2x - 3y + 7\right) + \lambda\left(7x + 4y + 2\right) = 0$

Line is passing through $\left(2, 3\right)$

$\left(4 - 9 + 7\right) + \lambda\left(14 + 12 + 2\right) = 0$

$\lambda = -\frac{1}{14}$

$\left(28x - 7x\right) + \left(-42y - 4y\right) + \left(98 - 2\right) = 0$

$21x - 46y + 96 = 0$

Hence, the correct answer is option B.

Solution 57

We Know,

latus rectum $= \frac{2b^2}{a}$

$\Rightarrow 4 = \frac{2b^2}{a}$

$\Rightarrow b^2 = 2a,$

And, $c = ae$

$c^2 = \frac{9}{16}a^2$

We know, $a^2 = b^2 + c^2$

So, $a^2 = 2a + \frac{9}{16}a^2 \Rightarrow a = \frac{32}{7}$

$b^2 = \frac{64}{7}$

Equation of ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Putting values of a and b

$\frac{49x^2}{1024} + \frac{7y^2}{64} = 1$

Hence, the correct answer is option B.

Solution 58

Let the equation of circle be

$(x-h)^2+(y-k)^2=r^2 \qquad \ldots(i)$

Putting $3,4$ in i

$(3-h)^2+(4-k)^2=r^2$

$9+h^2-6h+16+k^2-8k=r^2 \qquad \ldots(ii)$

Putting $(0,-6)$ in (i)

$(0-h)^2+(-6-k)^2=r^2$

$h^2+36+k^2+12k=r^2 \qquad \ldots(iii)$

Putting $\left(1,0\right)$ in $\left(i\right)$

$(1-h)^2+(0-k)^2=r^2$

$1+h^2-2h+k^2=r^2 \qquad \ldots(iv)$

Subtracting (iii) from (ii)

$9+h^2-6h+16+k^2-8k-\left(h^2+36+k^2+12k\right)=r^2-r^2$

$9-6h+16-8k-36-12k=0$

$6h+20k+11=0 \qquad \ldots(v)$

Subtracting (iii) from (iv)

$1+h^2-2h+k^2-\left(h^2+36+k^2+12k\right)=r^2-r^2$

$1-2h-36-12k=0$

$2h+12k+35=0 \qquad \ldots(vi)$

Solving for k and h from (v) and (vi)

$k=\frac{-47}{8}$ and $h=\frac{71}{4}$

Center $\left(\frac{71}{4},\frac{-47}{8}\right)$

Using distance formula between centre and point $\left(1,0\right)$ radius will be

$=\sqrt{\left(\frac{71}{4}-1\right)^2+\left(\frac{-47}{8}-0\right)^2}$

$=\sqrt{\left(\frac{67}{4}\right)^2+\left(\frac{-47}{8}\right)^2}$

$=\sqrt{\frac{4489}{16}+\frac{2209}{64}}$

$=\sqrt{\frac{17956+2209}{64}}$

$=\sqrt{\frac{20165}{64}}$

Putting values of centre and radius in $\left(i\right)$

$(x-h)^2+(y-k)^2=r^2$

$\left(x-\frac{71}{4}\right)^2+\left(y-\frac{-47}{8}\right)^2=\left(\sqrt{\frac{20165}{64}}\right)^2$

$x^2+\frac{5041}{16}-\frac{71x}{2}+y^2+\frac{2209}{64}+\frac{47y}{4}=\frac{20165}{64}$

$\frac{64x^2+64y^2-2272x+752y+2208}{64}=0$

$64x^2+64y^2-2272x+752y+2208=0$

$4x^2+4y^2-142x+47y+138=0$

Hence, the correct answer is option C.

Solution 59

Equation of plane is

$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1$

It pases through $\left(a, b, c\right)$

$\frac{a}{p} + \frac{b}{q} + \frac{c}{r} = 1.....\left(1\right)$

Let centre of sphere be $\left(h, k, l\right)$

$(h-p)^2 + k^2 + l^2 = h^2 + k^2 + l^2$

$p^2 = 2hp$

$h = \frac{p}{2} \Rightarrow p = 2h$

Similarly

$q = 2k,\ r = 2l$

Putting values in $\left(1\right)$

$\frac{a}{2h} + \frac{b}{2k} + \frac{c}{2l} = 1$

$\frac{a}{h} + \frac{b}{k} + \frac{c}{l} = 2$

Locus of center of sphere

$\frac{\mathrm{a}}{x} + \frac{\mathrm{b}}{y} + \frac{\mathrm{c}}{z} = 2$

Hence, the correct answer is option C.

Solution 60

$P1 : x + y + z - 1 = 0$

$P2 : 2x + 3y + 4z - 7 = 0$

Equation of plane pasing through the line of intersection of $P1$ and $P2$ is given by

$x + y + z - 1 + \lambda\,(2x + 3y + 4z - 7) = 0$

$x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 + 4\lambda) - 1 - 7\lambda = 0$

This is perpendicular to $x - 5y + 3z - 5 = 0$

$1(1 + 2\lambda) - 5(1 + 3\lambda) + 3(1 + 4\lambda) = 0$

$1 + 2\lambda - 5 - 15\lambda + 3 + 12\lambda = 0$

$-\lambda - 1 = 0$

$\lambda = -1$

Equation of plane is

$-x - 2y - 3z - 1 + 7 = 0$

$x + 2y + 3z - 6 = 0$

Hence, the correct answer is option A.

Solution 61

$y = 5^{\log x}$

$\log y = \left(\log x\right)\left(\log 5\right)$

$\log x = \frac{\log y}{\log 5}$

$\log x = \log y^{\frac{1}{\log(5)}}$

$x = y^{\frac{1}{\log 5}},\ y > 0$

Hence, the correct answer is option A.

Solution 62

$$f(x)=\begin{cases}-\frac{x}{\sqrt{x^2}}, & x\neq 0\\ 0, & x=0\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}-\frac{x}{|x|}, & x\neq 0\\ 0, & x=0\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}-\frac{-x}{x}=1, & x<0\\ 0, & x=0\\ -\frac{x}{x}=-1, & x>0\end{cases}$$

Clearly f(x), being a constant function for $x < 0$ and $x > 0$, is continuous for all $x < 0$ and all $x > 0$.
Consider the point $x = 0$.
We have,

$$LHL=\lim_{x\to 0^-} f(x)=\lim_{x\to 0}(1)=1$$

$$RHL=\lim_{x\to 0^+} f(x)=\lim_{x\to 0}(-1)=-1$$

$$\therefore LHL\neq RHL$$

So, $f(x)$ is not continuous at $x = 0$.
Hence, the correct answer is option C.

Solution 63

$$y = \left(\cos\ x\right)^y$$

$$\log\ y = y\ \log\ \cos\ x$$

$$\text{Differentiating both sides,}$$

$$\frac{1}{y}\ \frac{dy}{dx} = y\left(-\ \tan x\right) + \log\ \left(cosx\right)\frac{dy}{dx}$$

$$\left(\frac{1}{y}-\log\ \cos\ x\right)\frac{dy}{dx} = -y\ \tan x$$

$$\frac{dy}{dx} = \frac{-y^2\ \tan\ x}{1-y\ \log\ \cos\ x}$$

Hence, the correct answer is option A.

Solution 64

1. A polynomial function is continuous at every point in R, hence $x + x^2$ is continuous at $x = 0$
2. We know that cos $\frac{1}{x}$ is discontinuous at $x = 0$, so x + cos $\frac{1}{x}$ is discontinuous at $x = 0$
3. Since cos $\frac{1}{x}$ is discontinuous at $x = 0$, so x^2 + cos $\frac{1}{x}$ is discontinuous at $x = 0$

Hence, the correct answer is option A.

Solution 65

1. *dy*/*dx* at a point on the curve gives slope of the tangent at that point: Correct
2. If $a(t)$ denotes acceleration of a particle, then $\int a(t)dt + c$ gives velocity of the particle: Correct

3. If $s(t)$ gives displacement of a particle at time t, then *ds*/*dt* gives its acceleration at that instant: Incorrect
As *ds*/*dt* gives its velocity at that instant.

Hence, the correct answer is option A.

Solution 66

$$y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$$

$$y=\cos^{-1}\left(\frac{x-1}{x+1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right) = \frac{\pi}{2}$$

$$\frac{dy}{dx}=0$$

Hence, the correct answer is option A.

Solution 67

$\int \tan^{-1}(\sec x + \tan x)dx$

$(\sec x + \tan x) \ = \ \frac{1+\sin x}{\cos \ x}$

$= \frac{\left(\sin \frac{x}{2}+\cos \frac{x}{2}\right)^2}{\left(\cos \frac{x}{2}+\sin \frac{x}{2}\right)\left(\cos \frac{x}{2}-\sin \frac{x}{2}\right)}$

$= \frac{\left(\cos \frac{x}{2}+\sin \frac{x}{2}\right)}{\left(\cos \frac{x}{2}-\sin \frac{x}{2}\right)} = \frac{\left(1+\tan \frac{x}{2}\right)}{\left(1-\tan \frac{x}{2}\right)} = \tan \ \left(\frac{\pi}{4} - \frac{x}{2}\right)$

$\int \tan^{-1} \ \tan \ \left(\frac{\pi}{4} - \frac{x}{2}\right)dx \ = \int \left(\frac{\pi}{4} - \frac{x}{2}\right) \ dx = \ \frac{\pi x}{4} - \frac{x^2}{4} + c$

Hence, the correct answer is option D.

Solution 68

$$f(x) = \begin{Bmatrix} 1 - x^2 \ \text{for} \ 0 < x \leq 1 \\ \ln \ x \ \text{for} \ 1 < x \leq 2 \\ \ln \ 2 - 1 + 0 \ .5 \ x \ \text{for} \ 2 < x < \infty \end{Bmatrix}$$

$f'(x) \ = \ -2x \ \text{for} \ 0 < x \leq 1$

Hence, the correct answer is option B.

Solution 69

$f(x) \ = \ x(x-1)(x+1)$

$f(x) \ = \ x^3 \ - \ x$

$f'(x) \ = \ 3x^2 - 1$

$f''(x) \ = \ 6x$

$\text{At} \ f'(x) \ = \ 0 \ \Rightarrow \ x \ = \ \pm \frac{1}{\sqrt{3}}$

$\max \ f(x) \ = \ f\left(-\frac{1}{\sqrt{3}}\right) = \frac{2}{3\sqrt{3}}$

$\min \ f(x) \ = \ f\left(\frac{1}{\sqrt{3}}\right) = -\frac{2}{3\sqrt{3}}$

Local maximum value is larger than local minimum value.
Hence, the correct answer is option A.

Solution 70

1. Derivative of *f*(*x*) may not exist at some point: True

2. Derivative of *f*(*x*) may exist finitely at some point: True

3. Derivative of *f*(*x*) may be infinite (geometrically) at some point: True

Hence, correct answer is option D.

Solution 71

$f\left(x\right) = \frac{\ln\ x}{x}$

$f'\left(x\right) = \frac{\left(\frac{1}{x}\right)x - \ln x}{x^2} = \frac{1-\ln x}{x^2}$

Putting $f'\left(x\right) = 0$

$\Rightarrow\ \ln x = 1\ \Rightarrow x = e$

$f''\left(x\right)\ =\ \frac{-\frac{1}{x}x^2\ -\ 2x(1-\ln x)}{x^4}$

$= \frac{-x-2x(1-\ln x)}{x^4}\ =\ \frac{-(1+2-2\ln x)}{x^3} = \frac{-(3-2\ln x)}{x^3}$

$f''\left(e\right)\ =\ \frac{-(3-2)}{e^3}\ =\ -\frac{1}{e^3} < 0$

$\therefore f\left(x\right)_{max}\ = f\left(e\right)\ = \frac{\ln\ e}{e} = \frac{1}{e}$

Hence, the correct answer is option B.

Solution 72

$f(x) = |x| - x^3$

$f(-x) = |-x| - \left(-x\right)^3\ =\ \left|-x\right|\ +\ x^3$

f(*x*) is neither even nor odd.
Hence, the correct answer is option D.

Solution 73

$I_1\ =\ \frac{d}{dx}\left(e^{\sin\ x}\right)$

$I_2\ =\ \lim\limits_{h\to 0}\ \frac{e^{\sin\ (x+h)}\ -\ e^{\sin\ x}}{h}\ = \frac{d}{dx}\left(e^{\sin\ x}\right)\ =\ I_1$

$I_3\ =\ \int e^{\sin\ x} \cos\ x\ dx\ = \int e^t\ dt = e^t\ + c = e^{\sin\ x} + c$

$\frac{d}{dx}\left(I_3\right) = \frac{d}{dx}\left(e^{\sin\ x} + c\right) = \frac{d}{dx}\left(e^{\sin\ x}\right) = I_2$

Hence, the correct answer is option B.

Solution 74

$\frac{dy}{dx}\ = \frac{ax+h}{by+k}$

$\Rightarrow \left(by + k\right)\ dy = \left(ax + h\right)\ dx$

Integrating both sides, we get

$\Rightarrow \int\left(by + k\right)\ dy = \int\left(ax + h\right)\ dx$

$\Rightarrow \frac{by^2}{2} + ky = \frac{ax^2}{2} + hx + c$

$\Rightarrow \frac{ax^2}{2} - \frac{by^2}{2} + hx - ky + c = 0$

For an equation to represent a circle,
Coefficient of x^2 = Coefficient to y^2
$\Rightarrow a = -b \neq 0$

Hence, the correct answer is option B.

Solution 75

$$l=\lim_{x\to\frac{\pi}{2}}\ \frac{\sin\ x}{x}\ =\ \frac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}}=\frac{1}{\frac{\pi}{2}}=\frac{2}{\pi}$$

$$m=\lim_{x\to\infty}\ \frac{\cos\ x}{x},$$

$$\text{Let}\ x\ =\frac{1}{t}$$

$$\text{so,}\ \ \text{when}\ x\to\infty,\ t\ \to 0$$

$$m=\lim_{t\to 0}\ \frac{\cos\left(\frac{1}{t}\right)}{\frac{1}{t}}=\lim_{t\to 0}\ t\times\cos\left(\frac{1}{t}\right)=0$$

Hence, the correct answer is option C.

Solution 76

$$\int_0^{2\pi}\sqrt{1+\sin\frac{x}{2}}\,dx$$

$$=\ \int_0^{2\pi}\ \left|\sin\frac{x}{4}+\cos\frac{x}{4}\right|dx$$

$$=4\left(\sin\frac{x}{4}-\cos\frac{x}{4}\right)\Big|_0^{2\pi}$$

$$=4\left[\left(\sin\frac{2\pi}{4}-\cos\frac{2\pi}{4}\right)-\left(\sin\frac{0}{4}-\cos\frac{0}{4}\right)\right]$$

$$=4\left\{1-\left(-1\right)\right\}$$

$$=8$$

Hence, the correct answer is option A.

Solution 77

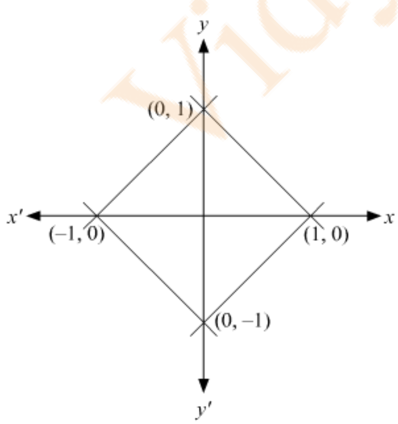
The curve $|x|+|y|=1$ represents the following equations:

$x + y = 1$

$-x + y = 1$

$x - y = 1$

$x + y = -1$



By the graphs of these equations, we can observe that area bounded between the lines is a square having diagonals of length 2.

Area = $\frac{1}{2} \times (2)^2 = 2 \text{ sq unit}$

Hence, the correct answer is option C.

Solution 78

$\text{Let } y = \frac{x^2}{1+x^4}$

$\Rightarrow y \geq 0$

$\text{Also, } y = \frac{x^2}{1+x^4} = \frac{1}{x^2+\frac{1}{x^2}}$

$\text{Now, } x^2 + \frac{1}{x^2} \geq 2 \quad \text{As}\left(\text{A.M.} \geq \text{G.M.}\right)$

$\text{So, } y \leq \frac{1}{2}$

$\Rightarrow y \in \left[0, \frac{1}{2}\right]$

Hence, the correct answer is option C.

Solution 79

$L.H.D$

$= \lim_{h\to0} \frac{[k-h]\sin\pi(k-h)-[k]\sin k\pi}{-h}$

$= \lim_{h\to0} \frac{(k-1)\sin\pi(k-h)-k\sin k\pi}{-h}$

$= \lim_{h\to0} \frac{(k-1)\sin\pi(k-h)}{-h} \qquad [\because \sin k\pi = 0 \ \forall\, k \in \boldsymbol{Z}\,]$

$= \lim_{h\to0} \frac{(k-1)\{\sin\pi k\cos\pi h-\cos\pi k\sin\pi h\}}{-h}$

$= \lim_{h\to0} \frac{(k-1)\{-\cos\pi k\sin\pi h\}}{-h} \qquad [\because \sin k\pi = 0 \ \forall\, k \in \boldsymbol{Z}]$

$= \lim_{h\to0} \frac{(k-1)\left\{-(-1)^k\sin(\pi h)\right\}}{-h} \qquad \left[\because \cos\pi k = (-1)^k \ \forall\, k \in \boldsymbol{Z}\right]$

$= \lim_{h\to0} \frac{(k-1)\left\{(-1)^{k+1}\sin(\pi h)\right\}}{-h}$

$= \lim_{h\to0} \frac{\pi(k-1)\left\{(-1)^k\sin(\pi h)\right\}}{\pi h}$

$= \pi\,(k-1)(-1)^k \lim_{h\to0} \frac{\sin(\pi h)}{\pi h}$

$= (-1)^k\,(\text{k}-1)\pi$

Hence, the correct answer is the option A.

Solution 80

$$f\left(x\right) = \frac{x}{2} - 1 \ , \ [0, \pi]$$

$$\tan[f(x)] = \tan\left[\frac{x}{2} - 1\right]$$

$\frac{1}{f(x)} = \frac{1}{\frac{x}{2}-1}$ *is discontinuous at* $x = 2$, $\because \frac{x}{2} = 1$ *at* $x = 2$ *and the denominator will become* 0

$\tan[f(x)]$ *is continuous in* $[0, \pi]$ $\because [f(x)]$ *in* $[0, \pi]$ *will have only two elements in it's range* $\{\tan(-1), 0\}$

Let $y = \frac{x}{2} - 1$

$\Rightarrow x = 2(y+1)$

$f^{-1}\left(x\right) = 2(x+1)$ *which is continuos in the interval* $[0, \pi]$

Hence, the correct answer is the option **B**

Solution 81

The differential equation given is

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \rho^2 \left[\frac{d^2y}{dx^2}\right]^2$$

The highest order differential is $\frac{d^2y}{dx^2}$, whose order is 2.

The power to which $\frac{d^2y}{dx^2}$ is raised is 2, hence the Degree is 2 as well.

Hence, the correct answer is option B

Solution 82

$$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$y = \frac{\pi}{2} - \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$y = \frac{\pi}{2} - 2\tan^{-1}x \ ,$$

$$\frac{dy}{dx} = 0 - \frac{2}{1+x^2} = \frac{-2}{1+x^2} \ \forall |x| < 1$$

Hence, the correct answer is option A.

Solution 83

$$f(x) = \sqrt{1 - e^{-x^2}}$$

$$f'(x) = \frac{-2x\left(-e^{-x^2}\right)}{2\sqrt{1-e^{-x^2}}} = \frac{xe^{-x^2}}{\sqrt{1-e^{-x^2}}}$$

which is defined $\forall \, \boldsymbol{x} \in \boldsymbol{R}$, except $x = 0$

$\Rightarrow f(x)$ is differentiable on $(-\infty, 0) \cup (0, \infty)$.

Hence, the correct answer is the option C.

Solution 84

For the function f(x) = *a*sin*x* + *b*cos*x*

The maximum value = $\sqrt{a^2 + b^2}$

Apply this in all parts

A. $\sqrt{1^2 + 1^2} = \sqrt{2}$

B. $\sqrt{3^2 + 4^2} = 5$

C. $\sqrt{2^2 + 1^2} = \sqrt{5}$

D. $\sqrt{1^2 + 3^2} = \sqrt{10}$

Hence, the correct answert is option B.

Solution 85

L.H.L = $\lim\limits_{x->0^-} x\left(\sqrt{x}-\sqrt{x+1}\right) = 0$

R.H.L = $\lim\limits_{x->0^+} x\left(\sqrt{x}-\sqrt{x+1}\right) = 0$

f(0) = 0

So, *f*(*x*) is continuous at *x* = 0

L.H.D. =

$\lim\limits_{h->0} \frac{f(x-h)-f(x)}{-h}$

$\lim\limits_{h->0} \frac{(x-h)\left(\sqrt{x-h}-\sqrt{x-h+1}\right)-x\left(\sqrt{x}-\sqrt{x+1}\right)}{-h}$

= −1

R.H.D. =

$\lim\limits_{h->0} \frac{f(x+h)-f(x)}{h}$

$\lim\limits_{h->0} \frac{(x+h)\left(\sqrt{x+h}-\sqrt{x+h+1}\right)-x\left(\sqrt{x}-\sqrt{x+1}\right)}{h}$

= −1

so, *f*(*x*) is differentiable at *x* = 0

Hence, the correct answer is option D.

Solution 86

$f\left(x\right)=\frac{x}{x},\ x\neq 0$

f(*x*) = 1 for all real values except *x* = 0

At *x* = 0 it is not defined.

Hence, the correct answer is option C.

Solution 87

Given: $f\left(n\right)=\left[\frac{1}{4}+\frac{n}{1000}\right]$

$\sum_{n=1}^{1000} f(n)=\left[\frac{1}{4}+\frac{1}{1000}\right]+\left[\frac{1}{4}+\frac{2}{1000}\right]+\left[\frac{1}{4}+\frac{1}{1000}\right]+\ldots+\left[\frac{1}{4}+\frac{1000}{1000}\right]$

$=0+0+0+\ldots+\left[\frac{1}{4}+\frac{750}{1000}\right]+\left[\frac{1}{4}+\frac{750}{1000}\right]+\ldots+\left[\frac{1}{4}+\frac{1000}{1000}\right]$

$=1+1+1+\ldots\ 1\,(251\ \text{times})$

This sum would be 0 till $\left[\frac{1}{4}+\frac{749}{1000}\right]$ and after that it will become 1 for each term uptil 1000.

Hence, we would have to add 1+1+1...251 times(including 750 and 1000 both)

Therefore the correct answer is 251.

Hence, the correct answer is option A.

Solution 88

$I=\int(\ln\ x)^{-1}dx-\int(\ln\ x)^{-2}dx$

Putting ln*x* = *t*

So, *x* = e^t ; *dx* = e^t *dt*

$I=\int\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt=\frac{e^t}{t}+c$

= $\frac{x}{\ln x}+C$

= $x(\ln x)^{-1}+c$

Hence, the correct answer is option A.

Solution 89

Let r be the radius and h be the height of a cyclinder of given surface area S. Then,

$S=\pi r^2+2\pi rh$ ___(1)

$h=\frac{S-\pi r^2}{2\pi r}$

Let V be the volume of the cyclinder. Then,

$V=\pi r^2 h$

$V=\pi r^2\left(\frac{S-\pi r^2}{2\pi r}\right)$

$\frac{dV}{dr}=\frac{S}{2}-\frac{3\pi r^2}{2}$

Critical value is obtained by $\frac{dV}{dr}=0$

So, $S=3\pi r^2$ ____(2)

Comparing 1 and 2, we get $r=h$.

Also $\frac{d^2V}{dr^2}$ is negetive, therefore V is maximum.

The radius and the height of the base of an open cylinder of a given surface area and maximum volume are equal, i.e., radius = height.

So, the diameter would be 2 times the height.

Hence k =2.

Hence, the correct answer is option B.

Solution 90

$I=\int_0^{\frac{\pi}{4}}\sqrt{\tan x}dx+\int_0^{\frac{\pi}{4}}\sqrt{\cot x}dx$

$=\int_0^{\pi/4}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)\mathrm{d}x$

$\sqrt{2}\int_0^{\pi/4}\frac{\sin x+\cos x}{\sqrt{1-(\sin x-\cos x)^2}}\mathrm{d}x$

Putting $\sin x-\cos x=t$

$dt=(\sin x+\cos x)dx$

when $x=0$, $t=-1$

and when $x=\frac{\pi}{4}$, $t=0$

So, we have

$I=\sqrt{2}\int_{-1}^{0}\frac{1}{\sqrt{1-t^2}}\mathrm{d}t=\left[\sqrt{2}\sin^{-1}t\right]_{-1}^{0}$

$\sqrt{2}\left(0-\left(-\frac{\pi}{2}\right)\right)=\frac{\pi}{\sqrt{2}}$

Hence, the correct answer is option D.

Solution 91

$g(x)=[x]$

$f(x) = [x]^2 - [x]$

Since $[x]$ is discontinuous at intergers so, $f(x)$ will also be discontinous at all integers except 1.

For $x = 1$

LHL =

$$\lim_{h \to 0} f(x) = \lim_{h \to 0} [1-h]^2 - \lim_{h \to 0} [1-h] = 0 - 0 = 0$$

RHL =

$$\lim_{h \to 0} f(x) = \lim_{h \to 0} [1+h]^2 - \lim_{h \to 0} [1+h] = 1 - 1 = 0$$

$f(1) = 0$

So, $f(x)$ is discontinous at alll integers except $x = 1$

Hence, the correct answer is option D.

Solution 92

$$y = A[\sin(x+c) + \cos(x+c)]$$
$$\frac{dy}{dx} = A[\cos(x+c) - \sin(x+c)]$$
$$\frac{d^2y}{dx^2} = -A[\sin(x+c) + \cos(x+c)] = -y$$
$$\frac{d^2y}{dx^2} + y = 0$$

$\Rightarrow y'' + y = 0$

Hence, the correct anser is option D.

Solution 93

$f(x) = x - \sin x$

$f'(x) = 1 - \cos x > 0$ for all $x > 0$

So, $f(x)$ is an increasing function.

Also, $f(0) = 0$

$x - \sin x > 0$, for all $x > 0$

$x > \sin x$ for all $x > 0$

So, statements I and II are correct but statement II is not the correct explaination of statement I.

Hence, the correct answer is option B.

Solution 94

$\frac{dy}{dx} - y\frac{\varphi'(x)}{\varphi(x)} = \frac{-y^2}{\varphi(x)}$

$\frac{1}{y^2}\left(\frac{dy}{dx}\right) - \frac{1}{y} \times \frac{\varphi'(x)}{\varphi(x)} = -\frac{1}{\phi(x)}$

Let $-\frac{1}{y} = t$

We get

$\frac{dt}{dx} = \frac{1}{y^2}\left(\frac{dy}{dx}\right)$

$\frac{dt}{dx} + \frac{\varphi'(x)}{\varphi(x)}t = -\frac{1}{\phi(x)}$

$\text{IF} = e^{\int \frac{\varphi'(x)}{\varphi(x)}dx} = e^{\log\,\varphi(x)} = \varphi\left(x\right)$

Solution of Differential equation will be

$t \times \varphi\left(x\right) = \int\left(-\frac{1}{\varphi(x)}\right) \times \varphi\left(x\right)dx$

$= \frac{-1}{y}\varphi\left(x\right) = -x$

$y = \frac{\varphi(x)}{x} + c$

Hence, correct answer is option B.

Solution 95

$f(x) = \frac{4x + x^4}{1 + 4x^3}$

$g(x) = \ln\left(\frac{1+x}{1-x}\right)$

$g\left(\frac{e-1}{e+1}\right) = \ln\left(\frac{1+\frac{e-1}{e+1}}{1-\frac{e-1}{e+1}}\right) = \ln\left(\frac{e+1+e-1}{e+1-e+1}\right) = \ln\left(\frac{2e}{2}\right) = 1$

$f\left(g\left(\frac{e-1}{e+1}\right)\right) = f(1) = \frac{4+1}{1+4} = \frac{5}{5} = 1$

Hence, the correct answer is option B.

Solution 96

Let $\Delta = \begin{vmatrix} 1-\alpha & \alpha-\alpha^2 & \alpha^2 \\ 1-\beta & \beta-\beta^2 & \beta^2 \\ 1-\gamma & \gamma-\gamma^2 & \gamma^2 \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$\Delta = \begin{vmatrix} 1 & \alpha-\alpha^2 & \alpha^2 \\ 1 & \beta-\beta^2 & \beta^2 \\ 1 & \gamma-\gamma^2 & \gamma^2 \end{vmatrix}$

$C_2 \to C_2 + C_3$

$\Delta = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix}$

Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$, we have:

$\Delta = \begin{vmatrix} 0 & \alpha-\gamma & \alpha^2-\gamma^2 \\ 0 & \beta-\gamma & \beta^2-\gamma^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix} = (\gamma-\alpha)(\beta-\gamma)\begin{vmatrix} 0 & -1 & -\alpha-\gamma \\ 0 & 1 & \beta+\gamma \\ 1 & \gamma & \gamma^2 \end{vmatrix}$

Applying $R_1 \to R_1 + R_2$, we have:

$$\Delta = (\gamma - \alpha)(\beta - \gamma)\begin{vmatrix} 0 & 0 & -\alpha+\beta \\ 0 & 1 & \beta+\gamma \\ 1 & \gamma & \gamma^2 \end{vmatrix}$$

$$= (\alpha - \beta)(\gamma - \alpha)(\beta - \gamma)\begin{vmatrix} 0 & 0 & -1 \\ 0 & 1 & \beta+\gamma \\ 1 & \gamma & \gamma^2 \end{vmatrix}$$

Expanding along C_1, we have:

$$\Delta = (\alpha - \beta)(\gamma - \alpha)(\beta - \gamma)\begin{vmatrix} 0 & -1 \\ 1 & \beta+\gamma \end{vmatrix} = (\alpha - \beta)(\gamma - \alpha)(\beta - \gamma)$$

Hence, the correct answer is option B.

Solution 97

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$C_{11} = 1 \quad C_{12} = -2 \quad C_{13} = 6$

$C_{21} = 6 \quad C_{22} = 1 \quad C_{23} = -3$

$C_{31} = -2 \; C_{32} = 4 \quad C_{33} = 1$

$$Adj\ A = \begin{bmatrix} 1 & 6 & -2 \\ -2 & 1 & 4 \\ 6 & -3 & 1 \end{bmatrix}$$

Hence, the correct answer is option B.

Solution 98

$$A = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}$$

$$= -4\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$$

$$= -4A$$

Hence, the correct answer is option B.

Solution 99

Let $z = x + iy$

$z^2 - i = x^2 - y^2 + 2xyi - i$

$= x^2 - y^2 + (2xy - 1)i \qquad \ldots\ldots(1)$

$\text{Re}\,(z^2 - i) = 2 \qquad \text{(Given)} \qquad \ldots\ldots(2)$

From (1) and (2)

$x^2 - y^2 = 2$

This is the equation of a rectangular hyperbola.

Hence, the correct answer is option C.

Solution 100

Given: $p + q + r = 0$

$\Rightarrow p^3 + q^3 + r^3 = 3pqr$

Also, $a + b + c = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

$$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr\left(a^3 + b^3 + c^3\right) - abc\left(p^3 + q^3 + r^3\right)$$

$$= pqr\,(3abc) - abc\,(3pqr) = 0$$

Hence, the correct answer is option A.

Solution 101

E: Committee will have exactly 1 woman

$n(S) = {}^4C_2 = \frac{4\times3}{2} = 6$

$n(E) = {}^2C_1 \times {}^2C_1 = 2 \times 2 = 4$

$\mathrm{P}(E) = \frac{4}{6} = \frac{2}{3}$

Hence, the correct answer is option B.

Solution 102

Since probability of even faces is double of odd faces.

P(1) = P(3) = P(5) = $\frac{1}{9}$

P(2) = P(4) = P(6) = $\frac{2}{9}$

Prime Number in case of rolling a die = 2, 3, 5

P(Prime Number) = $\frac{2}{9} + \frac{1}{9} + \frac{1}{9} = \frac{4}{9}$

Hence, correct answer is option C.

Solution 103

S = {0, 1, 2, ..., 50}
X = {3, 6, 9, ...48}
$n(X) = 16$

$Y = \{1, 3, 5, \ldots, 49\}$

$n(Y) = 25$

$\mathrm{P}(X) = \frac{16}{51}$

$\mathrm{P}(Y) = \frac{25}{51}$

Hence, the correct answer is option D.

Solution 104

Given: $\mathrm{P}(A) = \frac{1}{2}, \mathrm{P}\left(A \cup B\right) = \frac{2}{3}$ and $\mathrm{P}\left(A \cap B\right) = \frac{1}{6}$

$\mathrm{P}\left(A \cup B\right) = \mathrm{P}\left(A\right) + \mathrm{P}\left(B\right) - \mathrm{P}\left(A \cap B\right)$

$\mathrm{P}\left(B\right) = \frac{2}{3} - \frac{1}{2} + \frac{1}{6} = \frac{1}{3}$

$\mathrm{P}\left(\overline{A} \cap B\right) = \mathrm{P}\left(B\right) - \mathrm{P}\,(A \cap B)$

$= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

Hence, the correct answer is option A.

Solution 105

Mean deviation is the least when measured about mean and not median.
so, statement 3 is incorrect.
And statement 1 and statement 2 are correct.

Hence, the correct answer is option A.

Solution 106

Arithmetic Mean = 24
Standard deviation = 0
Since the standard deviation is given to be 0, therefore all observations are equal to 24.
Average of any five observations will be 24.
So, arithmetic mean of the smallest five observations in the data will be 24.

Hence, the correct answer is option D.

Solution 107

Regresion coefficient of *y* on *x* = Regression coefficient of *x* on *y*
So, (*x, y*) lies on *y* = *x* line.

Hence, the correct answer is option A.

Solution 108

$\text{Given}: \ \mathrm{P}(A) = \frac{1}{3}, \ \mathrm{P}(\mathrm{B}) = \frac{1}{6}, \ \mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \frac{1}{12}$

$\mathrm{P}\left(\mathrm{B} \middle| \overline{\mathrm{A}}\right) = \frac{\mathrm{P}\left(B \cap \overline{A}\right)}{\mathrm{P}\left(\overline{\mathrm{A}}\right)}$

$\mathrm{P}\left(B \cap \overline{A}\right) = \mathrm{P}(B) - \mathrm{P}(A \cap B) = \frac{1}{6} - \frac{1}{12} = \frac{1}{12}$

$\mathrm{P}\left(\overline{A}\right) = \frac{2}{3}$

$\mathrm{P}\left(B \middle| \overline{A}\right) = \frac{\mathrm{P}\left(B \cap \overline{A}\right)}{\mathrm{P}\left(\overline{\mathrm{A}}\right)} = \frac{\frac{1}{12}}{\frac{2}{3}} = \frac{1}{8}$

Hence, the correct anser is option C.

Solution 109

$\text{Mean}, \ \mu_x = np = \frac{2}{3}$

$\text{Variance}, \ \delta_x{}^2 = np(1-p) = npq = \frac{5}{9}$

$np = \frac{2}{3}, \ npq = \frac{5}{9}$

$q = \frac{5}{9} \times \frac{3}{2} = \frac{5}{6}$

$p = \frac{1}{6}, \ n = 4$

$p\left(X = 2\right) = {}^4C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2$

$= 6 \times \frac{1}{36} \times \frac{25}{36} = \frac{25}{216}$

Hence, the correct answer is option C.

Solution 110

Probability that a ship safely reaches a port is $\frac{1}{3}$
Probability that a ship does not safely reaches a port is $\frac{2}{3}$

$\mathrm{P}\left(\text{all ships reach safely}\right) = \left(\frac{1}{3}\right)^5 = \frac{1}{243}$

$\mathrm{P}\left(4 \text{ ships reach safely}\right) = {}^5\mathrm{C}_4 \times \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) = \frac{10}{243}$

$\mathrm{P}\left(\text{atleast } 4 \text{ ships reach safely}\right) = \mathrm{P}(\text{all ships reach safely}) + \mathrm{P}(4 \text{ ships reach safely})$

$= \frac{1}{243} + \frac{10}{243} = \frac{11}{243}$

Hence, the correct answer is option C.

Solution 111

First person can have birthday in any one the 12 months so, $\frac{12}{12}$
Second person can have birthday in any of the remaining 11 months so, $\frac{11}{12}$
Third person can have birthday in any of the remaining 10 months so, $\frac{10}{12}$

$\mathrm{P}\left(\text{none is born in same month}\right) = \frac{12\times11\times10}{12\times12\times12}$

$\mathrm{P}\left(\text{at least two born in same month}\right) = 1 - \mathrm{P}\left(\text{none born in same month}\right)$

$= 1 - \frac{12\times11\times10}{12\times12\times12}$

$= \frac{144-110}{144} = \frac{17}{72}$

Hence, the correct answer is option B.

Solution 112

$\overline{X} = 10,\ \overline{Y} = 90$

$\sigma_x = 3,\ \sigma_y = 12$

$r_{xy} = 0.8$

Regresion equation of x on y is

$X - \overline{X} = r\frac{\sigma_x}{\sigma_y}\left(Y - \overline{Y}\right)$

$X - 10 = r\frac{\sigma_x}{\sigma_y}\left(Y - 90\right)$

$X - 10 = 0.8 \times \frac{3}{12}\left(Y - 90\right)$

$X - 10 = 0.2\left(Y - 90\right)$

$X = -8 + 0.2Y$

Hence, the correct answer is option C.

Solution 113

$\mathrm{P}\left(B \cap \overline{C}\right) = \mathrm{P}\left(A \cap B \cap \overline{C}\right) + \mathrm{P}\left(\overline{A} \cap B \cap \overline{C}\right)$

$\Rightarrow \mathrm{P}\left(B \cap \overline{C}\right) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

Now, $\mathrm{P}\left(B\right) = \mathrm{P}\left(B \cap C\right) + \mathrm{P}\left(B \cap \overline{C}\right)$

$\Rightarrow \mathrm{P}\left(B \cap C\right) = \mathrm{P}\left(B\right) - \mathrm{P}\left(B \cap \overline{C}\right)$

$\Rightarrow \mathrm{P}\left(B \cap C\right) = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

Hence, the correct answer is option A.

Solution 114

Total expenditure of A = 10,000
Total expenditure of B = 8,100
So area of A : area of B = 10,000 : 8,100 = 100 : 81

$\frac{\pi {r_A}^2}{\pi {r_B}^2} = \frac{100}{81}$

$\Rightarrow \frac{{r_A}^2}{{r_B}^2} = \frac{100}{81}$

$\Rightarrow \frac{r_A}{r_B} = \frac{10}{9}$

Radii of A : Radii of B = 10 : 9
Hence, the correct answer is option B.

Solution 115

Arithmetic mean always lie between minimum and maximum value. So, out of given options, arithmetic mean will be $\frac{n}{2}$.

Hence, the correct answer is option D.

Solution 116

$\text{P}\left(\text{knowing the correct answer}\right) = p$

$\text{P}\left(\text{guessed answer is correct}\right) = (1-p)\times\left(\frac{1}{m}\right)$

$\text{P}\left(\text{correct answer}\right) = p+\frac{(1-p)}{m}$

$\text{So, required probabilty} = \frac{p}{p+\frac{1-p}{m}} = \frac{mp}{1+p(m-1)}$

Hence, the correct answer is option B.

Solution 117

$\frac{x_1+x_2}{2} - \sqrt{x_1x_2} > 1$

$\frac{x_1+x_2}{2} > \sqrt{x_1x_2} + 1$

$x_1 + x_2 > 2\sqrt{x_1x_2} + 2$

$x_1 + x_2 - 2\sqrt{x_1x_2} > 2$

$\left(\sqrt{x_1}-\sqrt{x_2}\right)^2 > 2$

$\left|\sqrt{x_1} - \sqrt{x_2}\right| > \sqrt{2}$

Hence, the correct answer is option C.

Solution 118

Variance is independent of change of origin but not change of scale.
So, Statement 1 is incorrect and Statement 2 is correct.

Hence, the correct answer is option B.

Solution 119

By triangular inequality we can observe, any three lengths can form a triangle except any combination with1 feet stick and the combination 3, 5, 9.

$\text{Required Probability} = \frac{^4C_3-1}{^5C_3} = \frac{3}{10} = 0.3$

Hence, the correct answer is option C.

Solution 120

$\text{Coefficient of correlation } \left(r\right) = \sqrt{r_a\times r_b} = \sqrt{0.2\times1.8} = 0.6$

Hence, the correct answer is option C.