Solution 14

(i)

$$\text{Cost price matrix} = \begin{bmatrix} 2000 & 3500 & 1800 \\ 1500 & 2800 & 1600 \\ 1800 & 3200 & 2000 \end{bmatrix} \begin{bmatrix} 4 \\ 4.50 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2000 \times 4 + 3500 \times 4.50 + 1800 \times 4 \\ 1500 \times 4 + 2800 \times 4.50 + 1600 \times 4 \\ 1800 \times 4 + 3200 \times 4.50 + 2000 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8000 + 15750 + 7200 \\ 6000 + 12600 + 6400 \\ 7200 + 14400 + 8000 \end{bmatrix}$$

$$= \begin{bmatrix} ₹30,950 \\ ₹25,000 \\ ₹29,600 \end{bmatrix}$$

(ii)

$$\text{Sale price matrix} = \begin{bmatrix} 2000 & 3500 & 1800 \\ 1500 & 2800 & 1600 \\ 1800 & 3200 & 2000 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 4.50 \end{bmatrix}$$

$$= \begin{bmatrix} 2000 \times 5 + 3500 \times 6 + 1800 \times 4.50 \\ 1500 \times 5 + 2800 \times 6 + 1600 \times 4.50 \\ 1800 \times 5 + 3200 \times 6 + 2000 \times 4.50 \end{bmatrix}$$

$$= \begin{bmatrix} 10000 + 21000 + 8100 \\ 7500 + 16800 + 7200 \\ 9000 + 19200 + 9000 \end{bmatrix}$$

$$= \begin{bmatrix} ₹39,100 \\ ₹31,500 \\ ₹37,200 \end{bmatrix}$$

(iii)

$$\text{Profit matrix} = \text{Sale price matrix} - \text{Cost price matrix}$$

$$= \begin{bmatrix} ₹39,100 \\ ₹31,500 \\ ₹37,200 \end{bmatrix} - \begin{bmatrix} ₹30,950 \\ ₹25,000 \\ ₹29,600 \end{bmatrix}$$

$$= \begin{bmatrix} ₹8,150 \\ ₹6,500 \\ ₹7,600 \end{bmatrix}$$

Total profit = ₹ (8,150 + 6,500 + 7,600) = ₹22,250.

(iv)

$$\text{Profit matrix} = \begin{bmatrix} ₹8,150 \\ ₹6,500 \\ ₹7,600 \end{bmatrix}$$

Revenue in Agra is ₹6,500.

(v) Revenue is Delhi of all items is ₹8,150
Revenue in Lucknow of all items is ₹7,600

Difference = ₹(8,150 – 7,600) = ₹550.

Solution 15

(i) X = {A, B, C} and Y = {1, 2, 3, 4, 5, 6}
n(X) = 3 = p(say) and n(Y) = 6 = q(say)

So, number of relation from X to Y = 2^{pq}

= 2^{18}

(ii)

$R = \{(1, 2), (2, 4), (3, 6)\}$

Reflexive: Clearly $(1, 1), (2, 2) \notin R$.

So, R is not reflexive.

Symmetric: Since $(1, 2) \in R$, but $(2, 1) \notin R$

So, R is not symmetric.

Transitive: Since $(1, 2) \in R$, $(2, 4) \in R$ but $(1, 4) \notin R$,

So, R is not transitive.

(iii)

$R : X \to X$, $R = \{(x, y) : x \text{ and } y \text{ have same age}\}$

Reflexive: $(x, x) \in R \, \forall x \in X$

So, R is reflexive.

Symmetric: Let $(x, y) \in R$

$\Rightarrow$ x & y have same age

$\Rightarrow$ $(y, x) \in R$

So, R is symmetric.

Transitive: Let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow$ x & y have same age

and y and z have same age

$\Rightarrow$ x and z must have same age

$\Rightarrow$ $(x, z) \in R$

So, R is transitive.

$\therefore$ R is an equivalence relation.

(iv)

Number of functions from X to Y $= [n(Y)]^{n(x)}$

$= n^m$

$= 6^3$

$= 216$

(v)

$\mathrm{R : X \rightarrow Y}$

$\mathrm{R = \{(A,\ 2),\ \ (C,\ 4),\ \ (B,\ 3),\ \ (A,\ 1),\ \ (B,\ 5)\}}$

For $\mathrm{A \in X}$,

1 and $2 \in \mathrm{Y}$

such that $\mathrm{(A,\ 1)}$ and $\mathrm{(A,\ 2) \in R}$

$\Rightarrow$ R is not a function.

Solution 16

(i) The matrix associated with the items is given as $A = \begin{bmatrix} 120 & 150 & 135 \\ 115 & 140 & 150 \\ 125 & 135 & 140 \end{bmatrix}$.

Sales price matrix = $A \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$

$$= \begin{bmatrix} 120 & 150 & 135 \\ 115 & 140 & 150 \\ 125 & 135 & 140 \end{bmatrix} \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1200 + 2250 + 1620 \\ 1150 + 2100 + 1800 \\ 1250 + 2025 + 1680 \end{bmatrix}$$

$$= \begin{bmatrix} ₹5070 \\ ₹5050 \\ ₹4955 \end{bmatrix}$$

(ii) The matrix associated with the items is given as $A = \begin{bmatrix} 120 & 150 & 135 \\ 115 & 140 & 150 \\ 125 & 135 & 140 \end{bmatrix}$.

Cost price matrix = $A\begin{bmatrix}7\\10\\8\end{bmatrix}$

$$=\begin{bmatrix}120 & 150 & 135\\115 & 140 & 150\\125 & 135 & 140\end{bmatrix}\begin{bmatrix}7\\10\\8\end{bmatrix}$$

$$=\begin{bmatrix}840+1500+1080\\805+1400+1200\\875+1350+1120\end{bmatrix}$$

$$=\begin{bmatrix}₹3420\\₹3405\\₹3345\end{bmatrix}$$

(iii) Since sales price matrix is $\begin{bmatrix}₹5070\\₹5050\\₹4955\end{bmatrix}$ and cost price matrix is $\begin{bmatrix}₹3420\\₹3405\\₹3345\end{bmatrix}$,

∴ Profit Matrix = Sales price matrix − Cost price matrix

$$=\begin{bmatrix}₹5070\\₹5050\\₹4955\end{bmatrix}-\begin{bmatrix}₹3420\\₹3405\\₹3345\end{bmatrix}$$

$$=\begin{bmatrix}₹1650\\₹1645\\₹1610\end{bmatrix}$$

Thus, the total monthly profit is ₹(1650 + 1645 + 1610) = ₹4905.

(iv) Since sales price matrix is $\begin{bmatrix}₹5070\\₹5050\\₹4955\end{bmatrix}$ and cost price matrix is $\begin{bmatrix}₹3420\\₹3405\\₹3345\end{bmatrix}$,

∴ Profit Matrix = Sales price matrix − Cost price matrix

$$= \begin{bmatrix} ₹5070 \\ ₹5050 \\ ₹4955 \end{bmatrix} - \begin{bmatrix} ₹3420 \\ ₹3405 \\ ₹3345 \end{bmatrix}$$

$$= \begin{bmatrix} ₹1650 \\ ₹1645 \\ ₹1610 \end{bmatrix}$$

Thus, monthly revenue on items sold in Meerut is ₹1645.
Therefore, the annual revenue on items sold in Meerut is ₹(12 × 1645) = ₹19,740.

(v) Revenue generated by Kanpur is ₹1650, by Meerut is ₹1645 and by Prayagraj is ₹1610.

Thus, percent of revenue by Kanpur and Prayagraj is

$$\frac{₹(1650+1610)}{₹(1650+1645+1610)} \times 100$$

$$= \frac{3260}{4905} \times 100$$

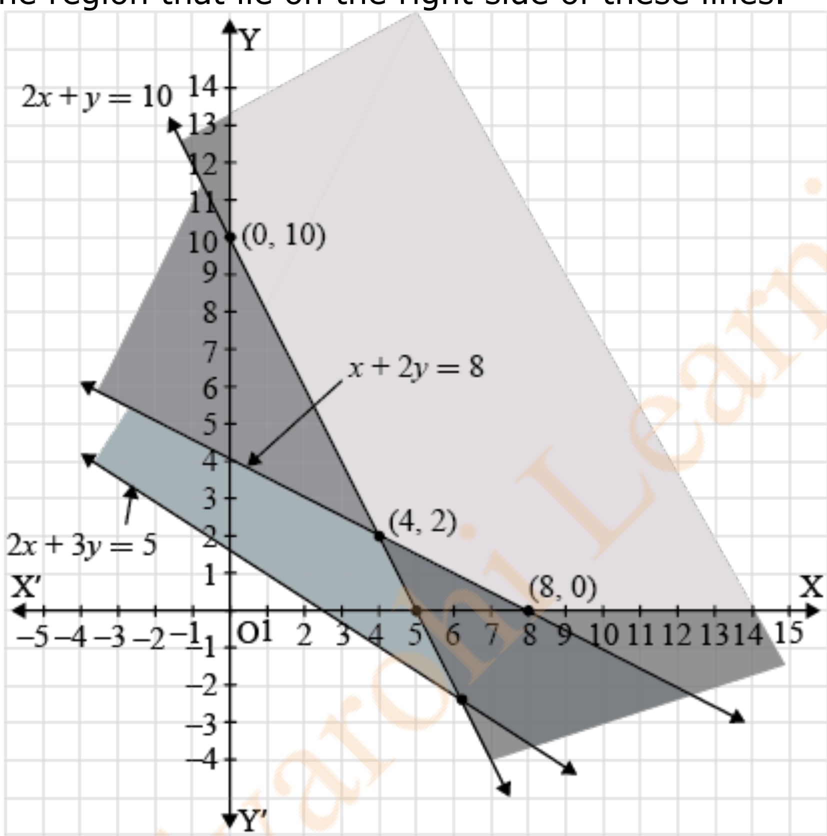
$$\approx 66.5\%$$

Solution 17

(i) Protein: $x + 2y = 8$ .....(1)
Mineral: $2x + y = 10$ .....(2)
Multiplying (1) by 2, we get
$2x + 4y = 16$ .....(3)
Subtracting (2) from (3), we get
$(2x + 4y) - (2x + y) = 16 - 10$
$\Rightarrow 3y = 6$
$\Rightarrow y = 2$
Substituting $y = 2$ in (1), we get
$x + 2(2) = 8$
$\Rightarrow x = 4$
Thus, the point of intersection is (4, 2).

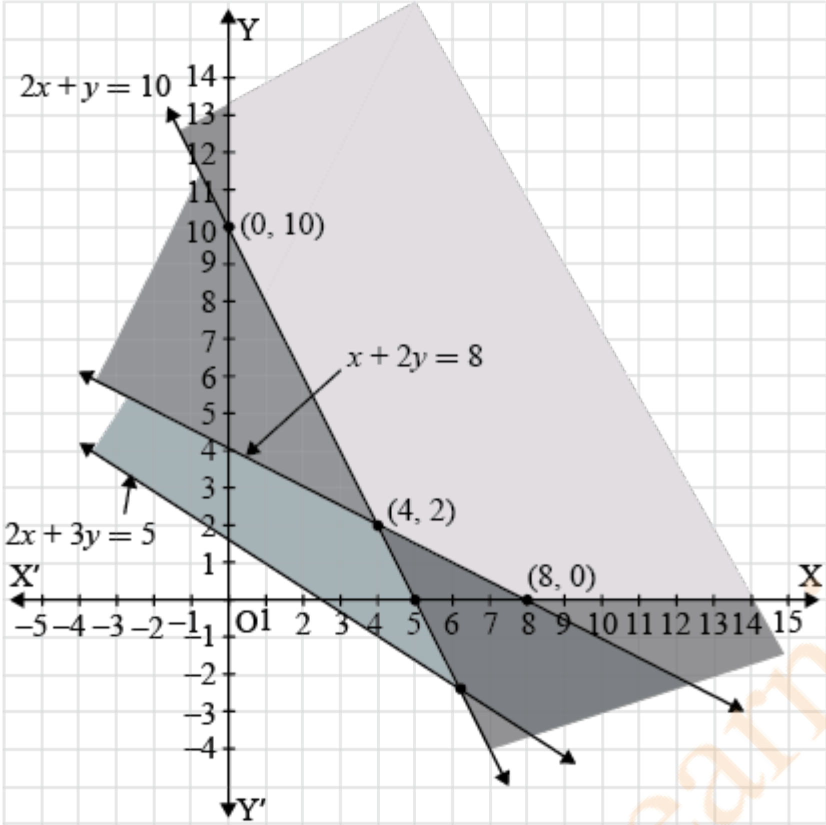
(ii) The given linear programming problem is:
Maximize: $Z = 60x + 72y$
Subject to constraints:
$2x + 3y \geq 5$
$x + 2y \geq 8$
$2x + y \geq 10$
$x \geq 0$ and $y \geq 0$

Draw the lines $x = 0$, $y = 0$, $2x + 3y = 5$, $x + 2y = 8$ and $2x + y = 10$ and shade the region that lie on the right side of these lines.



From the graph, the corner points of the feasible region can be observed as (0, 10), (4, 2) and (8, 0).

(iii) The feasible region for the given LPP is given by



Thus, (10, 0), (10, 5) and (5, 5) lies in the feasible region. However (5, 0) does not lie in it.

(iv) Given: $Z = 60x + 72y$

Corner Point	Value of $Z = 60x + 72y$
(0, 10)	60 × 0 + 72 × 10 = 720
(4, 2)	60 × 4 + 72 × 2 = 384
(8, 0)	60 × 8 + 72 × 0 = 480

Thus, the maximum cost ₹720 occurs at (0, 10).

(v) Given: $Z = 60x + 72y$

Corner Point	Value of $Z = 60x + 72y$
(0, 10)	60 × 0 + 72 × 10 = 720
(4, 2)	60 × 4 + 72 × 2 = 384
(8, 0)	60 × 8 + 72 × 0 = 480

Thus, the minimum cost ₹384 occurs at (4, 2).
The mixture contains 4 kg of Food A and 2 kg of Food B.
Therefore, the total mixture contains (4 + 2 = 6) kg.