Differential Equations

Q.No.1:
At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers *x* is given by $\frac{dP}{dx}=100-12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is :

JEE 2013

A. 2500
B. 3000
C. 3500
D. 4500

Q.No.2: Let *y*(*x*) be the solution of the differential equation $(x\log x)\frac{dy}{dx}+y=2x\log x,\ \ (x\geq 1)$. Then *y*(*e*) is equal to : **JEE 2015**

A. *e*
B. 0
C. 2
D. 2*e*

Q.No.3: If a curve *y* = *f* (*x*) passes through the point (1, –1) and satisfies the differential equation, *y* (1 + *xy*) *dx* = *xdy*, then $f\left(-\frac{1}{2}\right)$ is equal to : **JEE 2016**

A. $-\frac{4}{5}$
B. $\frac{2}{5}$
C. $\frac{4}{5}$
D. $-\frac{2}{5}$

Q.No.4: If (2 + sin x) $\frac{dy}{dx}$ + (y + 1) cos x = 0 and y(0) = 1, then y $\left(\frac{\pi}{2}\right)$ is equal to : **JEE 2017**

A. $\frac{1}{3}$

B. $-\frac{2}{3}$

C. $-\frac{1}{3}$

D. $\frac{4}{3}$

Q.No.5: Let $y = y(x)$ be the solution of the differential equation $\sin x\frac{dy}{dx} + y \cos x = 4x,\ x \in (0,\ \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to : **JEE 2018**

A. $-\frac{8}{9}\pi^2$

B. $-\frac{4}{9}\pi^2$

C. $\frac{4}{9\sqrt{3}}\pi^2$

D. $\frac{-8}{9\sqrt{3}}\pi^2$

Q.No.6: If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to: **JEE 2019**

A. $\frac{7}{64}$

B. $\frac{1}{4}$

C. $\frac{49}{16}$

D. $\frac{13}{16}$

Q.No.7:

Let $f : [0, 1] \to R$ be such that $f(xy) = f(x) \cdot f(y)$, for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right)$ is equal to:

JEE 2019

A. 3

B. 4
C. 2
D. 5

Q.No.8: If $\frac{dy}{dx} + \frac{3}{\cos^2 x} y = \frac{1}{\cos^2 x}$, $x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals: **JEE 2019**

A. $\frac{1}{3} + e^6$
B. $\frac{1}{3}$
C. $-\frac{4}{3}$
D. $\frac{1}{3} + e^3$

Q.No.9: The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2)\, dx + 2xy\, dy = 0$ which passes through (1, 1), is: **JEE 2019**

A. a circle with centre on the *x*-axis.
B. an ellipse with major axis along the *y*-axis.
C. a circle with centre on the *y*-axis.
D. a hyperbola with transverse axis along the *x*-axis.

Q.No.10: Let *f* be a differentiable function such that $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}$, $(x > 0)$ and $f(1) \neq 4$. Then $\lim\limits_{x \to 0^+} x\, f\left(\frac{1}{x}\right)$: **JEE 2019**

A. exists and equals $\frac{4}{7}$.
B. exists and equals 4.
C. does not exist.
D. exists and equals 0.