JEE Main 25 Jan 2023(Second Shift)

Total Time: 180

Total Marks: 300.0

Solution 1

A is correct because $F = G\frac{m_1 m_2}{r^2}$ ...(i)

D is correct because $T^2 \propto r^3$

$\therefore$ A and D are correct

Hence, the correct answer is option (4).

Solution 2

A. Isothermal process → II ($\Delta U = 0$)
B. Adiabatic process → I ($\Delta Q = 0$)
C. Isochoric process → IV ($w = 0$)
D. Isobaric process → III ($\Delta Q = \Delta U + w$)

Hence, the correct answer is option (3).

Solution 3

$\therefore\ E_x = 2\text{ cm} = 0$

$\frac{1}{4\pi\varepsilon_0}\frac{(10\ \mu\text{C})}{(2\ \text{cm})^2} = \frac{1}{4\pi\varepsilon_0}\frac{(40\ \mu\text{C})}{[(a-2)\ \text{cm}]^2}$

$\Rightarrow \left(\frac{a-2}{2}\right)^2 = 4$

$\Rightarrow \frac{a-2}{2} = 2$

$\boxed{\text{a} = 6\ \text{cm}}$

Hence, the correct answer is option (2).

Solution 4

From graph, we can say that.

$\frac{t_P - 30}{150} = \frac{t_Q - 0}{100}$

Hence, the correct answer is option (2).

Solution 5

Statement I: is correct as stopping potential is independent of power of light used.
Statement II: is correct as maximum kinetic energy of photoelectron depends on wavelength of light.

Hence, the correct answer is option (1).

Solution 6

The image will be **erect** and **laterally inverted**.
Hence, the correct answer is option (1).

Solution 7

Both the objects will have the same range because, $\alpha + \beta = 90°$. i.e., α, β are complementary angles.
Hence, the correct answer is option (4).

Solution 8

→ 10 km over Earth's surface – Troposphere
→ 100 km over Earth's surface – E-part of stratosphere
→ 300 km over Earth's surface – F_2-part of thermosphere
→ 65 - 75 km over Earth's surface – D-part of stratosphere

Hence, the correct answer is option (4).

Solution 9

$\because \lambda = \frac{hc}{\Delta E}$

$\lambda = \frac{1240}{10} \text{nm}$

$\approx 124 \text{ nm.}$

∴ D is the transition required.

Hence, the correct answer is option (1).

Solution 10

$\because\ \theta = \left(\frac{NBA}{K}\right)I$

$A = \frac{\theta K}{NBI}$

$= \frac{0.05\times4\times10^{-5}}{(200)\times(0.01)\times(10\times10^{-3})}$

$= 1\ \text{cm}^2$

Hence, the correct answer is option (1).

Solution 11

$x = A\sin(\omega t)$

$x = \frac{A}{2} = A\sin\left(\omega t\right)$

$\frac{1}{2} = \sin\left(\omega t\right)$

$t = \left(\frac{\pi}{6\omega}\right) = 2$

$\boxed{\frac{\pi}{\omega} = 12\ \text{sec}}$

$x = A = A\sin\left(\omega t\right)$

$\omega t = \left(\frac{\pi}{2}\right)$

$t = \left(\frac{\pi}{2\omega}\right) = 6\ \text{second}$

$\text{time} = 6 - 2 = 4\ \text{seconds}$

Hence, the correct answer is option (4).

Solution 12

$\epsilon = BVl$
$= 2 \times 8 \times 1$
$= 16$ V

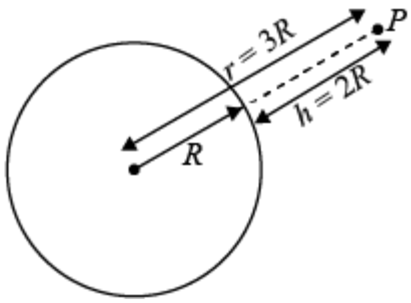
Hence, the correct answer is option (2).

Solution 13

$x = 4t^2$
$\frac{dx}{dt} = 8t$
v at t = 5s is 40 m/s

Hence, the correct answer is option (3).

Solution 14



$$V_{\text{surface}} = -\left(\frac{GMm}{R_e}\right)$$

$$V_p = -\frac{GMm}{3R_e}$$

$$\Delta V = \frac{GMm}{R_e}\left(1 - \frac{1}{3}\right)$$

$$= \frac{2GMm}{3(R_e^2)} \times R_e$$

$$\Delta V = \frac{2}{3} mgR_e$$

Hence, the correct answer is option (3).

Solution 15

$$\text{(A) } [Y] = \left[\frac{\text{MLT}^{-2}}{\text{L}^2}\right] = \left[\text{ML}^{-1}\,\text{T}^{-2}\right] \quad \ldots(\text{III})$$

$$\frac{F}{A} = \eta\left(\frac{dV}{dY}\right)$$

$$\text{(B) } [\eta] = \left[\frac{\text{MLT}^{-2} \times \text{L}}{\text{L}^2 \times \text{LT}^{-1}}\right] = \left[\text{ML}^{-1}\,\text{T}^{-1}\right] \quad \ldots(\text{I})$$

$$hv = E = \left[\text{ML}^{-1}\,\text{T}^{-1}\right]$$

$$\text{(C) } [h] = \left[\text{ML}^2\,\text{T}^{-1}\right] \quad \ldots(\text{II})$$

$$\text{(D) } \phi = \left[\text{ML}^2\,\text{T}^{-2}\right] \quad \ldots(\text{IV})$$

Hence, the correct answer is option (2).

Solution 16

Gauss's law $\oint \overrightarrow{E} \cdot \overrightarrow{ds} = \frac{q}{\epsilon_0}$ $(\text{A} \rightarrow \text{IV})$

Faraday's law $\oint \overrightarrow{E} \cdot \overrightarrow{d}\, l = -\frac{d\phi_B}{dt}$ $(\text{B} \rightarrow 1)$

Gauss's law in magnetism $\oint \overrightarrow{B} \cdot \overrightarrow{dA} = 0 \qquad (\mathrm{C} \rightarrow \mathrm{II})$

Ampere's-Maxwell law $\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 i_c + \mu_0 \in_0 \frac{d\phi_E}{dt} \qquad (\mathrm{D} \rightarrow \mathrm{III})$

Hence, the correct answer is option (3).

Solution 17

According to the equipartition of energy degree of freedom of diatomic gas is *f* = 7, (2 degree of freedom is added for every vibrational mode) So,
$C_V = \frac{f}{2} R = \frac{7R}{2}$

Hence, the correct answer is option (2).

Solution 18

Force required to push
$F_1 = mg\sin\theta + \mu mg\cos\theta = \frac{mg}{\sqrt{2}} (1 + \mu)$
Force required to prevent from sliding
$F_2 = (mg\sin\theta - \mu mg\cos\theta) = \frac{mg}{\sqrt{2}} (1 - \mu)$
Given $F_1 = 2F_2$
$1 + \mu = 2(1 - \mu)$
$\mu = \frac{1}{3} = 0.33$

Hence, the correct answer is option (4).

Solution 19

Statement I is correct but in statement II we cannot detect the current through ammeter thus the statement II is incorrect.
Hence, the correct answer is option (2).

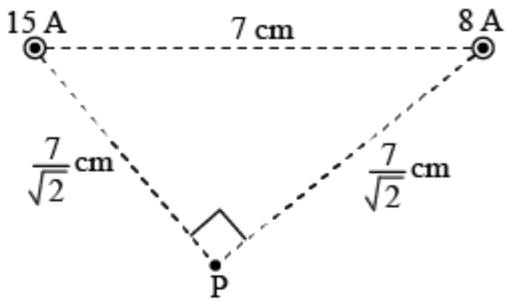
Solution 20

$R' = n^2 R$
$= 5^2 \times 5\ \Omega$
$= 125\ \Omega$

Hence, the correct answer is option (3).

Solution 21

From question



$\overrightarrow{B}_{15A} = \frac{\mu_0 \times 15}{2\pi\left(\frac{7}{\sqrt{2}}\ \text{cm}\right)}$, $\overrightarrow{B}_{8A} = \frac{\mu_0 \times 8}{2\pi\left(\frac{7}{\sqrt{2}}\right)\text{cm}}$

$\overrightarrow{B}_{15A}$ & $\overrightarrow{B}_{8A}$ are perpendicular to each other.

Hence $\overrightarrow{B}_P = \overrightarrow{B}_{15A} + \overrightarrow{B}_{8A}$

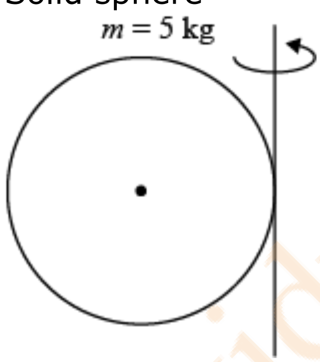
$= \sqrt{(B_{15A})^2 + (B_{8A})^2}$

$= \frac{\mu_0}{2\pi\left(\frac{7}{\sqrt{2}}\right)\text{cm}}\sqrt{18^2 + 8^2} = \frac{\mu_0 17}{2\pi \times \left(\frac{7}{\sqrt{2}}\right) \times 10^{-2}}$

$= \frac{4\pi \times 17 \times 10^{-7}}{2\pi \times \frac{7}{\sqrt{2}} \times 10^{-2}} = 68 \times 10^{-6}$

Solution 22

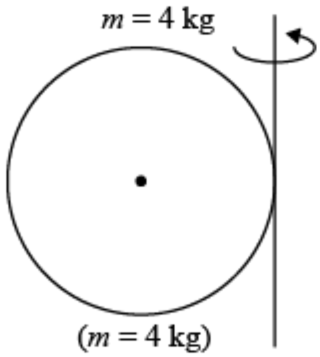
Solid sphere



$l_{\text{tangent}} = l_{\text{cm}} + mR^2$

$= \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2$

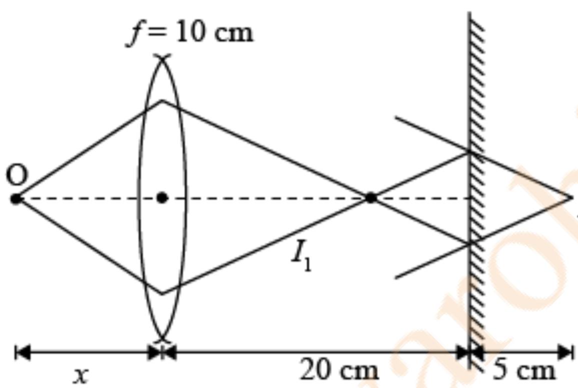
$= 7R^2 \qquad (m = 5kg)$



$$l_{\text{disc}} = l_{\text{cm}} + mR^2$$
$$= \frac{mR^2}{4} + mR^2$$
$$= \frac{5}{4} mR^2$$
$$= 5R^2$$

$$\frac{l_{\text{disc}}}{l_{\text{tangent}}} = \frac{5}{7}$$

Solution 23



From diagram

I_1 is image formed by lens and I_2 is image formed by mirror.

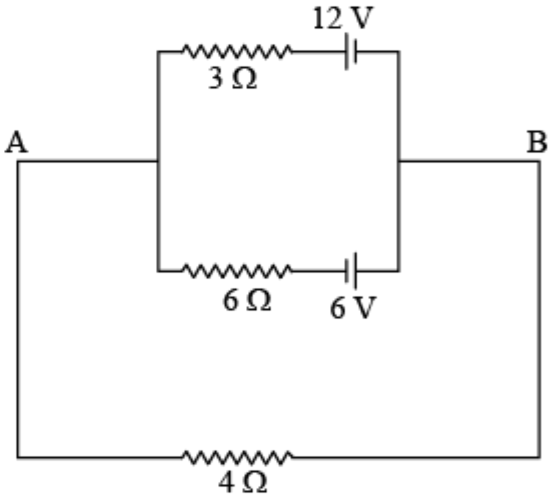
Location of I_1 and I_2 from mirror will be equal = 5 cm

Hence I_1 = 15 cm from lens

From $\frac{1}{v} - \frac{1}{u} = \frac{1}{10}$; $\quad u = -x,\ v = 15$

$\frac{1}{x} = \frac{1}{10} - \frac{1}{15} \Rightarrow x = 30\ \text{cm}$

Solution 24



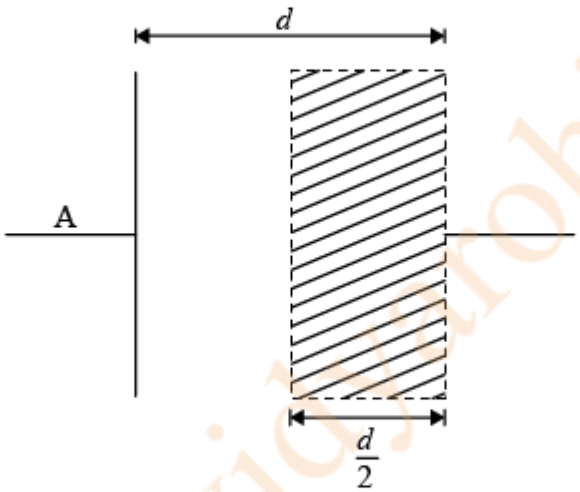
KCL at *A* gives

$$\frac{6-V_A}{4} + \frac{0-V_A}{6} + \frac{18-V_A}{3} = 0$$

$$V_A = 10$$

So current through $4\ \Omega = \frac{10-6}{4} = 1\text{A}$

Solution 25



$$C = \frac{\varepsilon_0 A}{d} K$$

When completely air filled

$$C = 5\mu F = \frac{\varepsilon_0 A}{d} \qquad \ldots (1)$$

When half filled with K = 1.5

$$\frac{1}{C_{eq}} = \frac{\frac{d}{2}}{\varepsilon_0 A} + \frac{\frac{d}{2}}{\varepsilon_0 AK}$$

$$C_{eq} = \left(\frac{2K}{K+1}\right)\frac{\varepsilon_0 A}{d} \qquad \ldots \left(2\right)$$

From $\left(1\right)$ & $\left(2\right)$

$$C_{eq} = \left(\frac{2\times1.5}{1.5+1}\right)5\ \mu\text{F} = 6\ \mu\text{F}$$

Solution 26

$$f{=}f_0\left(\frac{v}{v-v_s}\right)$$

$$f{=}320\left(\frac{330}{330-66}\right)$$

$$=320 \times \frac{330}{264}$$

$$=400\ \text{Hz.}$$

Solution 27

$m_1\ v_1 = m_2\ v_2$

$\Rightarrow \left(\frac{m_1}{m_2}\right) = \frac{v_2}{v_1} = \left(\frac{2}{3}\right)$

$m \propto A$

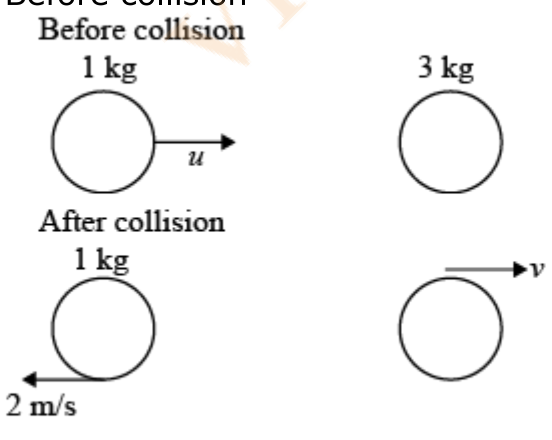
$\frac{A_1}{A_2} = \left(\frac{2}{3}\right)$

$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} = \left(\frac{2}{3}\right)^{\frac{1}{3}}$

$x = 2$

Solution 28

Before collision



Momentum conservation

$u + 0 = 3\ v - 2$

$\boxed{3v - u = 2} \quad \ldots(1)$

also,

$\frac{v+2}{u} = 1 \Rightarrow v + 2 = u$

$\boxed{u - v = 2} \quad \ldots\left(2\right)$

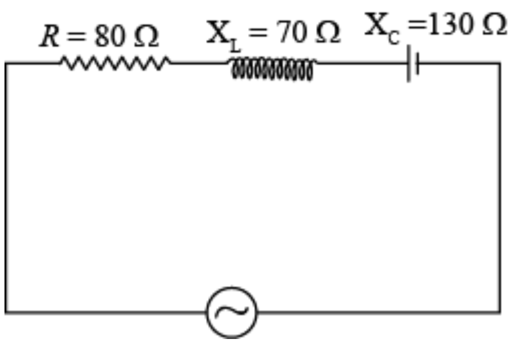
Adding (1) and (2)

$2v = 4$

$v = 2$ m/s

$\boxed{u = 4\ \mathrm{m/s}}$

Solution 29



Power factor = $\cos\phi = \frac{R}{Z} = \frac{80}{\sqrt{80^2+60^2}}$

$\frac{8}{10} = \frac{x}{10} \Rightarrow \boxed{x = 8}$

Solution 30

$\frac{u_f}{u_i} = \frac{\text{Area of final drop}}{\text{Area of initial drop}}$

$\frac{u_f}{u_i} = \frac{1000 \times 4\pi r_f^2}{4\pi r_i^2} = \frac{1000\left(r_f^2\right)}{\left(r_i^2\right)}$

$1000 \times \frac{4}{3} 4\pi r_f^3 = \frac{4}{3}\pi r_i^3$

$r_i = 10 r_f$

$\frac{u_f}{u_i} = \frac{1000 r_f^2}{100 r_f^2} = 10$

$\frac{10}{x} = 10 \Rightarrow x = 1$

Solution 31

Metallic character of an element is directly proportional to its electropositivity. Of the given elements silicon is least electro positive and potassium is most electropositive whereas beryllium and magnesium have intermediate values in

the increasing order.
Therefore, correct order of metallic character is Si < Be < Mg < K.

Hence, the correct answer is option (4).

Solution 32

Aromatic amines are less basic than aliphatic amines. Among given aliphatic amines, 2° amine is most basic, followed by 3° amine and 1° amine. Therefore the correct basic strength (K_b) order of the given amines is

$$\underset{(C)}{CH_3CH_2NHCH_2CH_3} > \underset{(D)}{(CH_3CH_2)_3N} > \underset{(B)}{CH_3CH_2NH_2} > \underset{(A)}{C_6H_5NH_2}$$

The pK_b order of the given amines will be just the opposite of their basic strength order. The correct matching is
A – III, B – IV, C – II, D – I

Hence, the correct answer is option (4).

Solution 33

Carbon monoxide is neutral and CO_2 is acidic in nature because with the increase in oxidation state of carbon, acidic strength increases. So, Assertion is correct.
CO_2 combines with water to form carbonic acid while CO is sparingly soluble in water. So, Reason is also correct and is the correct explanation of Assertion.

Hence, the correct answer is option (1).

Solution 34

The dipole moment is a vector quantity and is depicted by an arrow with tail on the positive centre and head pointing towards the negative centre. So, Statement-I is incorrect.
The crossed arrow of the dipole moment symbolizes the direction of the shift of charges in the molecules. So, Statement-II is correct.

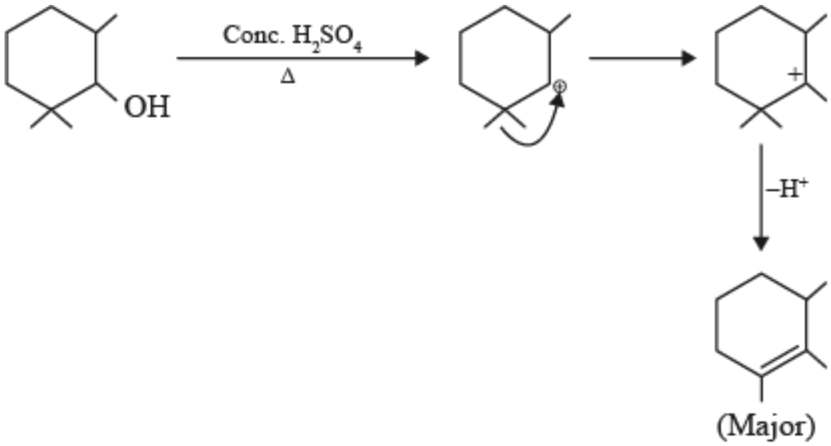
Hence, the correct answer is option (1).

Solution 35

Among the given alkali metals, sodium metal is the weakest reducing agent as its standard reduction potential $\left(E^{o}_{Na^+/Na} = -2.719\right)$ is least negative.

Hence, the correct answer is option (3).

Solution 36



Hence, the correct answer is option (2).

Solution 37

Ammonium salts produce haze in atmosphere.
Ozone is produced when atmospheric oxygen reacts with oxygen atoms and not chlorine atoms.
Polychlorinated biphenyls have number of applications including their use as cleansing solvents.
'Blue baby' syndrome occurs due to the presence of excess of nitrate ions and not sulphate ions in water.

Hence, the correct answer is option (1).

Solution 38

$$Cl^{-}\left(aq\right) + AgNO_3\left(aq\right) \xrightarrow{\text{dil } HNO_3} \underset{(A)}{AgCl\downarrow}$$

$$AgCl\downarrow + 2\,NH_4\,OH\left(aq\right) \rightarrow \underset{(B)}{[Ag(NH_3)]_2}\,Cl\left(aq\right) + 2H_2O$$

∴ (A) is AgCl and (B) is $[Ag(NH_3)_2]Cl$

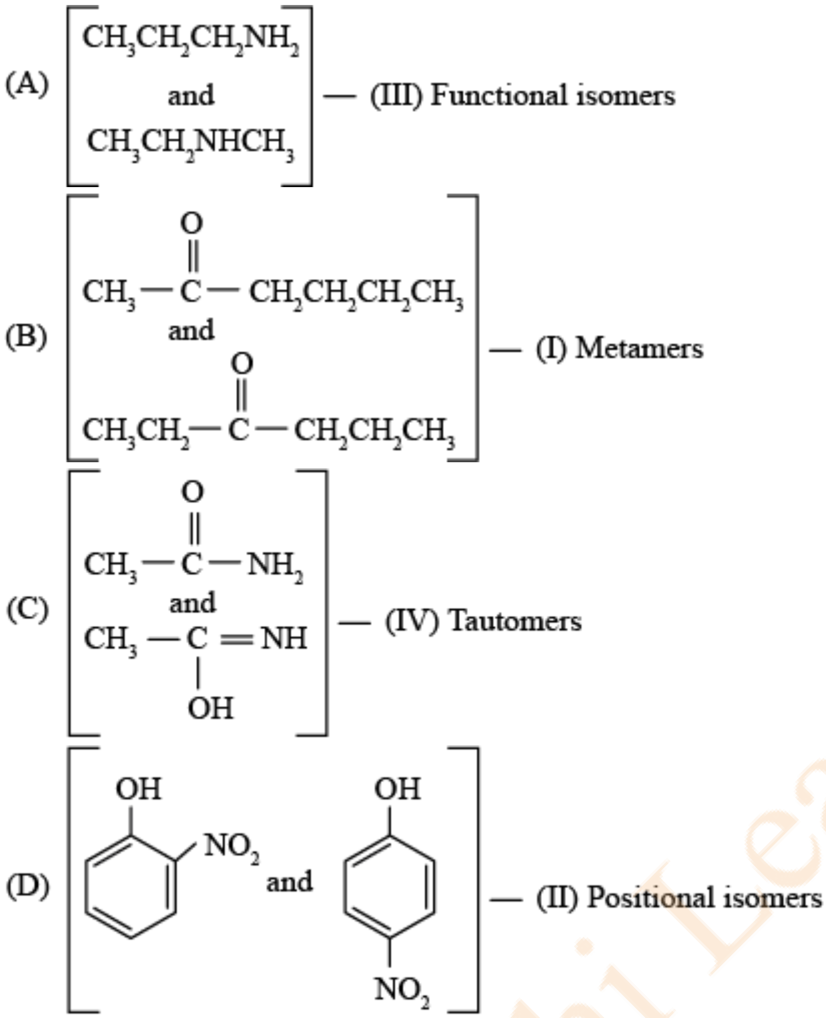
Hence, the correct answer is option (3).

Solution 39

(A) Glyptal — (III) Paints and Lacquers
(B) Neoprene — (IV) Gaskets
(C) Acrilan — (II) Synthetic wool
(D) LDP — (I) Flexible pipes

Hence, the correct answer is option (2).

Solution 40



Hence, the correct answer is option (1).

Solution 41

$K_2Cr_2O_7$ acts as a strong oxidising agent in acidic medium. During this process, oxidation state of Cr changes from +6 to +3.

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$$

Hence, the correct answer is option (4).

Solution 42

Butylated hydroxy anisole is added to butter to increase its shelf life from months to years as it is more reactive towards oxygen than food. Therefore, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

Hence, the correct answer is option (3).

Solution 43

Let the initial concentration of H^+ be 1

$\therefore [H^+]_i = 1 \Rightarrow pH = 0$

It changes by 1000 units

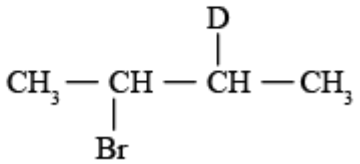
∴ $[H^+]_f = 10^3 \Rightarrow pH = -3$

∴ pH decreases by 3 units

Hence, the correct answer is option (4).

Solution 44

The isomeric deuterated bromide with molecular formula C_4H_8DBr having two chiral carbon atoms is



2-Bromo-3-deuterobutane

Hence, the correct answer is option (3).

Solution 45

Co-ordination compounds absorb a particular wavelength following certain rules.

$$\text{Wavelength of light absorbed} \propto \frac{1}{\text{Oxidation state of metal ion}}$$

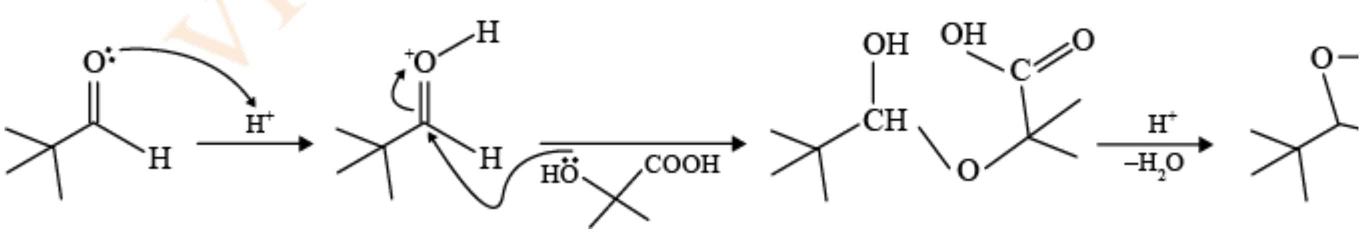
$$\text{Wavelength of light absorbed} \propto \frac{1}{\text{Strength of ligand}}$$

Ligand field strength : $CN^- > NH_3 > H_2O > Cl^-$

C.	$[Co^{III}(CN)_6]^{3-}$	I.	310
B.	$[Co^{III}(NH_3)_6]^{3+}$	II.	475
A.	$[Co^{III}Cl(NH_3)_5]^{2+}$	III.	535
D.	$[Cu^{II}(H_2O)_4]^{2+}$	IV.	600

Hence, the correct answer is option (4).

Solution 46



Hence, the correct answer is option (1).

Solution 47

Assertion is not correct because alkali metals and their salts impart characteristic colour to oxidising part of flame and not reducing part of flame. Reason is correct because all alkali metals can be detected by their flame tests.

Hence, the correct answer is option (3).

Solution 48

(A) Hydrogen reacts with carbon monoxide in presence of cobalt catalyst to give methanol

$$2H_2\left(g\right)+CO\left(g\right)\xrightarrow{\text{cobalt catalyst}}CH_3OH\left(l\right)$$

(B) Syn gas is produced from coal and the process is called coal gasification.

$$C\left(s\right)+H_2O\left(g\right)\xrightarrow{1270K}\underset{\text{(syn gas)}}{CO(g)+H_2(g)}$$

(C) Reaction of steam with hydrocarbons or coke at high temperature in presence of nickel catalyst gives a mixture of CO and H_2, called water gas

$$CH_4\left(g\right)+H_2O\left(g\right)\underset{Ni}{\xrightarrow{1270K}}\underset{\text{(water gas)}}{CO(g)+3H_2(g)}$$

(D) Electrolysis of brine solution produces H_2 gas at cathode and Cl_2 gas at anode

$NaCl(aq) \rightarrow Na^+(aq) + Cl^-(aq)$

Cathode: $2H_2O(l) + 2e^- \rightarrow H_2(g) + 2OH^-(aq)$

Anode: $2Cl^-(aq) \rightarrow Cl_2(g) + 2e^-$

Hence, the correct answer is option (4).

Solution 49

Statement I is false because the rotating paddle in froth floatation method agitates the mixture to generate froth and not to drive air out of it. Statement II is true because iron is commercially extracted from haematite ore and not from iron pyrites to minimize environmental pollution.

Hence, the correct answer is option (1).

Solution 50

Molality of aq. ethylene glycol solution = 0.25 m

Mass of ethylene glycol required for 1000 g water$=\frac{62}{4}=15.5\ \text{gm}$

Mass of solution = 1015.5 gm

Mass of ethylene glycol in 500 gm solution $=\frac{15.5\times500}{1015.5}=7.63\ \text{gm}$

Assuming density of solution as 1 gm/mL.

Mass of ethylenc glycol in 250 mL $=\frac{7.63}{2}=3.815\ \text{gm}$

∴ Mass ratio of ethylene glycol for making 500 gm of 0.25 m solution and 250 mL of 0.25 m solution = 2 : 1

Hence, the correct answer is option (1).

Solution 51

$\underset{1-\alpha}{A\ A} \longrightarrow \text{Products}$

$k = 4.6 \times 10^{-3} s^{-1}$

$kt = \ln \frac{1}{1-\alpha}$

$\alpha = 1 - e^{-kt}$

Reaction completes at infinite time

$\text{Half} - \text{life} = \frac{0.693}{4.6 \times 10^{-3}} = 150.65\ s$

$t_{10\%} = \frac{2.303}{k} \log \frac{100}{90} = \frac{2.303 \times 0.04}{k}$

$t_{90\%} = \frac{2.303}{k} \log \frac{100}{10} = \frac{2.303}{k}$

$t_{10\%} = 0.04 \times t_{90\%}$

Units of rate and rate constant are different

∴ Number of correct statements = 1

Solution 52

The correct statements are:

(A) Water vapours are adsorbed by anhydrous calcium chloride

(D) Adsorption is accompanied by decrease in entropy of the system.

The number of incorrect statements from the following are **2**.

Solution 53

Molar mass of a hydrocarbon (A) = 84 g/mol

Mass of carbon in 1 mol of (A) $= \frac{85.8}{100} \times 84 = 72\ \text{gm}$

Mass of hydrogen in 1 mol of (A) = 12 gm

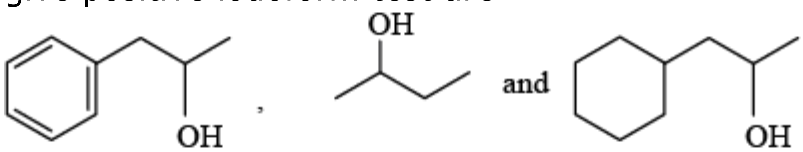
∴ Number of H-atoms in a molecule of (A) = 12.

Solution 54

The orbitals having electron density along the axis are p_x, p_y, p_z, $d_{x^2-y^2}$ and d_{z^2}.

Solution 55

The compounds which give red colour with ceric ammonium nitrate and also give positive iodoform test are



Solution 56

The following pairs of solutions have same value of osmotic pressure

(A) 0.500 M C_2H_5OH (aq) i = 1 and 0.25 M KBr(aq) i = 2
(B) 0.100 M $K_4[Fe(CN)_6]$ (aq) i = 5 and 0.100 M $FeSO_4(NH_4)_2SO_4$ (aq) i = 5
(D) 0.15 M NaCl (aq) i = 2 and 0.10 M $BaCl_2$ (aq) i = 3
(E) 0.02 M $KCl.MgCl_2.6H_2O$ (aq) i = 5 and 0.05 M KCl (aq) i = 2

Solution 57

$$\underset{(16.8-x)}{CH_4(g)} + 2O_2\left(g\right) \longrightarrow \underset{(16.8-x)}{CO_2(g)} + 2H_2O\left(l\right)$$

$$\underset{x}{C_2H_4(g)} + 3O_2\left(g\right) \longrightarrow \underset{(2x)}{2\,CO_2(g)} + 2H_2O\left(l\right)$$

$$16.8 + x = 28 \qquad \Rightarrow x = 11.2\ L$$

No. of moles of CH_4 = 0.25 and that of C_2H_4 = 0.50

$$|\text{Total heat envolved}| = \left|-\frac{900}{4} - \frac{1400}{2}\right| = 925\ kJ\ mol^{-1}$$

Solution 58

$Pt(s)|H_2(g)(1bar)|H^+(aq)(1M)||M^{3+}(aq), M^+(aq)|Pt(s)$

$E_{cell} = 0.1115\ V$ at $298\ K$; $E^o_{M^{3+}/M^+} = 0.2\ V$

Cell reaction is $H_2 + M^{3+} \longrightarrow 2H^+ + M^+$

$$E_{cell} = E^o_{cell} - \frac{0.059}{2}\log\frac{[H^+]^2[M^+]}{[M^{3+}]}$$

$$0.1115 = 0.2 - \frac{0.059}{2}\log 10^a$$

$$a = 3$$

Solution 59

$$\left[Co\left(NH_3\right)_4 Cl_2\right] Cl \xrightarrow{AgNO_3} AgCl$$

$$\left[Ni\left(H_2O\right)_6\right] Cl_2 \xrightarrow{AgNO_3} 2\,AgCl$$

$$\left[Pd\left(NH_3\right)_4\right] Cl_2 \xrightarrow{AgNO_3} 2\,AgCl$$

Total moles of AgCl precipitated = 5

Solution 60

The correct statements are
(B) Surface tension is due to uneven forces acting on the molecules present on the surface
(D) The molecules on the surface are responsible for vapour pressure if the

system is a closed system

Solution 61

$$\because \ \mathrm{A} - \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$$

$$\therefore \ \mathrm{A} \cdot \mathrm{A}^{\mathrm{T}} = \mathrm{A}^{\mathrm{T}} \cdot \mathrm{A} = \mathrm{I}$$

$$\therefore \ \mathrm{AM}^{2023}\,\mathrm{A}^{\mathrm{T}} = \mathrm{B}^{2023}$$

$$= \begin{bmatrix} 1 & -2023\mathrm{i} \\ 0 & 1 \end{bmatrix}$$

Hence, the correct answer is option (2).

Solution 62

$$\text{also} \ \left|\hat{b}\right| = 1 \qquad\qquad \left(\text{by Lami's Theorem}\right)$$

$$\Rightarrow b_1 = \frac{-2}{3},\ b_2 = \frac{-2}{3} \text{and } b_3 = \frac{1}{3}$$

$$\Rightarrow -6\hat{b} = 4\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\Rightarrow \widehat{a} - 6\hat{b} = 3\left(\hat{i} + \hat{j} + \hat{k}\right)$$

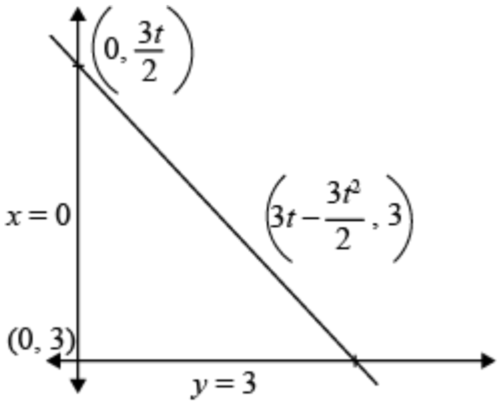
Solution 63

$(p \to q)\ \Delta\ (p\ \nabla\ q)$
$\Rightarrow (p' \vee q)\ \Delta\ (p\ \nabla\ q)$ .....(i)
If $\Delta = \vee$, $\nabla = \vee$
(i) becomes $(p' \vee q) \vee (p \vee q) = \mathrm{T}$

Hence, the correct answer is option (2).

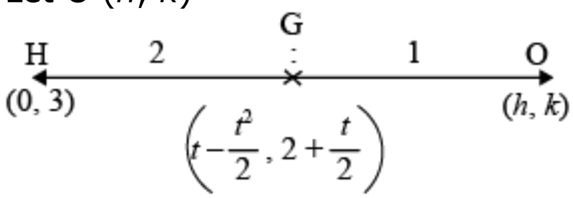
Solution 64

Third side of triangle
$ty = x + \frac{3}{2}t^2$



$\therefore\ \mathrm{H} = \left(0,\ 3\right)\quad \mathrm{G} = \left(t - \frac{t^2}{2},\ 2 + \frac{t}{2}\right)$

Let O (h, k)



$$\Rightarrow \frac{2h}{3} = t - \frac{t^2}{2} \text{ and } \frac{2k+3}{3} = 2 + \frac{t}{2}$$

$$\Rightarrow 4h = 6t - 3t^2 \text{ and } 4k = 6 + 3t$$

$$\Rightarrow 4h = 2\left(4k - 6\right) - 3\left(\frac{(4k-6)^2}{9}\right)$$

$$\Rightarrow 3h = 6k - 9 - \left(4k^2 + 9 - 12k\right)$$

$$\Rightarrow 4k^2 - 18k + 3h + 18 = 0$$

$$\Rightarrow 4y^2 - 18y + 3x + 18 = 0$$

Hence, the correct answer is option (3).

Solution 65

$$L_1 :\ \frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}}$$

$$L_2 :\ \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$$

$$\mathrm{S.D} = \left|\frac{\left(-\hat{i}+2\hat{j}-\hat{k}\right)\cdot\left(2\hat{i}-3\hat{j}+6\hat{k}\right)}{7}\right|$$

$$= \left|\frac{-2-6-6}{7}\right| = 2 \text{ units}$$

Hence, the correct answer is option (1).

Solution 66

$$\lim_{x\to\frac{\pi^-}{2}} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}} = e^{\lambda}$$

$$\lim_{x\to\frac{\pi^+}{2}} e^{\frac{\cot 6x}{\cot+4x}} = \lim_{x\to\frac{\pi^+}{2}} e^{\frac{\tan 4x}{\tan 6x}}$$

$$= e^{\frac{2}{3}}$$

$\lambda = \frac{2}{3},\ \mu = e^{\frac{2}{3}}$

$9\lambda + 6\ln\mu + \mu^6 - e^{6\lambda}$

$= 6 + 4 + e^4 - e^4$

$= 10$

Hence, the correct answer is option (3).

Solution 67

$\overrightarrow{AB} = -2\hat{i} + 6\hat{j} - 3\hat{k}$

$\overrightarrow{AC} = -5\hat{i} + 3\hat{j} + \hat{k}$

$\overrightarrow{AD} = 2\hat{i} + \left(4 - 2\alpha\right)\hat{j} + 2\hat{k}$

$$\begin{vmatrix} -2 & 6 & -3 \\ -5 & 3 & 1 \\ 2 & 4-2\alpha & 2 \end{vmatrix} = 0$$

$\Rightarrow 14b - 34\alpha = 0$

Or $\alpha = \frac{73}{17}$

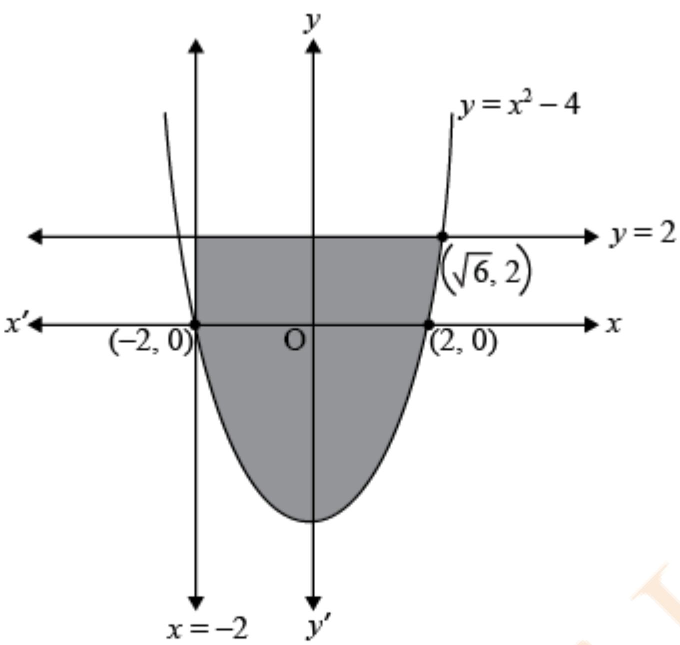
Hence, the correct answer is option (3).

Solution 68

$16(x+2)^2-(y-2)^2=16$

$\frac{(x+2)^2}{1}-\frac{(y-2)^2}{16}=1$

TA : $y = 2$
CA : $x = -2$



$$A=\left|\int_{-2}^{\sqrt{6}}\left(2-\left(x^2-4\right)\right)dx\right|$$

$$=6x-\frac{x^3}{3}\Big|_{-2}^{\sqrt{6}}$$

$$=\left(6\sqrt{6}-\frac{6\sqrt{6}}{3}\right)-\left(-12+\frac{8}{3}\right)$$

$$=\frac{12\sqrt{6}}{3}+\frac{28}{3}$$

Hence, the correct answer is option (1).

Solution 69

$\underline{5}\ \_\_\_\ \_\_\_\ \_\_\_ \Rightarrow {}^4C_3\cdot 3!=24$ ways

$\underline{7}\ \_\_\_\ \_\_\_\ \_\_\_ \Rightarrow {}^4C_3\cdot 3!=24$ ways

$9\ \_\_\_\ \_\_\_\ \_\_\_ \Rightarrow {}^4C_3\cdot 3!=24$ ways

Total ways = 72

Hence, the correct answer is option (3).

Solution 70

$\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$

$\mathrm{I.F.} = e^{\int \alpha dt} = e^{\alpha t}$

$\Rightarrow y \cdot e^{\alpha t} = \gamma \int e^{(\alpha-\beta)t} dt = \gamma \frac{e^{(\alpha-\beta)t}}{(\alpha-\beta)} + \mathrm{C}$

$\Rightarrow \mathrm{y} = \frac{\gamma}{(\alpha-\beta)} e^{-\beta t} + \mathrm{C} e^{-\alpha t}$

$\lim_{\mathrm{x}\to\infty} y\left(t\right) = \lim_{\mathrm{x}\to\infty} \left[\frac{\gamma}{(\alpha-\beta)} e^{-\beta t} + \mathrm{C} e^{-\alpha t}\right] = 0$

Hence, the correct answer is option (2).

Solution 71

$\sum_{k=0}^{6} {}^{51-k}\mathrm{C}_3 = {}^{51}\mathrm{C}_3 + {}^{50}\mathrm{C}_3 + {}^{49}\mathrm{C}_3 + {}^{48}\mathrm{C}_3 + {}^{47}\mathrm{C}_3 + {}^{46}\mathrm{C}_3 + \left({}^{45}\mathrm{C}_3 + {}^{45}\mathrm{C}_4\right) - {}^{45}\mathrm{C}$

$\mathrm{S} = {}^{51}\mathrm{C}_3 + {}^{50}\mathrm{C}_3 + {}^{49}\mathrm{C}_3 + {}^{48}\mathrm{C}_3 + {}^{47}\mathrm{C}_3 + \left({}^{46}\mathrm{C}_3 + {}^{46}\mathrm{C}_4\right) - {}^{45}\mathrm{C}_4$

$\Rightarrow \mathrm{S} = {}^{52}\mathrm{C}_4 - {}^{45}\mathrm{C}_4$

Hence, the correct answer is option (2).

Solution 72

$A^T = A$, $B^T = -B$, $C^T = -C$
$P = A^{13}B^{26} - B^{26}A^{13}$
$P^T = (A^{13}B^{26} - B^{26}A^{13})^T = (A^{13}B^{26})^T - (B^{26}A^{B})^T$
$= (B^{26})^T (A^{13})^T - (A^{13})^T - (A^{13})^T (B^{26})^T$
$= (B^T)^{26} (A^T)^{13} - (A^T)^{13}(A^T)^{26}$
$= B^{26}A^{13} - A^{13}B^{26} = -(A^{13}B^{26} - B^{26}A^{13}) = -P$
P is skew-symmetric matrix ⇒ S_1 is false
$Q = A^{26}C^{13} - C^{13}A^{26} = Q^T = (A^{26}C^{13} - C^{13}A^{26})^T$
$Q = (A^{26}C^{13})^T - (C^{13}A^{26})^T = (C^{13})^T(A^{26})^T - (A^{26})^T(C^{13})^T$
$= (C^T)^{13}(A^T)^{26} - (A^T)^{26}(C^T)^{13} = -C^{13}A^{26} + A^{26}C^{13}$
$= A^{26}C^{13} + C^{13}A^{26}$
⇒ $Q^T = Q$ ⇒ Q is symmetric matrix ⇒ S_2 is true.

Hence, the correct answer is option (2).

Solution 73

$f(x) = 2x^n + \lambda, \lambda \in R, n \in N$
$f(4) = 2 \cdot 4^n + \lambda = 133, f(5) = 2.5n + \lambda = 255$
$f(5) = f(4) = 2 \cdot (5^n \cdot 4^n) = 122 \Rightarrow n = 3$
$\Rightarrow f(3) - f(2) = 2 \cdot (3^n \cdot 2^n) = 2 \cdot (3^3 - 2^3) = 2 \times 19$
Required sum = 1 + 2 + 19 + 38 = 60

Hence, the correct answer is option (3).

Solution 74

$$\because f : \{1, 2, 3, 4\} \longrightarrow \{a \in \mathbb{Z} : |9| \le 8\}$$

and $f(n) + \frac{1}{n} f(n+1) = 1$

$$\Rightarrow nf(n) + f(n+1) = n \qquad \ldots(\text{i})$$

$$\therefore f(1) + f(2) = 1 \Rightarrow f(2) = 1 - f(1)$$

But $f(1) \in [-8, 8]$

Hence, $f(2) \in [-8, 8] \Rightarrow f(1) \in [-7, 8] \quad \ldots(\text{A})$

and $2f(2) + f(3) = 2 \Rightarrow f(3) = 2f(1)$

$$\therefore 2f(1) \in [-8, 8] \Rightarrow f(1) \in [-4, 4] \qquad \ldots(\text{B})$$

and $3f(3) + f(4) = 3 \Rightarrow f(4) = 3 - 6f(1)$

$$\therefore f(1) \in \left[-\frac{5}{6}, \frac{11}{6}\right] \qquad \ldots(\text{C})$$

From (A), (B) and (C) : $f(1) = 0$ or 1
∴ Only two functions are possible.

Hence, the correct answer is option (2).

Solution 75

$$f'(x) = 6x^2 + 2x(2p - 7) + 3(2p - 9)$$

$$x_1 < 0, \ x_2 > 0$$

$$\Rightarrow f'(0) < 0$$

$$\Rightarrow p < \frac{9}{2}$$

Hence, the correct answer is option (4).

Solution 76

$\text{I} = \int \frac{dx}{x^3(x^2+2)^2}$

$= \frac{1}{4}\int \frac{x}{x^2+2}dx + \frac{1}{4}\int \frac{x}{(x^2+2)^2} - \frac{1}{4}\int \frac{dx}{x} + \frac{1}{4}\int \frac{dx}{x^3}$

$= \frac{1}{8}\ln\left(x^2+2\right) - \frac{\ln x}{4} - \frac{1}{8(x^2+2)} - \frac{1}{8x^3}$

$\text{Now, } 16\int_1^2 \frac{dx}{x^3(x^2+2)^2} = 2\ln 6 - 2\ln 3 - 4\ln 2 + \frac{11}{6}$

$= \frac{11}{6} - \ln 4$

Hence, the correct answer is option (2).

Solution 77

We know that $\sin x - \cos x \in \left[-\sqrt{2}, \sqrt{2}\right]$

$\log_{\sqrt{M}}\left(\sqrt{2}\,(\sin x - \text{co}\,x) + M - 2\right) \in \left[\log_{\sqrt{M}}(M-4),\ \log_{\sqrt{M}} M\right]$

$\Rightarrow \log_{\sqrt{M}}(M-4) = 0 \Rightarrow M = 5$

Hence, the correct answer is option (4).

Solution 78

$n-2,\ \sqrt{3n},\ n+2\ \rightarrow \text{G.P.}$

$3n = n^2 - 4$

$\Rightarrow n^2 - 3n - 4 = 0$

$\Rightarrow n = 4, -1\left(\text{rejected}\right)$

$\text{P}\left(\text{S} = 4\right) = \frac{3}{36} = \frac{1}{12} = \frac{4}{48}$

$\therefore\ k = 4$

Hence, the correct answer is option (3).

Solution 79

$\left|\frac{z-2i}{z+i}\right| = 2,$

$\Rightarrow (z-2i)(\bar{z}+2i) = 4(z+i)(\bar{z}-i)$

$\Rightarrow 2\bar{z} + 2iz - 2i\bar{z} + 4 = 4\left(z\bar{z} - zi + \bar{z}i + 1\right)$

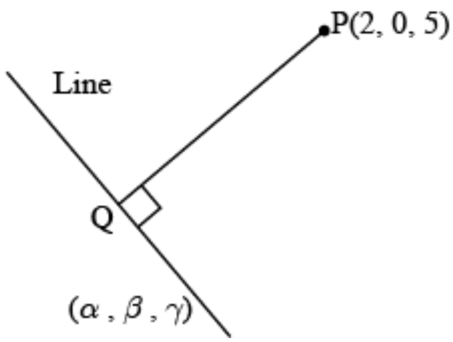
$\Rightarrow 3z\bar{z} - 6iz + 6i\bar{z} = 0$

$\Rightarrow 2\bar{z} - 2iz + 2i\bar{z} = 0$

$\therefore$ Centre $\left(-2i\right)$ or $\left(0, -2\right)$

Hence, the correct answer is option (4).

Solution 80



$\mathrm{L}: \frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} = t$

then,

$\alpha = 2t - 1$

$\beta = 5t + 1$

$\gamma = -t - 1$

$\text{for foot of } \perp^r \; 2\left(2t-3\right) + 5\left(5t+1\right) - \left(-t-6\right) = 0$

$\Rightarrow 30t + 5 = 0$

$\therefore\ t = \frac{1}{6}$

$\therefore\ \alpha = \frac{-1}{3} - 1 = \frac{-4}{3},\ \beta = \frac{1}{6},\ \gamma = \frac{-5}{6}$

So, $\frac{\beta}{\gamma} = \frac{\frac{1}{6}}{-\frac{5}{6}} = \frac{-1}{5}$

Hence, the correct answer is option (2).

Solution 81

$x^2 + 60^{\frac{1}{4}}x + a = 0$

$\therefore\ \alpha + \beta = -60^{\frac{1}{4}},\ \alpha\beta = a$

Now $\left(\alpha^2 + \beta^2\right)^2 - 2a^2 = -30$

$\Rightarrow \left[\left(\alpha + \beta\right)^2 - 2a\right]^2 - 2a^2 = -30$

$\Rightarrow \left(60^{\frac{1}{2}} - 2a\right)^2 - 2a^2 = -30$

$\Rightarrow 60 + 4a^2 - 4.60^{\frac{1}{2}}a - 2a^2 + 30 = 0$

$\Rightarrow 2a^2 - 8\sqrt{15a} + 90 = 0$

Product of value of a = 45

Solution 82

$\because\ a,\ b,\ \frac{1}{18} \rightarrow \text{G.P.}$

$\therefore\ b^2 = \frac{a}{18} \qquad \ldots\left(1\right)$

And $\frac{1}{a},\ 10,\ \frac{1}{b}\ \rightarrow\ \text{A.P.}$

$\therefore\ 20 = \frac{1}{\text{a}} + \frac{1}{\text{b}}$

$20ab\ =\ a\ +\ b$

By $\left(1\right)\ a\ = 18b^2$

$\therefore\ 20\ \times 18b^2 = 18b^2 + b$

$\because\ a,\ b\ > 0$

$360b^2 - 18b - 1 = 0$

$\Rightarrow 360b^2 - 30b + 12b - 1 = 0$

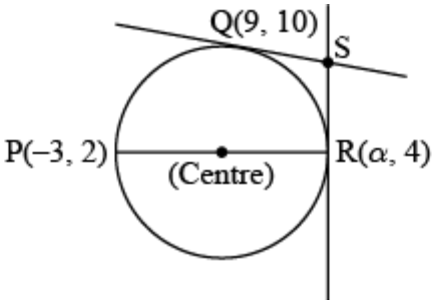
$\Rightarrow 30b\left(12b - 1\right) + 1\left(12b - 1\right) = 0$

$b = \frac{1}{12},\ b = \frac{-1}{30}x$

$\therefore\ 12b\ =\ 1,\ a\ =\ 18 \times \frac{1}{144} = \frac{2}{16}$

$\therefore\ 16a + 12b = 3$

Solution 83



Now, $\frac{10-2}{9+3} \times \frac{10-4}{9-\alpha} = -1$

$\Rightarrow \frac{8}{12}.6 = \alpha - 9 \Rightarrow \boxed{\alpha = 13}$

$\therefore\ 0 = \left(5,\ 3\right)$ So $\begin{matrix} m_{OQ} = \frac{7}{4} \\ m_{OR} = \frac{1}{8} \end{matrix}$

$\therefore\ Q\ :\ y\ -\ 10 = \frac{-4}{7}(x-9)$

$\Rightarrow 4x + 7y = 106$ .....(i)

Tangent at R : $y - 4\ =\ -8\,(x - 13)$

$8x\ +\ y\ =\ 108$ .....(ii)

By (i) and (ii) $S = \left(\frac{25}{2},\ 8\right)$, satisfies with the line

$\therefore\ \boxed{K = 3}$

Solution 84

Points $\left(1,\ 2,\ 3\right)$ and $\left(2,\ 3,\ 4\right)$

$L_1 : \frac{(x-1)}{1} = \frac{(y-2)}{1} = \frac{(2-3)}{1}$

$L_2 : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$

$\overrightarrow{b}_1 = \hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{b}_2 = 2\hat{i} - \hat{j} + 0\hat{k}$

$\overrightarrow{a}_1 - \overrightarrow{a}_2 = 0\hat{i} - 3\hat{j} - \hat{k}$

$d = \left|\frac{(\bar{a}_1-\bar{a}_2)\cdot(n_1\times n_2)}{|n_1\times n_2|}\right|$

$= \left|\frac{6-3}{\sqrt{9+1+4}}\right| = \frac{3}{\sqrt{14}} = \alpha$

$28\alpha^2 = \frac{28\times 9}{14} = 18$

Solution 85

Total 8 oranges, 5 white apple and 7 red apple.
5 fruits needs to be selected.
Case I: 3 orange + 1 red apple + 1white apple
= ${}^{8}C_{3} \times {}^{7}C_{1} \times {}^{5}C_{1} = 1960$
Case II: 2 oranges + 2 red apples + 1 white apple.
= ${}^{8}C_{2} \times {}^{7}C_{2} \times {}^{5}C_{1} = 2940$
Case III: 2 oranges + 1 red apples + 2 white apple.
= ${}^{8}C_{2} \times {}^{7}C_{1} \times {}^{5}C_{2} = 1960$
Total = 1960 + 2940 + 1960
= 6860

Solution 86

Probability of a person being smoker = $\frac{1}{4}$

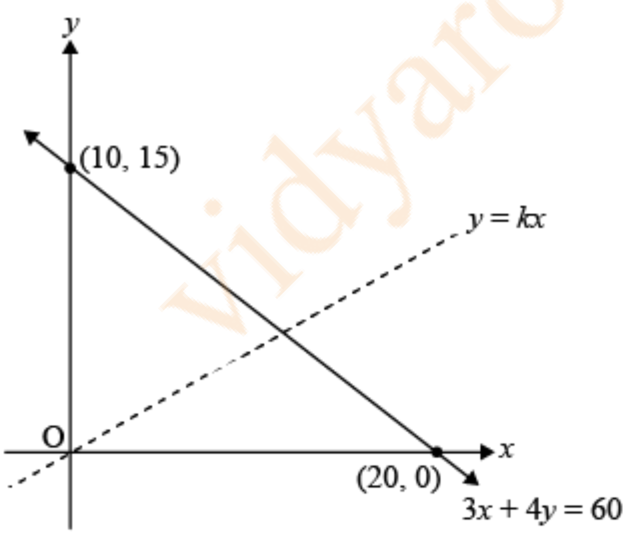
Probability of a person being non-smoker = $\frac{3}{4}$

$$P\left(\frac{\text{Person is smoker}}{\text{Person diagonsed with cancer}}\right) = \frac{\frac{1}{4}\cdot 27P}{\frac{1}{4}\cdot 27P + \frac{3P}{4}}$$

$$= \frac{9}{10} = \frac{k}{10}$$

$\Rightarrow \boxed{k = 9}$

Solution 87



As b is multiple of a the required point lie on
Line $y = kx$ ($k \in$ z)
$\therefore 3x + 4kx = 60$
$x = \frac{60}{3+4x}$
If $k = 1$ 8 integral points
$k = 2$ 5 integral points

$k = 3$ 3 integral points
$k = 4$ 3 integral points
$k = 5$ 2 integral points
$k = 6$ 2 integral points
$k = 7$ 1 integral points
$k = 8$ 1 integral points
: :
$k = 14$ 1 integral points

∴ Total 31 points

Solution 88

Let N = 2023

2023 is divisible by 7

∴ 2023^{2023} is divisible by 7

∴ Let N = 7α

$N = 2023^{2023} \equiv 3^{2023}(\text{mod } 5)$

$\equiv 3^3(\text{mod } 5) \equiv 2(\text{mod } 5)$

∴ N = 5β + 2

⇒ 7α = 5β + 2

7α = 5β + 7 − 5

7 (α − 1) = 5(β − 1)

α − 1 is divisible by 5

α = 5p + 1

N = 7α = 7(5p + 1) = 35p + 7

Solution 89

$2\cos 2\theta \cos\frac{\theta}{2} = 2\cos 3\theta \cos\frac{9\theta}{2}$

$\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}$

$\cos\frac{5\theta}{2} = \cos\frac{15\theta}{2}$

$\frac{15\theta}{2} = 2n\pi \pm \frac{5\theta}{2}$

$\frac{15\theta}{2} \pm \frac{5\theta}{2} = 2n\pi$

$10\theta = 2n\pi$ or $5\theta = 2n\pi$

$\theta = \frac{n\pi}{5}$ or $\theta = \frac{2n\pi}{5}$

$\Rightarrow \theta = \frac{n\pi}{5}$

$\theta = \pm\pi, \ \pm\frac{4\pi}{5}, \ \pm\frac{3\pi}{5}, \ \pm\frac{2\pi}{5}, \ \pm\frac{\pi}{5}$

$m = 5, \quad n = 5$

$mn = 25$

Solution 90

$$I=\int_{\frac{1}{3}}^{3}|\ln\ x|dx$$

$$=-\int_{\frac{1}{3}}^{1}\ln\ xdx+\int_{1}^{3}\ln\ \text{x}dx$$

$$=-[x\ \ln\ x-x]_{\frac{1}{3}}^{1}+x\ \ln\ x-x\Big]_{1}^{3}$$

$$=\ -\left[(0-1)-\left(\frac{1}{3}\ \text{In}\ 3-\frac{1}{3}\right)\right]+[(3\ \ln\ 3-3)-(0-1)]$$

$$=\frac{2}{3}-\frac{1}{3}\ \ln\ 3\ +\ 3\ \ln\ 3-2$$

$$=\frac{8}{3}\ln\ 3\ -\frac{4}{3}$$

$$=\frac{4}{3}\left(2\ \ln\ 3\ -\ \ln\ e\right)$$

$$=\frac{4}{3}\ \ln\ \left(\frac{3^2}{e}\right)$$

$m=4,\ n\ =\ 3$

$m^2+n^2=20$