JEE Main 25 July 2022(Second Shift)

Total Time: 180

Total Marks: 300.0

Solution 1

$A_{\max} = 6 \text{ V}$

$A_{\min} = 2 \text{ V}$

$\mu = \frac{A_{\max} - A_{\min}}{A_{\min} + A_{\min}} = \frac{6-2}{6+2} = 0.5$

$\mu = 50\%$

Hence, the correct answer is option D.

Solution 2

$B = \frac{n\mu_0 l}{2R}$

$B_1 = \frac{2\mu_0 l}{2R_1}$

$B_2 = \frac{5\mu_0 l}{2R_2}$

$R_2 = \frac{2R_1}{5}$

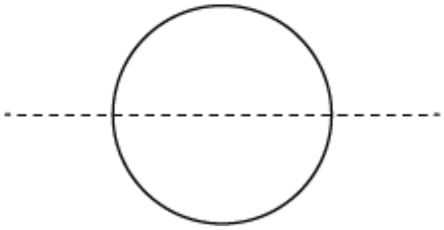
$\Rightarrow \frac{B_2}{B_1} = \frac{5}{2} \times \frac{R_1}{R_2} = \frac{25}{4}$

Hence, the correct answer is option B.

Solution 3

Balancing the forces on drop

$2\pi RT + \frac{4}{3}\pi R^3 \rho g = \frac{2}{3}\pi R^3 \sigma g$



$$\Rightarrow 2T = \frac{2R^2}{3}(\sigma - 2\rho) \times 10$$

$$\Rightarrow \frac{15\times10^{-2}\times3}{10(\sigma-2\rho)2} R^2$$

$$R=\frac{3}{2\times10}\sqrt{\frac{1}{(\sigma-2\rho)}}$$

$$=\frac{3}{20}\sqrt{\frac{1}{\sigma-2\rho}}\ (\text{in m})$$

$$(R)\ \text{in cm} = \frac{3\times100}{20}\sqrt{\frac{1}{\sigma-2\rho}} = 15 \times \frac{1}{\sqrt{\sigma-2\rho}}$$

$$\text{Now if } 2\rho > \sigma\,(R_{\text{in cm}}) = \frac{15}{\sqrt{2\rho-\sigma}}$$

Hence the correct answer is option A.

Solution 4

Change in momentum of one ball
= 2 × (0.05)(10) kg m/s
= 1 kg m/s

$$\Rightarrow F_{\text{avg}} = \frac{1}{\Delta t} = \frac{1}{0.005}\text{N}$$

$$=200\ \text{N}$$

Hence, the correct answer is option B.

Solution 5

Resultant of already applied forces = $-\hat{i} - \hat{j}$

⇒ Force required to balance = $\hat{i} + \hat{j}$

⇒ Force required = $\sqrt{2}$ N in magnitude at angle 45° with +ve *x*-axis

Hence, the correct answer is option A.

Solution 6

Initially = $C_0 = 4\pi\varepsilon_0 R_1$

finally $\frac{4\pi\varepsilon_0 R_1 R_2}{R_2-R_1}=nC_0=4\pi\varepsilon_0 nR_1$

$\frac{R_2}{R_2-R_1}=n$

$1-\frac{R_1}{R_2}=\frac{1}{n}$

$\frac{R_1}{R_2}=\frac{n-1}{n}$

$\frac{R_2}{R_1}=\frac{n}{n-1}$

Hence, the correct answer is option A.

Solution 7

$\lambda=\frac{h}{mv}=\frac{h}{\sqrt{2meV}}$

So $\frac{\lambda_p}{\lambda_d}=\frac{\sqrt{m_d V_d}}{\sqrt{m_p V_p}}=\frac{1}{\sqrt{2}}$

$\frac{2V_d}{V_p}=\frac{1}{2}$

$\frac{V_p}{V_d}=\frac{4}{1}$

Hence, the correct answer is option D.

Solution 8

The shift produced by the glass plate is

$d=t\left(1-\frac{1}{\mu}\right)=1\times\left(1-\frac{1}{1.5}\right)=\frac{1}{3}\ \text{cm}$

So final image must be produced at $\left(12-\frac{1}{3}\right)\text{cm}=\frac{35}{3}$ cm from lens so that glass plate must shift it to produce image at screen. So

$\frac{1}{12}-\frac{1}{-240}=\frac{1}{f}=\frac{1}{35/3}-\frac{1}{u}$

$\frac{1}{u}=\frac{3}{35}-\frac{1}{12}-\frac{1}{240}$

or $u=-560\ \text{cm}$

so shift $=5.6-2.4=3.2\ \text{m}$

Hence, the correct answer is option B.

Solution 9

c = 3 × 10^8 m/sec

$B=\frac{E}{c}=\frac{540}{3\times10^8}=18\times10^{-7}\ \mathrm{T}$

Hence, the correct answer is option A.

Solution 10

Metal detector works on the principle of resonance in ac circuits.

Hence, the correct answer is option B.

Solution 11

$T=\frac{2\pi m}{Bq}$

$\Rightarrow\text{Frequency } f=\frac{Bq}{2\pi m}$

$=\frac{10^{-4}\times1.6\times10^{-19}}{2\pi\times9\times10^{-31}}$

$\simeq 2.8\times10^6\ \mathrm{Hz}$

Hence, the correct answer is option C.

Solution 12

$\text{Effective } R=\left[5+\frac{5\times10}{5+10}+10\right]\ \mathrm{k\Omega}$

$=\frac{275}{15}\ \mathrm{k\Omega}$

$\Rightarrow\Delta V_{AB}=15\ \mathrm{mA}\times\frac{275}{15}\mathrm{k\Omega}$

$=275\ \mathrm{V}$

Hence, the correct answer is option D.

Solution 13

$g=\frac{GM}{(R+h)^2}=\frac{GM}{9R^2}=\frac{g_0}{9}$

$\Rightarrow T=2\pi\sqrt{\frac{l}{g}}=2\pi\sqrt{\frac{l}{\frac{g_0}{9}}}$

$\Rightarrow 2=2\pi\sqrt{\frac{9l}{g_0}}$

$\Rightarrow l=\frac{g_0}{9\pi^2}=\frac{1}{9}\ \mathrm{m}$

Hence, the correct answer is option D.

Solution 14

Molar mass $M=\frac{2\times4+n\times1}{2+n}\ .....(\mathrm{i})$

Also, $\gamma = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \frac{2\times 5R + n\times 7R}{2\times 3R + n\times 5R}$

$\Rightarrow \gamma = \frac{10+7n}{6+5n} \quad \ldots\ldots \text{(ii)}$

$\text{Given that } V_{\text{rms}} = \sqrt{2}\, V_{\text{sound}}$

$\Rightarrow \sqrt{\frac{3RT}{M}} = \sqrt{2}\sqrt{\frac{\gamma RT}{M}}$

$\Rightarrow \gamma = \frac{3}{2}$

$\Rightarrow n = 2$

Hence, the correct answer is option B.

Solution 15

$\eta_1 = 1 - \frac{420}{720} = \frac{300}{720}$

$\text{And } \eta_2 = 1 - \frac{320}{1220} = \frac{900}{1220}$

$\Rightarrow \frac{\eta_1}{\eta_2} = \frac{300}{720} \times \frac{1220}{900}$

$\simeq 0.56$

Hence, the correct answer is option B.

Solution 16

w = *mg*

$w' = \frac{mg}{\left(1+\frac{h}{R}\right)^2} = \frac{mg}{\left(\frac{5}{4}\right)^2} = \frac{16}{25} mg$

$\therefore\ \% \text{ decrease in weight}$

$= \left(1 - \frac{16}{25}\right) \times 100\%$

$= 36\%$

Hence, the correct answer is option A.

Solution 17

$$\text{Loss in } KE = \frac{1}{2} \times \frac{m_1 m_2}{m_1 + m_2} \times v^2$$
$$= \frac{1}{2} \times \frac{9.8 \times 0.2}{10} \times (10)^2$$
$$= 9.8 \text{ J}$$

Hence, the correct answer is option B.

Solution 18

$\because \tan\theta = \frac{4H}{R}$

$\Rightarrow \tan\theta = 4 \times 1$

$\Rightarrow \tan\theta = 4$

Hence, the correct answer is option D.

Solution 19

$\because H = i^2 Rt$

$$\because \% \text{ error in } H = 2 \times 2\% + 1\% + 3\%$$
$$= 8\%$$

Hence, the correct answer is option D.

Solution 20

$\because \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$

$\Rightarrow \frac{1}{\lambda R} = 1 - \frac{1}{n^2}$

$\Rightarrow \frac{1}{n^2} = 1 - \frac{1}{\lambda R} = \frac{\lambda R - 1}{\lambda R}$

$\Rightarrow n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$

Hence, the correct answer is option B.

Solution 21

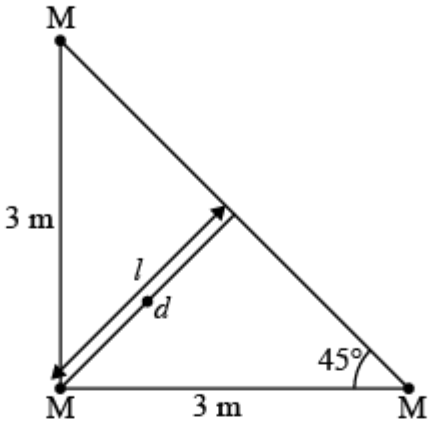
$\frac{dv}{dx} = 5 \text{ ms}^{-1}/\text{m}$

Acceleration of particle

when v = 20 m/s

$a = v\frac{dv}{dx} = 20\,(5)\ \text{m/s}^2 = 100\ \text{m/s}^2$

Solution 22



$$d_{\text{cm}} = 3 \ \sin 45^\circ = \frac{3}{\sqrt{2}}$$

$$d_{\text{cm}} = \frac{2}{3} \times \frac{3}{\sqrt{2}} = \sqrt{2} = \sqrt{x}$$

$$x = 2$$

Solution 23

Heat lost by water = Heat gained by ice
0.3 × 4200 × 25 = x × 3.5 × 10^5

$$x = \frac{0.3 \times 4200 \times 25}{3.5 \times 10^5}$$

$$= 90 \times 100 \times 10^5 \times 10^3 \text{ gram} = 90 \text{ gm}$$

Solution 24

$$E_n = -\frac{13.6}{n^2} \text{eV}$$

$$\frac{\frac{1}{2^2} - \frac{1}{3^2}}{\frac{1}{2^2}} = \frac{x}{x+4}$$

$$\Rightarrow \frac{9-4}{9 \times 4 \times \frac{1}{4}} = \frac{x}{x+4} = \frac{5}{9}$$

$$x = 5$$

Solution 25

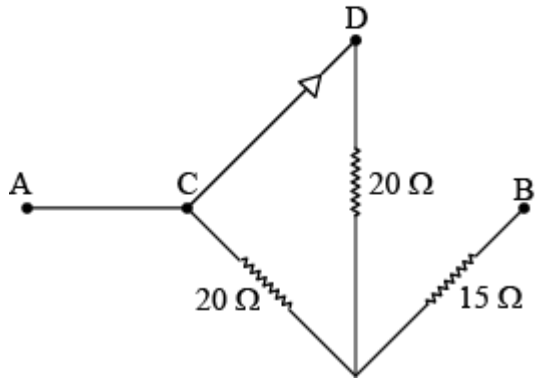
$$E \propto l$$

$$\frac{1.2}{1.8} = \frac{36}{l'}$$

$$l' = \frac{3}{2} \times 36 = 54 \text{ cm}$$

$$\Delta\, l = l' - l = 54 - 36 = 18 \ \text{ cm}$$

Solution 26



$R=\frac{20\times20}{40}+15=25\ \Omega$

Solution 27

$A_{\text{net}}=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi}$

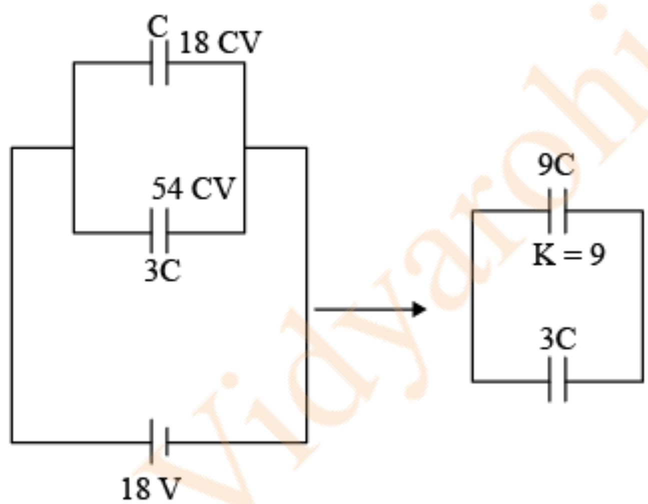
$\sqrt{3}A=\sqrt{A^2+A^2+2A^2\cos\phi}$

$3A^2=2A^2+2A^2\cos\phi$

$\cos\phi=\frac{1}{2}$

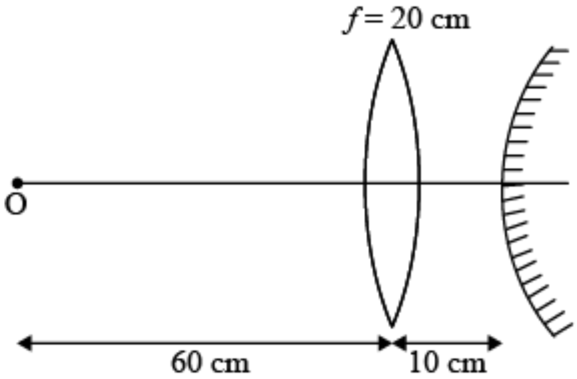
$\phi=60^{\circ}$

Solution 28



$V_{\text{common}}=\frac{18CV+54CV}{3C+9C}6\text{ V}$

Solution 29



$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\frac{1}{v}-\frac{1}{-60}=\frac{1}{20}$

$\frac{1}{v}=-\frac{1}{60}+\frac{1}{20}=\frac{-1+3}{60}=\frac{2}{60}$

$\Rightarrow v=+30\ \text{cm}$

∴ Radius of curvature of mirror = 30 – 10 = 20 cm

$\Rightarrow f_{\text{mirror}}=\frac{20}{2}=10\ \text{cm}$

Solution 30

$R=20\ \Omega$

$\phi=8t^2-9t+5$

$\varepsilon=\left|-\frac{d\phi}{dt}\right|=|16t-9|=|16\,(0.25)-9|=5$

$i=\frac{\varepsilon}{R}=\frac{5}{20}=0.25\ \text{A}=\frac{0.25}{10^3}\times 10^3\ \text{A}=250\ \text{mA}$

Solution 31

XeO_3 — sp^3, Pyramidal
XeF_2 — sp^3d, linear
$XeOF_4$ — sp^3d^2 , Square Pyramidal
XeF_6 — sp^3d^3, distorted octahedral

Hence, the correct answer is option A.

Solution 32

$$\Delta T_f = \mathrm{i\,K_f \times m}$$

$$\frac{\Delta T_{f(A)}}{\Delta T_{f(B)}} = \frac{1}{4}$$

$$\frac{i \times K_f \times \frac{1}{M_A} \times 1}{i \times K_f \times \frac{1}{M_B} \times 1} = \frac{1}{4}$$

$$\frac{M_B}{M_A} = \frac{1}{4}$$

$$M_A : M_B = 4 : 1 = 1 : 0.25$$

Hence, the correct answer is option B.

Solution 33

$$H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-} \quad K_{a_3}$$

$$H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^- \quad K_{a_1}$$

$$HC_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-} \quad K_{a_2}$$

$$K_{a_3} = \frac{[H^+]^2[C_2O_4^{2-}]}{[H_2C_2O_4]}$$

$$K_{a_1} = \frac{[H^+][HC_2O_4^-]}{[H_2C_2O_4]},\ K_{a_2} = \frac{[H^+][C_2O_4^-]}{[HC_2O_4^-]}$$

$$K_{a_3} = K_{a_1} \times K_{a_2}$$

Hence, the correct answer is option D.

Solution 34

$$\Lambda_{m_1} = \frac{\kappa_1 \times 1000}{M_1} = \frac{\kappa_1 \times 1000}{\frac{10}{0.02}}$$

$$\Lambda_{m_2} = \frac{\kappa_2 \times 1000}{\frac{20}{0.08}}$$

It is given that $\kappa_1 = \kappa_2$

$$\kappa_1 = \frac{\Lambda_{m_1}}{2} \qquad \kappa_2 = \frac{\Lambda_{m_2}}{4}$$

Applying the given condition on conductivity.

$$\frac{\Lambda_{m_1}}{2} = \frac{\Lambda_{m_2}}{4}$$
$$\Lambda_{m_2} = 2\Lambda_{m_1}$$

Hence, the correct answer is option A.

Solution 35

Micelle formation is an endothermic process with positive entropy change.

Hence, the correct answer is option C.

Solution 36

The first ionisation energy increase from left to right along 2^{nd} period with the following exceptions
IE_1 : Be > B and N > O
This is due to stable configuration of Be in comparison to B and that of N in comparison to O. Hence the correct order is N > O > Be > B

Hence, the correct answer is option D.

Solution 37

Cast iron is made by melting pig iron with scrap iron and coke using hot air blast.
Hence Statement-I is incorrect
But Pig iron has relatively more carbon content
Hence statement-II is incorrect.

Hence, the correct answer is option B.

Solution 38

High purity (>99.95%) H_2 is obtained by electrolysing warm aqueous $Ba(OH)_2$ solution between nickel electrodes.

Hence, the correct answer is option C.

Solution 39

Density of Sr = 2.63 g/cm^3
Density of Be = 1.84 g/cm^3
Density of Mg = 1.74 g/cm^3
Density of Ca = 1.55 g/cm^3

Hence, the correct answer is option C.

Solution 40

NO, N_2O, CO – neutral oxides
B_2O_3, N_2O_5, SO_3, P_4O_{10} – acidic oxides

Hence, the correct answer is option B.

Solution 41

Stronger is ligand attached to metal ion, greater will be the splitting between t_2g and eg (hence greater will be ΔU), $\therefore$ greater will be absorption of energy.
Hence correct order
$[Ni(en)_3]^{2+} > [Ni(NH_3)_6]^{2+} > [Ni(H_2O)_6]^{2+}$

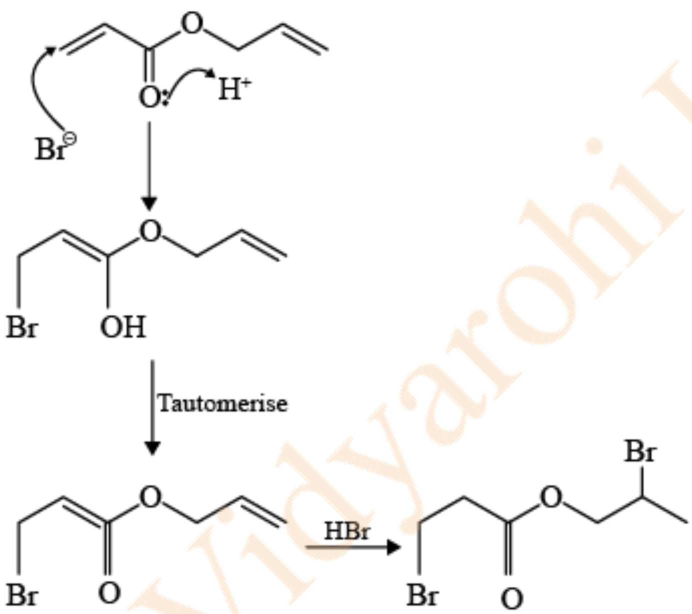
Hence, the correct answer is option A.

Solution 42

Sodium arsinite — Herbicide
Nicotine — Pesticide
Sulphate — Laxative effect
Fluoride — Bending of bones

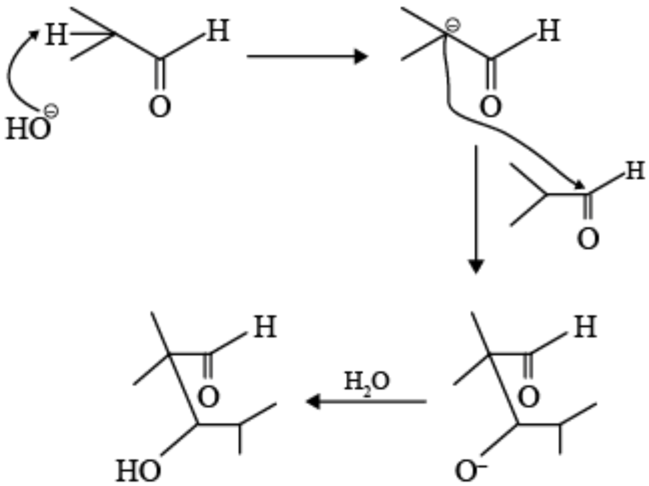
Hence, the correct answer is option C.

Solution 43



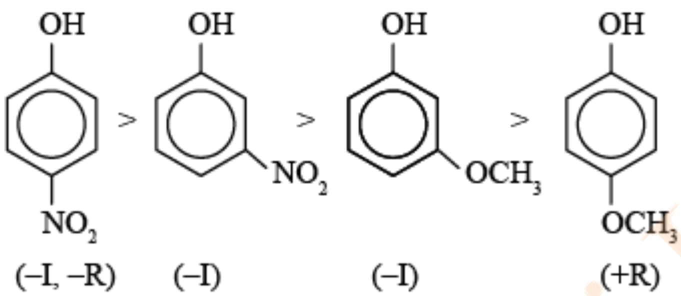
Hence, the correct answer is option D.

Solution 44



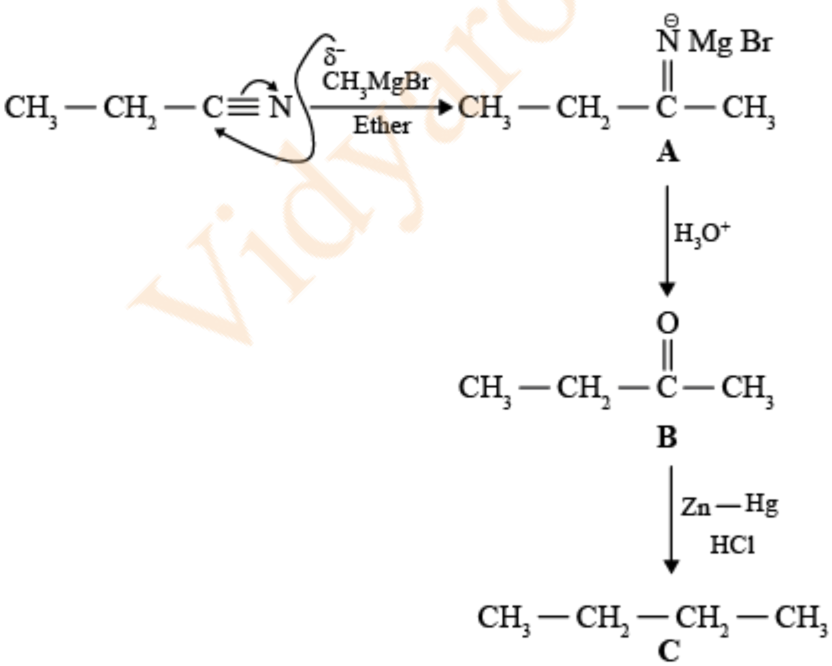
Hence, the correct answer is option B.

Solution 45



Hence, the correct answer is option A.

Solution 46



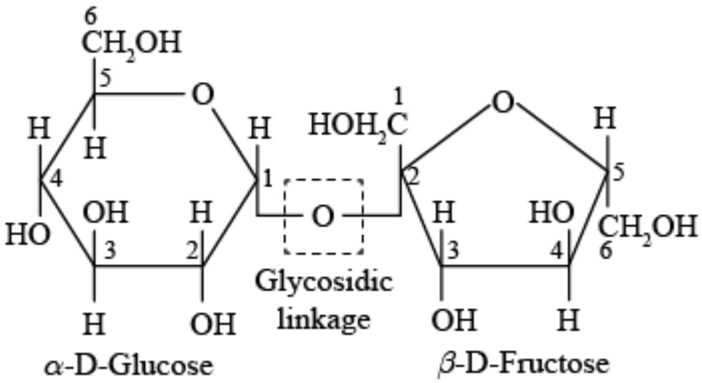
Hence, the correct answer is option A.

Solution 47

Nylon 6, 6 → used in making bristles of brushes
Low density polythene → used in making Toys
High density polythene → used in making Buckets
Teflon → used in making non-stick utensils

Hence, the correct answer is option B.

Solution 48



Sucrose

Hence in sucrose glycosidic linkage between C_1 of α-glucose and C_2 of β-D-fructose is found
Maltose ⇒ Glycosidic linkage between C_1 and C_4
Lactose ⇒ Glycosidic linkage between C_1 and C_4
Amylose ⇒ Glycosidic linkage between C_1 and C_4

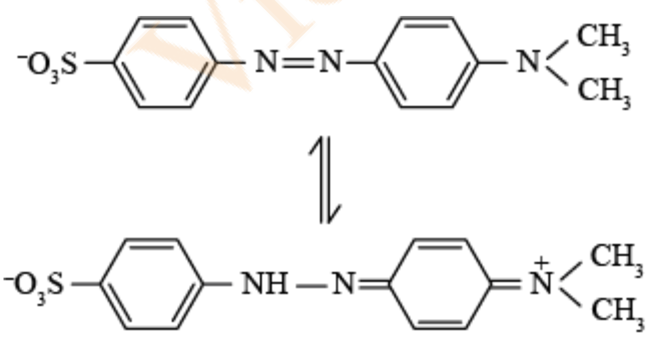
Hence, the correct answer is option B.

Solution 49

Some drugs do not bind to the enzyme's active site. These bind to a different site of enzyme which is called allosteric site.

Hence, the correct answer is option B.

Solution 50



(quinonoid form)

Hence at the end point methyl orange is present as quinonoid form.

Hence, the correct answer is option A.

Solution 51

$N_2(g) + 3H_2\ (g) \rightarrow 2NH_3\ (g)$
Since H_2 is in excess and 20 L of ammonia gas is produced.
Hence, 2 moles $NH_3 \equiv 1$ mole N_2 ($v \propto n$)
20 L $NH_3 \equiv 10$ L N_2
$\text{Volume of } N_2 \text{ left} = 56 - 10 = 46\ L$

Solution 52

From ideal gas equation,
PV = nRT
$P = 2 \times 10^6$ Pa
$V = 2\ dm^3 = 2 \times 10^{-3}\ m^3$
$R = 8.3\ JK^{-1}\ mol^{-1}$
$n = \frac{11}{44}$ mol
$2 \times 10^6 \times 2 \times 10^{-3} = \frac{11}{44} \times 8.3 \times T$
$T = 1927.7\ K$
$T\ (\text{in } ^\circ C) = 1927.7 - 273 \simeq 1655\ ^\circ C$

Solution 53

Since there is a single hydrogen atom, so only 5 → 4, 4 → 3, 3 → 2, 2 → 1 lines are obtained.

Solution 54

(I) HCl + NaOH → NaCl + H_2O
$\Delta H_1 = -57.3$ KJ mol^{-1}
(II) CH_3COOH + NaOH → CH_3COONa + H_2O
$\Delta H_2 = -55.3$ KJ mol^{-1}
Reaction (I) can be written as
(III) NaCl + H_2O → HCl + NaOH
$\Delta H_3 = 57.3$ KJ mol^{-1}
By adding (II) and (III)
CH_3COOH + NaCl → CH_3COONa + HCl ΔH_r
$\Delta H_r = \Delta H_3 + \Delta H_2 = 57.3 - 55.3 = 2\ kJ\ mol^{-1}$

Solution 55

For first order reaction,
ln A = ln A_0 − kt
Hence Slope = $-k$
$-k = -3.465 \times 10^4$

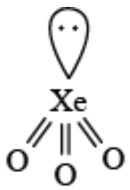
$$k = \frac{0.693}{t_{\frac{1}{2}}}$$

$$3.465 \times 10^4 = \frac{0.693}{t_{\frac{1}{2}}}$$

$$t_{\frac{1}{2}} = 2 \times 10^{-5}\ \mathrm{s}$$

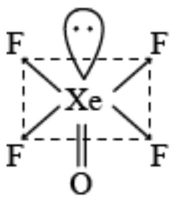
Solution 56

XeO_3⇒S.N. (Steric number) = $\frac{1}{2}\,[8] = 4 \Rightarrow sp^3$



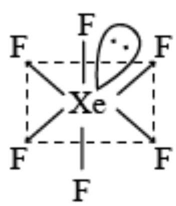
Lone pair
on central atom = 1

$XeOF_4$⇒S.N = $\frac{1}{2}$[8 + 4] = 6⇒sp^3d^2



Lone pair
on central atom = 1

XeF_6⇒S.N = $\frac{1}{2}$ [8 + 6] = 7⇒sp^3d^3



Lone pair
on central atom = 1

Sum of lone pairs = 3

Solution 57

Among the pairs given, Cr^{3+}/Cr^{2+} has negative reduction potential which is −0.41 V.
Cr (III)⇒d^3
Number of unpaired electrons = 3

$$\mu = \sqrt{3\,(3+2)} = \sqrt{15} \simeq 4\ \mathrm{B.M.}$$

Solution 58

R – OH + CH_3 MgI⇒R – OMgI + CH_4
moles of alcohol (ROH)≡moles of CH_4
At STP, [Assuming STP]
1 mole corresponds to 22.7 L
Hence, 3.1 mL≡ $\frac{3.1}{22700}$ mol
So, moles of alcohol = $\frac{3.1}{22700}$

$$\Rightarrow \frac{3.1}{22700} = \frac{4.5\times10^{-3}}{M}$$

$$M \simeq 33\ g/\ mol$$

Solution 59

$$R_f = \frac{\text{Distance travelled by the substance}}{\text{Distance travelled by the solvent front}}$$

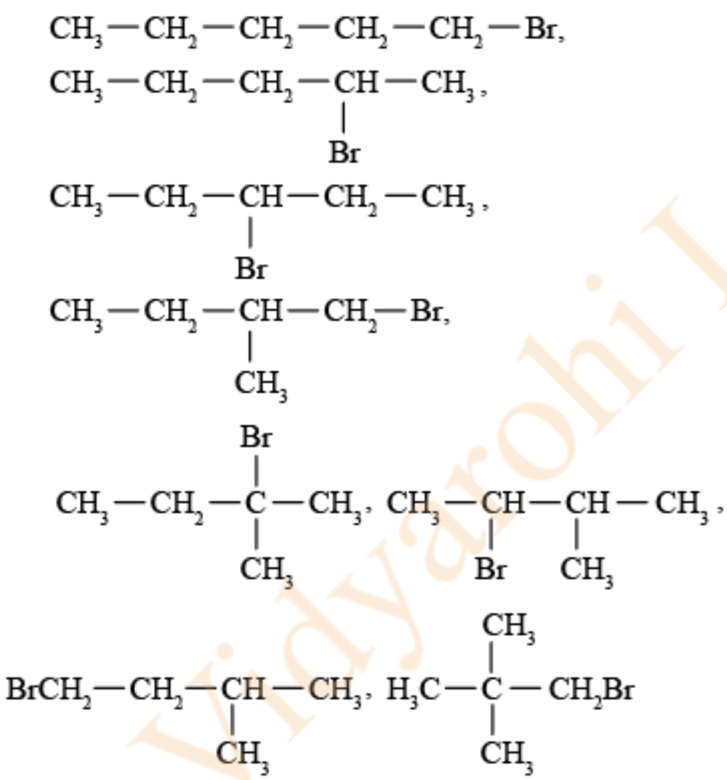
$$(R_f)_A = \frac{2.08}{3.25}$$

$$(R_f)_B = \frac{1.05}{3.25}$$

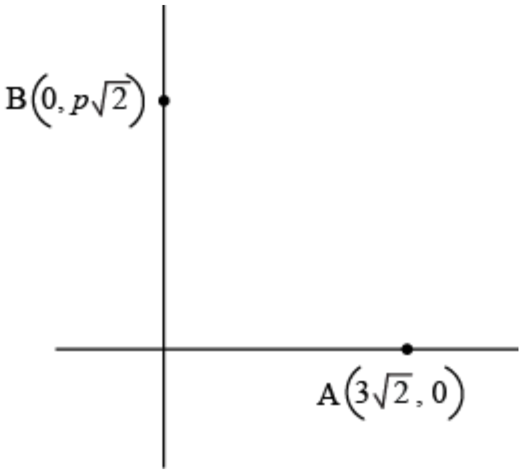
$$\frac{(R_f)_A}{(R_f)_B} \simeq 2$$

Solution 60

Total monobromo derivatives = 8



Solution 61



It is sum of distance of *z* from $\left(3\sqrt{2},\ 0\right)$ and $\left(0,\ p\sqrt{2}\right)$

For minimising, *z* should lie on *AB* and *AB* = $5\sqrt{2}$

$$(AB)^2 = 18 + 2p^2$$
$$p = \pm 4$$

Hence, the correct answer is option C.

Solution 62

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2 - |\lambda| \end{vmatrix} = 2\left(3\lambda^2 - 3|\lambda| - 1\right) + 3\left(\lambda^2 - |\lambda| + 3\right) + 5(-1-9)$$

$$= 9\lambda^2 - 9|\lambda| - 43$$
$$= 9|\lambda|^2 - 9|\lambda| - 43$$

$\Delta = 0$ for 2 values of $|\lambda|$out of which one is –ve and other is +ve
So, 2 values of λ satisfy the system of equations to obtain no solution

Hence, the correct answer is option C.

Solution 63

As function is one-one and onto, out of 50 elements of domain set 17 elements are following restriction

$$f(3) > f(9) > f(15) \ldots > f(99)$$

$$\text{So number of ways} = {}^{50}C_{17} \cdot 1 \cdot 33!$$
$$= {}^{50}P_{33}$$

Hence, the correct answer is option B.

Solution 64

Re $\left(\frac{(11)^{1011}+(1011)^{11}}{9}\right) = \text{Re}\left(\frac{2^{1011}+3^{11}}{9}\right)$

For $\text{Re}\left(\frac{2^{1011}}{9}\right)$

$$2^{1011} = (9-1)^{337} = {}^{337}C_0 9^{337}(-1)^0 + {}^{337}C_1 9^{336}(-1)^1 + {}^{337}C_2 9^{335}(-1)^2 + \ldots\ldots + {}^{337}C_{337} 9^0(-1)^{337}$$

so, remainder is 8

and $\text{Re}\left(\frac{3^{11}}{9}\right) = 0$

So, remainder is 8

Hence, the correct answer is option D.

Solution 65

$$\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)} = \frac{3}{4}\sum_{n=1}^{21} \frac{1}{4n-1} - \frac{1}{4n+3}$$
$$= \frac{3}{4}\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+\ldots\ldots+\left(\frac{1}{83}-\frac{1}{87}\right)\right]$$
$$= \frac{3}{4}\left[\frac{1}{3}-\frac{1}{87}\right]$$
$$= \frac{3}{4} \times \frac{84}{3\times 87}$$
$$= \frac{7}{29}$$

Hence, the correct answer is option B.

Solution 66

$$\lim_{x\to\frac{\pi}{4}} \frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{-7(\cos\ x + \sin\ x)^6(-\sin\ x + \cos\ x)}{-2\sqrt{2}\ \cos 2x}$$ using L–H Rule

$$= \lim_{x \to \frac{\pi}{4}} \frac{56(\cos\ x - \sin\ x)}{2\sqrt{2}\cos 2x} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{-56(\sin\ x + \cos\ x)}{-4\sqrt{2}\sin 2x} \quad \text{using L} - \text{H Rule}$$

$$= 7\sqrt{2} \cdot \sqrt{2} = 14$$

Hence, the correct answer is option A.

Solution 67

$$I = \lim_{n \to \infty} \frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}} + \frac{1}{\sqrt{1-\frac{2}{2^n}}} + \frac{1}{\sqrt{1-\frac{3}{2^n}}} + \ldots\ldots + \frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$$

Let $2^n = t$ and if $n \to \infty$ then $t \to \infty$

$$l = \lim_{n \to \infty} \frac{1}{t}\left(\sum_{r=1}^{t-1} \frac{1}{\sqrt{1-\frac{r}{t}}}\right)$$

$$l = \int_0^1 \frac{dx}{\sqrt{1-x}} = \int_0^1 \frac{dx}{\sqrt{x}} \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$= \left[2x^{\frac{1}{2}}\right]_0^1$$

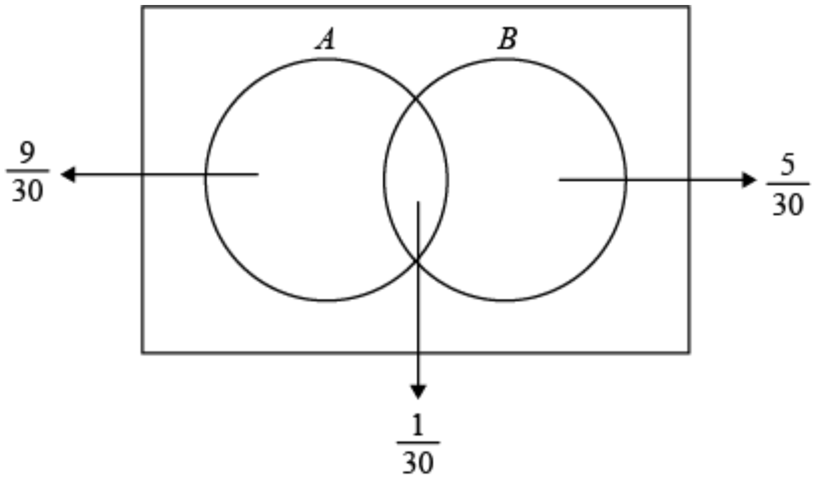
$$= 2$$

Hence, the correct answer is option C.

Solution 68

$$P(A) = \frac{1}{3},\ P(B) = \frac{1}{5}\ \text{ and }\ P(A \cup B) = \frac{1}{2}$$

$$\therefore\ P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{2} = \frac{1}{30}$$



Now, $P\left(A|B'\right)+P\left(B|A'\right)=\frac{P(A\cap B')}{P(B')}+\frac{P(B\cap A')}{P(A')}$

$$=\frac{\frac{9}{30}}{\frac{4}{5}}+\frac{\frac{5}{30}}{\frac{2}{3}}=\frac{5}{8}$$

Hence, the correct answer is option B.

Solution 69

$l=\int_{-3}^{101}\left([\sin(\pi x)]+e^{[\cos(2\pi x)]}\right)dx$

$[\sin\ \pi x]$ is periodic with period 2 and $e^{[\cos(2\pi x)]}$ s periodic with period 1.

So,

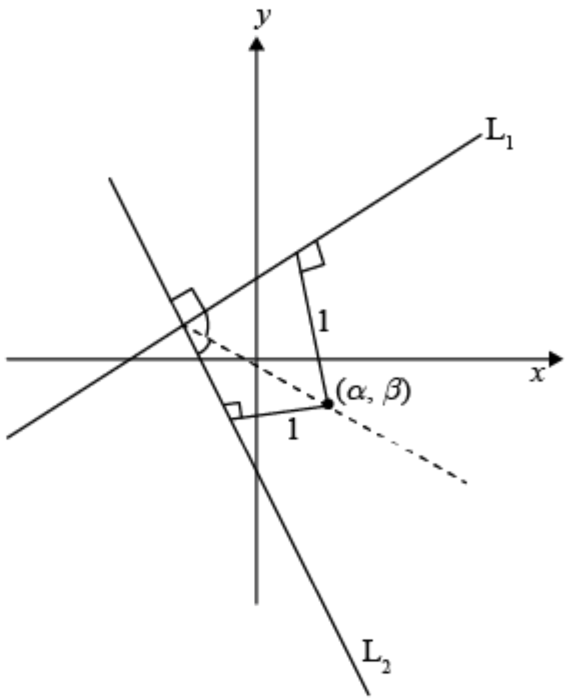
$l{=}52\int_0^2\left([\sin\ \pi x]+e^{[\cos 2\pi x]}\right)dx$

$=52\left\{\int_1^2 -1\ dx+\int_{\frac{1}{4}}^{\frac{3}{4}}e^{-1}dx+\int_{\frac{5}{4}}^{\frac{7}{4}}e^{-1}\ dx+\int_0^{\frac{1}{4}}e^0\ dx+\int_{\frac{3}{4}}^{\frac{5}{4}}e^0\ dx+\int_{\frac{7}{4}}^{2}e^0\ dx\right\}$

$=\frac{52}{e}$

Hence, the correct answer is option B.

Solution 70



$L_1 : 3x - 4y + 12 = 0$

$L_2 : 8x + 6y + 11 = 0$

Equation of angle bisector of L_1 and L_2 of angle containing origin

$2(3x - 4y + 12) = 8x + 6y + 11$

$2x + 14y - 13 = 0$ .....(i)

$\frac{3\alpha - 4\beta + 12}{5} = 1$
$\Rightarrow 3\alpha - 4\beta + 7 = 0 \quad .....(\text{ii})$

Solution of $2x + 14y - 13 = 0$ and $3x - 4y + 7 = 0$

gives the required point $P(\alpha, \beta), \ \alpha = \frac{-23}{25}, \ \beta = \frac{53}{50}$

$100(\alpha + \beta) = 14$

Hence, the correct answer is option D.

Solution 71

$\frac{dy}{dx} \propto \frac{-y}{x}$
$\frac{dy}{dx} = \frac{-ky}{x} \Rightarrow \int \frac{dy}{y} = -K \int \frac{dx}{x}$
$\ln|y| = -K \ln|x| + C$

If the above equation satisfy (1, 2) and (8, 1)
ln2 = $-K \times 0 + C \Rightarrow C = \ln 2$
ln1 = $-K \ln 8 + \ln 2 \Rightarrow K = \frac{1}{3}$
So, at $x = \frac{1}{8}$

$\ln|y| = -\frac{1}{3}\ln\left(\frac{1}{8}\right) + \ln 2 = 2\ln 2$

$|y| = 4$

Hence, the correct answer is option B.

Solution 72

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the line $\frac{x}{7} + \frac{y}{2\sqrt{6}} = 1$ on the *x*-axis

So, *a* = 7

and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the line $\frac{x}{7} - \frac{y}{2\sqrt{6}} = 1$ on the *y*-axis

So, $b = 2\sqrt{6}$

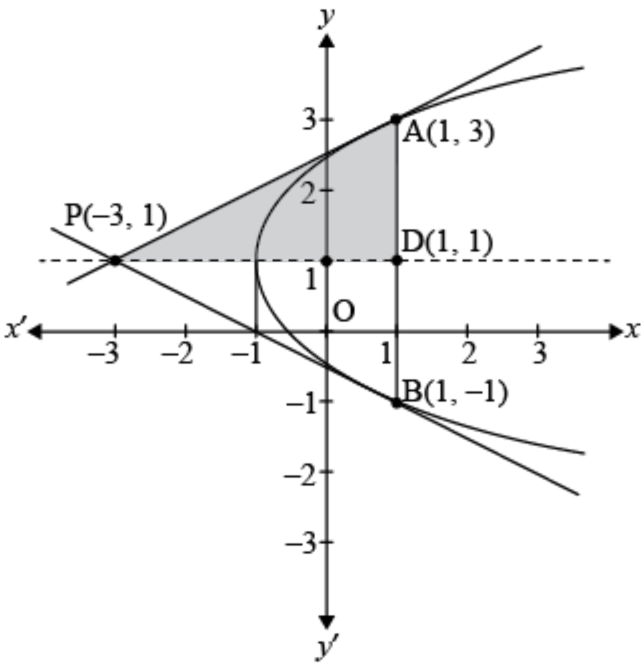
Therefore, $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{24}{49}$

$e = \frac{5}{7}$

Hence, the correct answer is option A.

Solution 73

Given curve : $y^2 - 2x - 2y = 1$.
Can be written as
$(y - 1)^2 = 2(x + 1)$



And, the given information
Can be plotted as shown in figure
Tangent at A : $2y - x - 5 = 0$ {using $T = 0$}
Intersection with $y = 1$ is $x = -3$
Hence, point P is $(-3, 1)$
Taking advantage of symmetry

$$\text{Area of } \Delta PAB = 2 \times \frac{1}{2} \times (1 - (-3)) \times (3 - 1)$$
$$= 8 \text{ sq. units}$$

Hence, the correct answer is option D.

Solution 74

Ellipse $: \frac{x^2}{16}+\frac{y^2}{7}=1$

Eccentricity $=\sqrt{1-\frac{7}{16}}=\frac{3}{4}$

Foci $\equiv(\pm a\ e,\ 0)\equiv(\pm 3,\ 0)$

Hyperbola $: \frac{x^2}{\left(\frac{144}{25}\right)}-\frac{y^2}{\left(\frac{\alpha}{25}\right)}=1$

Eccentricity $=\sqrt{1+\frac{\alpha}{144}}=\frac{1}{12}\sqrt{144+\alpha}$

Foc $\equiv(\pm ae,\ 0)\equiv\left(\pm\frac{12}{5}\cdot\frac{1}{12}\sqrt{144+\alpha},\ 0\right)$

If foci coincide then $3=\frac{1}{5}\sqrt{144+\alpha}\Rightarrow\alpha=81$

Hence, hyperbola is $\frac{x^2}{\left(\frac{12}{5}\right)^2}-\frac{y^2}{\left(\frac{9}{5}\right)^2}=1$

Length of latus rectum $=2\cdot\frac{\frac{81}{25}}{\frac{12}{5}}=\frac{27}{10}$

Hence, the correct answer is option D.

Solution 75

First plane, $P_1 = 2x - 2y + z = 0$, normal vector $\equiv\bar{n}_1=(2,\ -2,\ 1)$
Second plane, $P_2\equiv x-y+2z=4$, normal vector $\equiv\bar{n}_2=(1,\ -1,\ 2)$
Plane perpendicular to P_1 and P_2 will have normal vector $\bar{n}_3$
Where $\bar{n}_3=(\bar{n}_1\times\bar{n}_2)$
Hence, $\bar{n}_3=(-3,\ -3,\ 0)$
Equation of plane E through $P(1, -1, 1)$ and $\bar{n}_3$ as normal vector
$(x-1,\ y+1,\ z-1)\cdot(-3,\ -3,\ 0)=0$
$\Rightarrow x+y=0\equiv E$
Distance of PQ $(a, a, 2)$ from $E=\left|\frac{2a}{\sqrt{2}}\right|$

as given, $\left|\frac{2a}{\sqrt{2}}\right|=3\sqrt{2}\Rightarrow a=\pm 3$

Hence, $Q\equiv(\pm 3,\ \pm 3,\ 2)$

Distance $7Q=\sqrt{21}\Rightarrow(PQ)^2=21$

Hence, the correct answer is option C.

Solution 76

$L_1 \ : \ \frac{x+7}{-6} = \frac{y-6}{7} = \frac{z-0}{1}$

Any point on it $\overrightarrow{a}_1 (-7, \ 6, \ 0)$

and L_1 is parallel to $\overrightarrow{b}_1 (-6, \ 7, \ 1)$

$L_2 \ : \ \frac{x-7}{-2} = \frac{y-2}{1} = \frac{z-6}{1}$

Any point on it, $\overrightarrow{a}_2 (-7, \ 2, \ 6)$

and L_2 is parallel to $\overrightarrow{b}_2 (-2, \ 1, \ 1)$

Shortest distance between L_1 and L_2

$= \left| \frac{\left(\overrightarrow{a}_2 - \overrightarrow{a}_1\right) \cdot \left(\overrightarrow{b}_1 \times \overrightarrow{b}_2\right)}{\left|\overrightarrow{b}_1 \times \overrightarrow{b}_2\right|} \right| = \left| \frac{(-14, 4, -6) \cdot (3, 2, 4)}{\sqrt{9+4+16}} \right|$

$= 2\sqrt{29}.$

Hence, the correct answer is option A.

Solution 77

$\overrightarrow{a} = \hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{a} \times \overrightarrow{b} = 2\hat{i} - \hat{k}$

$\overrightarrow{a} \cdot \overrightarrow{b} = 3$

$\left|\overrightarrow{a} \times \overrightarrow{b}\right|^2 + \left|\overrightarrow{a} \cdot \overrightarrow{b}\right|^2 = \left|\overrightarrow{a}\right|^2 \cdot \left|\overrightarrow{b}\right|^2$

$\Rightarrow 5 + 9 = 6\left|\overrightarrow{b}\right|^2$

$\Rightarrow |b|^2 = \frac{7}{3}$

$\left|\overrightarrow{a} - \overrightarrow{b}\right| = \sqrt{\left|\overrightarrow{a}\right|^2 + \left|\overrightarrow{b}\right|^2 - 2\overrightarrow{a} \cdot \overrightarrow{b}} = \sqrt{\frac{7}{3}}$

projection of $\overrightarrow{b}$ on $\overrightarrow{a} - \overrightarrow{b} = \frac{\overrightarrow{b} \cdot \left(\overrightarrow{a} - \overrightarrow{b}\right)}{\left|\overrightarrow{a} - \overrightarrow{b}\right|}$

$$=\frac{\vec{b}\cdot\vec{a}-\left|\vec{b}\right|^2}{\left|\vec{a}-\vec{b}\right|}=\frac{3-\frac{7}{3}}{\sqrt{\frac{7}{3}}}$$

$$=\frac{2}{\sqrt{21}}$$

Hence, the correct answer is option A.

Solution 78

Median = $\frac{2k+12}{2}=k+6$

Mean deviation =$\sum\frac{|x_i-M|}{n}=6$

$$\Rightarrow\frac{(k+3)+(K+1)+(k-1)+(6-k)+(6-k)+(10-k)+(15-k)+(18-k)}{8}$$

$$\therefore\frac{58-2k}{8}=6$$

$$k=5$$

$$\text{Median}=\frac{2\times5+12}{2}=11$$

Hence, the correct answer is option D.

Solution 79

$$2\sin\frac{\pi}{22}\sin\frac{3\pi}{22}\sin\frac{5\pi}{22}\sin\frac{7\pi}{22}\sin\frac{9\pi}{22}$$

$$=2\sin\left(\frac{11\pi-10\pi}{22}\right)\sin\left(\frac{11\pi-8\pi}{22}\right)\sin\left(\frac{11\pi-6\pi}{22}\right)\sin\left(\frac{11\pi-4\pi}{22}\right)\sin\left(\frac{11\pi-2\pi}{22}\right)$$

$$=2\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}$$

$$=\frac{2\sin\frac{32\pi}{11}}{2^5\sin\frac{\pi}{11}}$$

$$=\frac{1}{16}$$

Hence, the correct answer is option B.

Solution 80

P : Ramu is intelligent
Q : Ramu is rich
R : Ramu is not honest
Given statement, “Ramu is intelligent and honest if and only if Ramu is not rich”
$=(P\wedge\sim R)\Leftrightarrow\sim Q$
So, negation of the statement is

$$\sim[(P \wedge \sim R) \Leftrightarrow Q]$$
$$= \sim[\{\sim(P \wedge \sim R) \vee \sim Q\} \wedge \{Q \vee (P \wedge \sim R)\}]$$
$$= ((P \wedge \sim R) \wedge Q) \vee (\sim Q \wedge (\sim P \vee R))$$

Hence, the correct answer is option D.

Solution 81

$\because \ (B \cup C)' = B' \cap C'$

B' is a set containing sub sets of *A* containing element 1 and not containing 2.
And *C′* is a set containing subsets of *A* whose sum of elements is not prime.
So, we need to calculate number of subsets of {3, 4, 5, 6, 7} whose sum of elements plus 1 is composite.
Number of such 5 elements subset = 1
Number of such 4 elements subset = 3 (except selecting 3 or 7)
Number of such 3 elements subset = 6 (except selecting {3, 4, 5}, {3, 6, 7}, {4, 5, 7} or {5, 6, 7}) Number of such 2 elements subset = 7 (except selecting {3, 7}, {4, 6}, {5, 7})
Number of such 1 elements subset = 3 (except selecting {4} or {6})
Number of such 0 elements subset = 1

$n\left(B' \cap C'\right) = 21 \Rightarrow n\left(B \cup C\right) = 2^7 - 21 = 107$

Solution 82

Let *f*(*x*) = (*x* −α) (*x* – β)

It is given that *f*(0) = *p* $\Rightarrow \alpha\beta$ = *p*

and $f\,(1) = \frac{1}{3} \Rightarrow (1 - \alpha)\,(1 - \beta) = \frac{1}{3}$

Now, let us assume that α is the common root of *f*(*x*) = 0 and *fofofof*(*x*) = 0

fofofof(*x*) = 0

$$\Rightarrow fofof(0) = 0$$
$$\Rightarrow fof(p) = 0$$

So, *f*(*p*) is either α or β.

$$(p - \alpha)\,(p - \beta) = \alpha$$
$$(\alpha\beta - \alpha)\,(\alpha\beta - \beta) = \alpha \Rightarrow (\beta - 1)\,(\alpha - 1)\,\beta = 1 \qquad (\because\ \alpha \neq 0)$$

So, β = 3

$(1-\alpha)(1-3)=\frac{1}{3}$

$\alpha=\frac{7}{6}$

$f(x)=\left(x-\frac{7}{6}\right)(x-3)$

$f(-3)=\left(-3-\frac{7}{6}\right)(3-3)=25$

Solution 83

$$A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+\begin{bmatrix}0&a&a\\0&0&b\\0&0&0\end{bmatrix}=I+B$$

$$B^2=\begin{bmatrix}0&a&a\\0&0&b\\0&0&0\end{bmatrix}+\begin{bmatrix}0&a&a\\0&0&b\\0&0&0\end{bmatrix}=\begin{bmatrix}0&0&ab\\0&0&0\\0&0&0\end{bmatrix}$$

B^3 = 0

$$\therefore\ A^n=(1+B)^n={}^nC_0 l+{}^nC_1\ B+{}^nC_2\ B^2+{}^nC_3\ B^3+\ldots..$$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+\begin{bmatrix}0&na&na\\0&0&nb\\0&0&0\end{bmatrix}+\begin{bmatrix}0&0&\frac{n(n-1)ab}{2}\\0&0&0\\0&0&0\end{bmatrix}$$

$$=\begin{bmatrix}1&na&na+\frac{n(n-1)}{2}ab\\0&1&nb\\0&0&1\end{bmatrix}=\begin{bmatrix}1&48&2160\\0&1&48\\0&0&1\end{bmatrix}$$

On comparing we get na = 48, nb = 96 and

$na+\frac{n(n-1)}{2}ab=2160$

$\Rightarrow a=4,\ n=12$ and $b=8$

$n+a+b=24$

Solution 84

$f(x)$ = $|5x-7|+[x^2+2x]$
$\quad = |5x-7|+[(x+1)^2]-1$

Critical points of

$$f(x) = \frac{7}{5},\ \sqrt{5}-1,\ \sqrt{6}-1,\ \sqrt{7}-1,\ \sqrt{8}-1,\ 2$$

∴ Maximum or minimum value of *f*(*x*) occur at critical points or boundary points

$$\therefore\ f\left(\frac{5}{4}\right) = \frac{3}{4} + 4 = \frac{19}{4}$$

$$f\left(\frac{7}{5}\right) = 0 + 4 = 4$$

as both |5*x* – 7| and x^2 + 2*x* are increasing in nature after $x = \frac{7}{5}$

$$\therefore\ f(2) = 3 + 8 = 11$$

$$\therefore\ f\left(\frac{7}{5}\right)_{\min} = 4 \text{ and } f(2)_{\max} = 11$$

Sum is 4 + 11 = 15

Solution 85

$$\frac{dy}{dx} = \frac{y}{x}\frac{(4y^2+2x^2)}{(3y^2+x^2)}$$

Put *y* = *vx* $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v\left(4v^2+2\right)}{\left(3v^2+1\right)}$$

$$\Rightarrow x\frac{dv}{dx} = v\left(\frac{\left(4v^2+2-3v^2-1\right)}{3v^2+1}\right)$$

$$\Rightarrow \int \left(3v^2+1\right)\frac{dv}{v^3+v} = \int \frac{dx}{x}$$

$$\Rightarrow \ln\left|v^3+v\right| = \ln x + c$$

$$\Rightarrow \ln\left|\left(\frac{y}{x}\right)^3 + \left(\frac{y}{x}\right)\right| = \ln\ x + C$$

$$\Big\downarrow\ y(1) = 1$$

$$\Rightarrow C = \ln 2$$

$$\therefore\ \text{for}\ y\left(2\right)$$

$$\ln\left(\frac{y^3}{8} + \frac{y}{2}\right) = 2\ln\ 2\ \Rightarrow \frac{\mathrm{y}^3}{8} + \frac{\mathrm{y}}{2} = 4$$

$$\Rightarrow [y(2)] = 2$$

$$\Rightarrow n = 3$$

Solution 86

$$\because\ f(x) + \int_0^x (x-t)\,f'(t)dt = \left(e^{2x} + e^{-2x}\right)\cos 2x + \frac{2x}{a} \qquad \ldots\ldots(\mathrm{i})$$

Here $f(0) = 2$ ...(ii)

On differentiating equation (i) w.r.t. *x* we get :

$$f'(x) + \int_0^x f'(t)dt + xf'(x) - xf'(x) = 2\left(e^{2x} - e^{-2x}\right)\ \cos 2x - 2\left(e^{2x} + e^{-2x}\right)\sin$$

$$\Rightarrow f'(x) + f(x) - f(0) = 2\left(e^{2x} - e^{-2x}\right)\cos 2x - 2\left(e^{2x} + e^{-2x}\right)\sin 2x + \frac{2}{a}$$

Replace *x* by 0 we get :

$$\Rightarrow 4 = \frac{2}{a} \Rightarrow a = \frac{1}{2}.$$

$$\therefore\ (2a+1)^5 \cdot a^2 = 2^5 \cdot \frac{1}{2^2} = 2^3 = 8$$

Solution 87

$$a_n = \int_{-1}^{n} \left(1 + \frac{x}{2} + \frac{x^2}{3} + ... + \frac{x^{n-1}}{n}\right) dx$$

$$= \left[x + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \ldots + \frac{x^n}{n^2}\right]_{-1}^{n}$$

$$a_n = \frac{n+1}{1^2} + \frac{n^2-1}{2^2} + \frac{n^3+1}{3^2} + \frac{n^4-1}{4^2} + \ldots + \frac{n^n + (-1)^{n+1}}{n^2}$$

Here $a_1 = 2,\ a_2 = \frac{2+1}{1} + \frac{2^2-1}{2} = 3 + \frac{3}{2} = \frac{9}{2}$

$a_3 = 4 + 2 + \frac{28}{9} = \frac{100}{9}$

$a_4 = 5 + \frac{15}{4} + \frac{65}{9} + \frac{255}{16} > 31.$

∴ The required set is {2, 3}.
∵ $a_n \in (2, 30)$
∴ Sum of elements = 5.

Solution 88

The circle $x^2 + y^2 + 6x + 8y + 16 = 0$ has centre (–3, –4) and radius 3 units.
The circle $x^2 + y^2 + 2\left(3 - \sqrt{3}\right)x + 2\left(4 - \sqrt{6}\right)y = k + 6\sqrt{3} + 8\sqrt{6},\ k > 0$ has centre $\left(\sqrt{3} - 3,\ \sqrt{6} - 4\right)$ and radius $\sqrt{k + 34}$

∵ These two circles touch internally hence $\sqrt{3 + 6} = \left|\sqrt{k + 34} - 3\right|$.
Here, $k = 2$ is only possible (∵ $k > 0$)
Equation of common tangent to two circles is
$2\sqrt{3x} + 2\sqrt{6}y + 16 + 6\sqrt{3} + 8\sqrt{6} + k = 0$
∵ $k = 2$ then equation is $x + \sqrt{2}y + 3 + 4\sqrt{2} + 3\sqrt{3} = 0$ .....(i)
∵ $(\alpha,\ \beta)$ are foot of perpendicular from $(-3,\ -4)$
To line (i) then

$$\frac{\alpha+3}{1} = \frac{\beta+4}{\sqrt{2}} = \frac{-\left(-3-4\sqrt{2}+3+4\sqrt{2}+3\sqrt{3}\right)}{1+2}$$

$\therefore \alpha + 3 = \frac{\beta+4}{\sqrt{2}} = -\sqrt{3}$

$\Rightarrow \left(\alpha + \sqrt{3}\right)^2 = 9$ and $\left(\beta + \sqrt{6}\right)^2 = 16$

$\therefore\ \left(\alpha + \sqrt{3}\right)^2 + \left(\beta + \sqrt{6}\right)^2 = 25$

Solution 89

$4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ differentiating both sides we get

$12x^2 - 3y^2 - 6xyy' + 12x - 5y - 5xy' - 16yy' + 9 = 0$

$\downarrow (-2,\ 3)$

$\Rightarrow 48 - 27 + 36y' - 24 - 15 + 10y' - 48y' + 9 = 0$

$\Rightarrow 2y' = -9$

$\Rightarrow m_T = \frac{-9}{2}\ \&\ m_N = \frac{2}{9}$

$T \equiv y - 3 = \frac{-9}{2}(x+2)\ \&\quad N \equiv y - 3 = \frac{2}{9}(x+2)$

$\downarrow y = 0 \qquad\qquad \downarrow y = 0$

$x = \frac{-4}{3} \qquad\qquad x = \frac{-31}{2}$

$\therefore\ \text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$

$\text{A} = \frac{1}{2} \times \left(\frac{-4}{3} + \frac{31}{2}\right)(3) = \frac{1}{2}\left(\frac{85}{6}\right) \cdot 3 = \frac{85}{4}$

$= 8\text{A} = 170$

Solution 90

$\because\ x = \sin\left(2\tan^{-1}\alpha\right) = \frac{2\alpha}{1+\alpha^2} \qquad \ldots.(\text{i})$

$\text{and}\ \ y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right) = \sin\left(\sin^{-1}\frac{1}{\sqrt{5}}\right) = \frac{1}{\sqrt{5}}$

Now, $y^2 = 1 - x$

$\frac{1}{5} = 1 - \frac{2\alpha}{1+\alpha^2}$

$\Rightarrow 1 + \alpha^2 = 5 + 5\alpha^2 - 10\alpha$

$\Rightarrow 2\alpha^2 - 5\alpha + 2 = 0$

$\therefore \alpha = 2,\ \frac{1}{2}$

$\therefore \sum_{\alpha \in S} 16\alpha^3 = 16 \times 2^3 + 16 \times \frac{1}{2^3}$

$= 130$