Board Paper of Class 10 Maths (Standard) Term-I 2021 Delhi(Set 4) - Solutions

Total Time: 90

Total Marks: 40.0

Section A

Solution 1

3750 = 5 × 5 × 5 × 5 × 3 × 2
= $5^4 \times 3^1 \times 2^1$
The exponent of 5 in the prime factorization of 3750 is 4.

Hence, the correct answer is option (b).

Solution 2

A polynomial with degree *n* has *n* zeroes and intersects the *x*-axis at atmost *n* points.
The graph of polynomial P(*x*) cuts the *x*-axis at 3 + 2 = 5 points therefore, it has 5 zeroes.

Hence, the correct answer is option (d).

Solution 3

Given equations are:

$$32x + 33y = 34 \qquad .....(1)$$
$$33x + 32y = 31 \qquad .....(2)$$

Subtracting (1) from (2), we get

$$x - y = -3$$
$$\Rightarrow y = x + 3 \qquad .....(3)$$

Substituting (3) in (1) we get,

$$32x + 33(x+3) = 34$$
$$\Rightarrow 32x + 33x + 99 = 34$$
$$\Rightarrow 65x = -65$$
$$\Rightarrow x = -1 \qquad .....(4)$$

Substituting (4) in (3) we get,
$$\Rightarrow y = -1 + 3$$
$$\Rightarrow y = 2$$

Hence, the correct answer is option (a).

Solution 4

The triangle ABC is an equilateral triangle.

Now, the distance between two points P(x_1, y_1) and P(x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Then,
AB = BC

$$\Rightarrow \sqrt{(0-3)^2 + \left(0-\sqrt{3}\right)^2} = \sqrt{(3-0)^2 + (k-0)^2}$$
$$\Rightarrow \sqrt{9+3} = \sqrt{9+k^2}$$
$$\Rightarrow k^2 + 9 = 12$$
$$\Rightarrow k^2 = 3$$
$$\Rightarrow k = \pm\sqrt{3}$$

Hence, the correct answer is option (c).

Solution 5

In ΔADE and ΔABC, since DE ∥ BC therefore,
∠ADE = ∠ABC (Corresponding angles)
∠AED = ∠ACB (Corresponding angles)
∠DAE = ∠BAC (Common)

Therefore, ΔADE ~ ΔABC by AA similarity criterion.

Then,
The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
Therefore,

$$\frac{\text{ar}(\Delta\,\text{ABC})}{\text{ar}(\Delta\,\text{ADE})}=\left(\frac{\text{AB}}{\text{AD}}\right)^2$$
$$\Rightarrow \frac{\text{ar}(\Delta\,\text{ABC})}{\text{ar}(\Delta\,\text{ADE})}=\frac{5^2}{2^2}=\frac{25}{4}$$

⇒ ar (ΔABC) : ar (ΔADE) = 25 : 4

Hence, the correct answer is option (d).

Solution 6

$$\cot\theta=\frac{1}{\sqrt{3}}$$
$$\Rightarrow \cot\theta=\cot\ 60^\circ$$
$$\Rightarrow \theta=60^\circ$$

So,
$$\begin{aligned}\sec^2\theta+\text{cosec}^2\,\theta&=(\sec 60^\circ)^2+(\text{cosec}\,60^\circ)^2\\&=2^2+\left(\frac{2}{\sqrt{3}}\right)^2\\&=4+\frac{4}{3}\\&=\frac{16}{3}\\&=5\frac{1}{3}\end{aligned}$$

Hence, the correct answer is option (d).

Solution 7

Let the radius of the circle be *r*.
Circumference of the circle$=2\pi r$
$$\Rightarrow 2\pi r=176$$
$$\Rightarrow r=\frac{176\times 7}{2\times 22}=28\ \text{m}$$

So, area of the quadrant = $\frac{1}{4}\pi r^2$

= $\frac{1}{4}\times\frac{22}{7}\times 28\times 28$

= 616 m^2

Hence, the correct answer is option (c).

Solution 8

Given that for an event *E*, $\mathrm{P}(\mathrm{E}) + \mathrm{P}(\overline{\mathrm{E}}) = x$.

The sum of the probabilities of an event and its complement is 1.
$\Rightarrow \mathrm{P}(\mathrm{E}) + \mathrm{P}(\overline{\mathrm{E}}) = 1$

Thus, $x = 1$.
$\Rightarrow x^3 - 3 = (1)^3 - 3$
$= 1 - 3$
$= -2$

Hence, the correct answer is option (a).

Solution 9

The greatest possible speed at which the girl can walk in an exact number of minutes is the highest common factor of both the distances.

Now,
95 = 5 × 19
171 = 3 × 3 × 19
⇒ HCF(95, 171) = 19

Thus, the greatest possible speed at which the girl can walk in an exact number of minutes is 19 m/min.

Hence, the correct answer is option (b).

Solution 10

Since the polynomial P(*x*) cuts the *x*-axis at three distinct points.
Therefore, the polynomial P(*x*) has three zeroes.

Hence, the correct answer is option (c).

Solution 11

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel if
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Here, $a_1 = 3,\ b_1 = -2$ and $c_1 = -5$.

(a) $9x + 8y - 7 = 0$
So, $a_2 = 9,\ b_2 = 8$ and $c_2 = -7$.
$\Rightarrow \frac{3}{9} \neq \frac{-2}{8} \neq \frac{-5}{-7}$
Therefore, these lines are not parallel.

(b) $-12x - 8y - 7 = 0$
So, $a_2 = -12,\ b_2 = -8$ and $c_2 = -7$.

$\Rightarrow \frac{3}{-12} \neq \frac{-2}{-8} \neq \frac{-5}{-7}$
Therefore, these lines are not parallel.

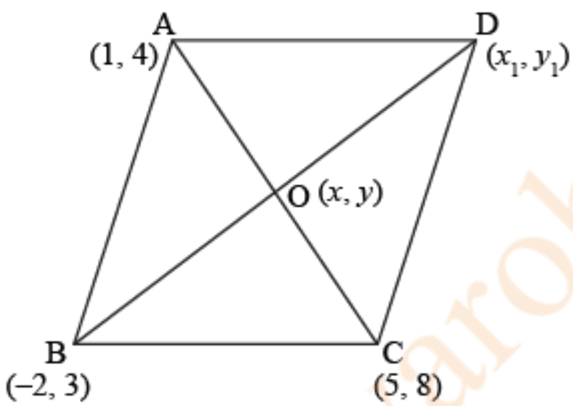
(c) $-12x + 8y - 7 = 0$
So, $a_2 = -12,\ b_2 = 8$ and $c_2 = -7.$
$\Rightarrow \frac{3}{-12} = \frac{-2}{8} \neq \frac{-5}{-7}$
Therefore, these lines are parallel.

(d) $12x + 8y - 7 = 0$
So, $a_2 = 12,\ b_2 = 8$ and $c_2 = -7.$
$\Rightarrow \frac{3}{12} \neq \frac{-2}{8} \neq \frac{-5}{-7}$
Therefore, these lines are not parallel.

Hence, the correct answer is option (c).

Solution 12

Let the centre of the parallelogram be O(*x*, *y*) and the fourth vertex D have the coordinates $(x_1,\ y_1)$.



The diagonals of a parallelogram bisects each other. Thus, the centre of the parallelogram divides the diagonals in the ratio 1 : 1.

Using the midpoint formula,
$(x,\ y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Consider the points A(1, 4) and C(5, 8).
$\Rightarrow (x,\ y) = \left(\frac{1+5}{2}, \frac{4+8}{2}\right)$
$\Rightarrow (x,\ y) = (3,\ 6)$

Now, consider the points B(−2, 3) and D$(x_1,\ y_1)$.

$\Rightarrow (3,\ 6) = \left(\frac{-2+x_1}{2}, \frac{3+y_1}{2}\right)$

$\Rightarrow 3 = \frac{-2+x_1}{2},\ 6 = \frac{3+y_1}{2}$

$\Rightarrow x_1 = 6 + 2 = 8$ and $y_1 = 12 - 3 = 9$

Thus, the ordinate of the fourth vertex D is 9.

Hence, the correct answer is option (b).

Solution 13

Given that in ΔABC and ΔDEF, ∠F = ∠C, ∠B = ∠E and $\mathrm{AB} = \frac{1}{2}\mathrm{DE}$.

Since $\mathrm{AB} = \frac{1}{2}\mathrm{DE}$. Therefore, ΔABC is not congruent to ΔDEF.

Now, ∠F = ∠C and ∠B = ∠E. Therefore, the two triangles are similar by AA criterion.

Hence, the correct answer is option (b).

Solution 14

Consider a right angled triangle ΔABC.

Given that $\sin\mathrm{A} = \frac{7}{25}$.

Now, $\sin\mathrm{A} = \frac{7}{25} = \frac{\mathrm{BC}}{\mathrm{AC}}$.

So, let BC = 7*k* and AC = 25*k* where *k* is any natural number.

Now, $\cos\mathrm{C} = \frac{\mathrm{BC}}{\mathrm{AC}}$.

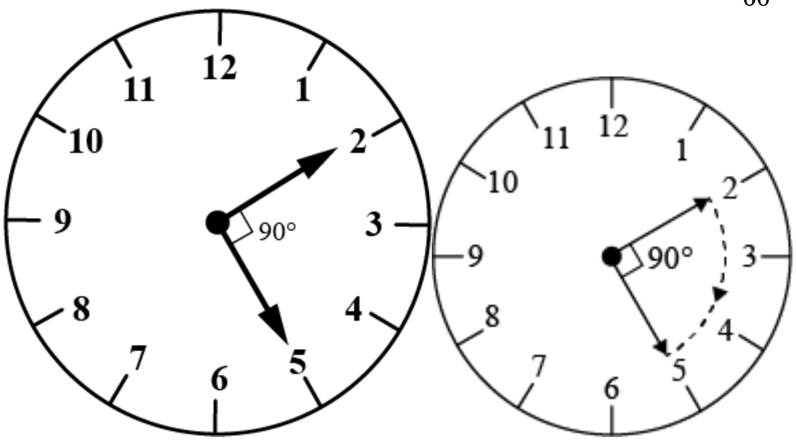
$\Rightarrow \cos\mathrm{C} = \frac{7k}{25k} = \frac{7}{25}$

$\Rightarrow \cos\mathrm{C} = \frac{7}{25}$

Hence, the correct answer is option (a).

Solution 15

10 : 00 am to 10 : 25 am = 15 minutes
In 1 hour (i.e., 60 minutes), the minute hand rotates 360°.
In 15 minutes, the minute hand will rotate = $\frac{360^\circ}{60^\circ} \times 15 = 90^\circ$



Therefore, the distance covered by the tip of the minute hand in 15 minutes will be the length of the arc of a sector with angle 90° in a circle of radius 84 cm.
Length of the arc of the sector with angle θ = $\frac{\theta}{360^\circ} \times 2\pi \mathrm{r}$
Length of the arc of the sector of 90°

$$= \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 84$$
$$= \frac{1}{4} \times 44 \times 12$$
$$= 44 \times 3$$
$$= 132 \text{ cm}$$

Therefore, the distance covered by the tip of the minute hand from 10 : 00 am to 10 : 25 am is 132 cm.

Hence, the correct answer is option (c).

Solution 16

Let E be the event for the drawn card is neither an ace nor a spade.
The total number of possible outcomes = 52
Number of spade = 13
Number of aces other than the ace of spade = 4 − 1 = 3
The number of possible outcomes to *E* = 52 − 13 − 3 = 36

$$P(E) = \frac{\text{Number of favourable outcomes to } E}{\text{Total number of possible outcomes}}$$
$$= \frac{36}{52}$$
$$= \frac{9}{13}$$

Thus, the probability that the card is drawn from a pack of 52 cards is neither an ace nor a spade is $\frac{9}{13}$.
Hence, the correct answer is option (a).

Solution 17

Given: Three alarm clocks first beep together at 12 noon.
As the three-alarm clocks ring their alarms at regular intervals of 20 min, 25 min, and 30 min.
They will ring together after LCM of 20, 25, and 30.
Now,
$20 = 2 \times 2 \times 5$
$25 = 5 \times 5$
$30 = 2 \times 3 \times 5$

$\therefore$ LCM of 20, 25, and 30 $= 2 \times 2 \times 3 \times 5 \times 5 = 300$
Thus, they will beep again for the first time after 300 minutes,i.e., 5 hours.
($\because$ 60 min = 1 h)

Therefore, three-alarm clocks first beep together at 12 noon, then at 5 : 00 pm they will beep again for the first time.

Hence, the correct answer is option (c).

Solution 18

Given: Sum of zeroes = 8 and product of zeroes = 5

If α and β are the zeroes of a quadratic equation, then $k[x^2 - (\alpha + \beta)x + (\alpha\beta)]$ is the required quadratic equation where k is any constant.

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

Here, $\alpha + \beta = 8$ and $\alpha\beta = 5$

$\therefore$ The required quadratic equation is $k[x^2 - 8x + 5]$

Hence, the correct answer is option (a).

Solution 19

Given: A$(-1, y)$ and B(5, 7) lie on the circle with centre O$(2, -3y)$

Thus, OA and OB are the radii of the same circle.

Here, $x_1 = 2$, $y_1 = -3y$, $x_2 = -1$, $y_2 = y$ and $x_3 = 5$, $y_3 = 7$

$\therefore$ OA = OB

$$\Rightarrow \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{(x_1-x_3)^2+(y_1-y_3)^2} \qquad \Big[\text{Using dis}$$

$$\Rightarrow \sqrt{(2+1)^2+(-3y-y)^2}=\sqrt{(2-5)^2+(-3y-7)^2}$$

$$\Rightarrow (3)^2+(-4y)^2=(-3)^2+(-3y-7)^2$$

$$\Rightarrow 9+16y^2=9+9y^2+49+42y$$

$$\Rightarrow 16y^2-9y^2-49-42y=0$$

$$\Rightarrow 7y^2-42y-49=0$$

$$\Rightarrow y^2-6y-7=0$$

$$\Rightarrow y^2-7y+y-7=0$$

$$\Rightarrow y\,(y-7)+1\,(y-7)=0$$

$$\Rightarrow (y-7)\,(y+1)=0$$

$$\Rightarrow (y-7)=0 \text{ or } (y+1)=0$$

$$\Rightarrow y=7 \text{ or } y=-1$$

Thus, the values of y are 7 and −1.

Hence, the correct answer is option (b).

Solution 20

$$\text{Given : } \sec\theta=\sqrt{2}$$

$$\cos\theta=\frac{1}{\sqrt{2}} \qquad \left(\because \cos\theta=\frac{1}{\sec\theta}\right)$$

$$\because \sin^2\theta=1-\cos^2\theta$$

$$\Rightarrow \sin^2\theta=1-\left(\frac{1}{\sqrt{2}}\right)^2$$

$$\Rightarrow \sin^2\theta=1-\frac{1}{2}$$

$$\Rightarrow \sin^2\theta=\frac{1}{2}$$

$$\Rightarrow \sin\theta=\frac{1}{\sqrt{2}}$$

$$\begin{aligned}\frac{1+\tan\theta}{\sin\theta} &= \frac{1}{\sin\theta} + \frac{\tan\theta}{\sin\theta} \\ &= \frac{1}{\sin\theta} + \frac{\frac{\sin\theta}{\cos\theta}}{\sin\theta} \qquad \left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right) \\ &= \frac{1}{\sin\theta} + \frac{1}{\cos\theta} \\ &= \sqrt{2} + \sqrt{2} \\ &= 2\sqrt{2}\end{aligned}$$

Hence, the correct answer is option (a).

Section B

Solution 21

Let the required number be *x*.
According to the question, when the numbers 1251, 9377 and 15628 are divided by *x* leaves the remainder 1, 2 and 3 respectively.

$$\begin{aligned}\therefore x &= \text{HCF}\,(1251-1,\ 9377-2,\ 15628-3) \\ &= \text{HCF}\,(1250,\ 9375,\ 15625) \\ &= \text{HCF}\,(2\times 625,\ 15\times 625,\ 25\times 625) \\ &= 625\end{aligned}$$

Hence, the correct answer is option (d).

Solution 22

The probability of an event is always greater than or equal to zero and less than or equal to 1 i.e. $0 \le P(E) \le 1$.

(a) $0 < 0.01 < 1$
(b) $0 < 0.03 < 1$ $\left(\because 3\% = \frac{3}{100}\right)$
(c) $0 < \frac{16}{17} < 1$
(d) $\frac{17}{16} > 1$
Thus, $\frac{17}{16}$ cannot be the probability of an event.

Hence, the correct answer is option (d).

Solution 23

Distance covered in one revolution is equal to the circumference of the wheel.
Let the total number of revolutions to cover the distance of 132 km be *x*.

$\therefore$ Distance covered in one revolution $\times x = 132$ km
$\Rightarrow 2\pi r x = 132 \times 10^5$ cm
$\Rightarrow \left(2 \times \frac{22}{7} \times \frac{42}{2} x\right)$ cm $= 132 \times 10^5$ cm
$\Rightarrow 132x = 132 \times 10^5$
$\Rightarrow x = 10^5$

Hence, the correct answer is option (b).

Solution 24

$\tan\ \theta + \cot\ \theta = 2$
$\Rightarrow \tan\ \theta + \frac{1}{\tan\ \theta} = 2$
$\Rightarrow \tan^2\theta + 1 = 2\tan\ \theta$
$\Rightarrow \tan^2\theta - 2\tan\ \theta + 1 = 0$
$\Rightarrow (\tan\ \theta - 1)^2 = 0$
$\Rightarrow \tan\ \theta = 1$
$\Rightarrow \theta = \frac{\pi}{4}$

Substitute $\theta = \frac{\pi}{4}$ in $\sin^3\theta + \cos^3\theta$.

$$\begin{aligned}\sin^3 45^\circ + \cos^3 45^\circ &= \left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3 \\ &= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

Hence, the correct answer is option (c).

Solution 25

If a line divides the line segment joining the points $(x_1,\ y_1)$ and $(x_2,\ y_2)$ in the ratio *m* : *n*, then the point of division is given by $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$.

Let the given line divide the line segment joining the points (1, 3) and (2, 7) in the ratio *k* : 1.

Coordinates of point of division: $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

Substitute $x = \frac{2k+1}{k+1}$ and $y = \frac{7k+3}{k+1}$ in the line $3x + y - 9 = 0$.

$3\left(\frac{2k+1}{k+1}\right) + \left(\frac{7k+3}{k+1}\right) - 9 = 0$

$\Rightarrow 3\left(2k+1\right) + 7k + 3 - 9\left(k+1\right) = 0$

$\Rightarrow 6k + 3 + 7k + 3 - 9k - 9 = 0$

$\Rightarrow 4k - 3 = 0$

$\Rightarrow k = \frac{3}{4}$

Thus, the given line divides the line segment joining the points (1, 3) and (2, 7) in the ratio 3 : 4.

Hence, the correct answer is option (c).

Solution 26

$a + b = 4 \qquad .....(1)$

By factor theorem, if $x - a$ is a factor of $p\left(x\right)$, then $p\left(a\right) = 0$.

Since $x - 1$ is a factor of $p\left(x\right)$,

$p\left(1\right) = 0$

$\Rightarrow 1^3 + a \times 1^2 + 2b = 0$

$\Rightarrow 1 + a + 2b = 0$

$\Rightarrow a + 2b = -1 \qquad .....(2)$

Subtract (1) from (2) to solve the pair of linear equations by elimination method.

$a + 2b - a - b = -1 - 4$

$\Rightarrow b = -5$

$\therefore a - 5 = 4 \qquad [\text{From } (1)]$

$\Rightarrow a = 4 + 5$

$\Rightarrow a = 9$

Hence, the correct answer is option (b)

Solution 27

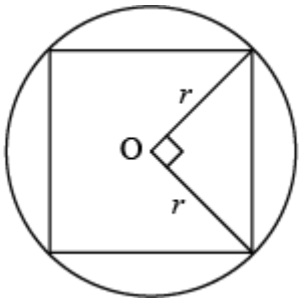
If *a* and *b* are two coprime numbers, then HCF(*a*, *b*) = 1.
This means that there is no factor common to *a* and *b* other than 1.

Thus, for any real value of k, HCF of a^k and b^k is also 1.
$\therefore$ HCF(a^3, b^3) = 1
This implies a^3 and b^3 are coprime numbers.

Hence, the correct answer is option (a).

Solution 28

Given that, area of circle = $\frac{1408}{7}$ cm^2



Now, area of circle = πr^2
$\Rightarrow \frac{22}{7} \times r^2 = \frac{1408}{7}$
$\Rightarrow 22 \times r^2 = 1408$
$\Rightarrow r^2 = 64$
$\Rightarrow r = 8\ \text{cm}$

$\text{Side of square} = r\sqrt{2} = 8\sqrt{2}\ \text{cm}$ (Using pythagoras theorem)

$\therefore \text{Area of square} = (\text{side})^2$
$= 8\sqrt{2} \times 8\sqrt{2}$
$= 128\ \text{cm}^2$

Thus, the area of the square is 128 cm^2.

Hence, the correct answer is option (c).

Solution 29

Given that, vertices of $\triangle$ ABC are $A(4,-2),\ B(7,-2),\ C(7,9)$
We know, distance between points (x_1, y_1) *and* (x_2, y_2) is
$\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$\therefore\ AB = \sqrt{(-2+2)^2 + (7-4)^2} = 3$

$BC = \sqrt{(9+2)^2 + (7-7)^2} = 11$

$CA = \sqrt{(-2-9)^2 + (4-7)^2} = \sqrt{130}$

$$\text{Now, } (AB)^2 + (BC)^2 = (3)^2 + (11)^2$$
$$= 130$$

Since, $(AB)^2 + (BC)^2 = (CA)^2$
Therefore, $\triangle \mathrm{ABC}$ is a right angled triangle.

Hence, the correct answer is option (c)

Solution 30

Given that, α and β are the zeros of p(x) = $x^2 - (k+6)x + 2(2k-1)$.
Now, using the relations of zeros of polynomial
$\alpha + \beta = k + 6$

$\alpha\beta = 2(2k-1)$

Since, $(\alpha + \beta) = \frac{1}{2}(\alpha\beta)$ (Given)

$\therefore\ k + 6 = \frac{1}{2} \times 2 \times (2k-1)$

$\Rightarrow k + 6 = 2k - 1$

$\Rightarrow 2k - k = 6 + 1$

$\Rightarrow k = 7$

Thus, the value of k is 7.

Hence, the correct answer is option (b).

Solution 31

Since n is a natural number, therefore values of $2(5^n + 6^n)$ at

$n = 1 \Rightarrow 2(5+6) = 22$

$n = 2 \Rightarrow 2(5^2 + 6^2) = 122$

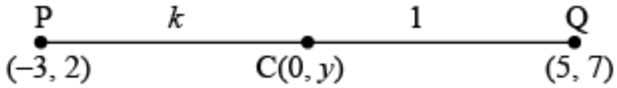
$n = 3 \Rightarrow 2(5^3 + 6^3) = 682$

Thus, $2(5^n + 6^n)$ always ends with 2.

Hence, the correct answer is option (d).

Solution 32

Let C(0, y) be any point on the y-axis, which divides line segment PQ in the ratio k : 1.



$\therefore\ (0,y)=\left(\frac{5k-3}{k+1},\frac{7k+2}{k+1}\right)$ (∵ By division formula)

$\Rightarrow \frac{5k-3}{k+1}=0$

$\Rightarrow 5k-3=0$

$\Rightarrow k=\frac{3}{5}$

$\Rightarrow k:1=3:5$

Thus, the y-axis divides the line segment joining the points P(−3, 2) and Q(5, 7) in the ratio 3 : 5.

Hence, the correct answer is option (d)

Solution 33

Given that, $p=acot\theta+b\cos ec\theta$ and $q=b\text{cot}\theta+a\text{cosec}\theta$

Now,

$$p^2-q^2=(a\cot\theta+b\,\text{cosec}\,\theta)^2-(b\text{cot}\theta+a\text{cosec}\theta)^2$$
$$=\left(a^2\cot^2\theta+b^2\,\text{cosec}^2\,\theta+2ab\cot\theta\,\text{cosec}\,\theta\right)-\left(b^2\cot^2\theta+a^2\,\text{cosec}^2\,\theta+2a\right.$$
$$=\left(a^2-b^2\right)\cot^2\theta+\left(b^2-a^2\right)\text{cosec}^2\,\theta+2ab\cot\theta\,\text{cosec}\,\theta-2ab\cot\theta\,\text{cosec}$$
$$=\left(b^2-a^2\right)\left(\text{cosec}^2\,\theta-\cot^2\theta\right)$$
$$=\left(b^2-a^2\right)\qquad\left(\because\ \text{cosec}^2\,\theta-\cot^2\theta=1\right)$$

Hence, the correct answer is option (b).

Solution 34

Given that,

Perimeter of circle = $\frac{1}{2}$ × (Perimeter of a square)

Now,

Perimeter of circle = 2πr

Perimeter of square = 4 $\times$ side

$\therefore\ 2\pi r=\frac{1}{2}\times(4\times\text{side})$

$\Rightarrow \pi r=\text{side}$

$\Rightarrow \frac{r}{\text{side}}=\frac{1}{\pi}$

$$\therefore \ \frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{(\text{side})^2}$$

$$= \pi\left(\frac{r}{\text{side}}\right)^2$$

$$= \pi \times \left(\frac{1}{\pi}\right)^2$$

$$= \frac{1}{\pi}$$

$$= \frac{7}{22}$$

$$= 7 : 22$$

Hence, the correct answer is option (d).

Solution 35

Total outcomes when a dice is rolled twice = 6 × 6 = 36

Outcomes when 5 will come at least one time are (1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 5).

Thus, the number of outcomes when 5 will come at least one time = 11

So, the number of outcomes that 5 will not come either time = 36 − 11 = 25

$$P(E) = \frac{\text{Number of favourable outcomes to } E}{\text{Total number of possible outcomes}}$$

$$= \frac{25}{36}$$

Thus, the probability that 5 will not come either time = $\frac{25}{36}$.

Hence, the correct answer is option (d).

Solution 36

Given that the LCM of two numbers is 2400.
Since HCF of two numbers is always the factor of LCM.

300 is the factor of 2400.
So, 300 can be their HCF.

400 is the factor of 2400.
So, 400 can be their HCF.

500 is not the factor of 2400.
So, 500 cannot be their HCF.

600 is the factor of 2400.
So, 600 can be their HCF.

Hence, the correct answer is option (c).

Solution 37

Given: PA, QB, and RC are each perpendicular to AC.
PA is perpendicular to AC ⇒ ∠PAB = 90º
QB is perpendicular to AC ⇒ ∠QBA = ∠QBC = 90º
RC is perpendicular to AC ⇒ ∠RCB = 90º

Since, ∠PAB + ∠QBA = 90º + 90º
= 180º (Cointerior angles)
⇒ PA∥QB

Similarly, ∠QBC + ∠RCB = 180º
⇒ RC∥QB

In ΔPAC, PA∥QB
By Basic Proportionality Theorem,

$$\frac{\mathrm{QB}}{\mathrm{PA}}=\frac{\mathrm{BC}}{\mathrm{AC}}$$

$$\Rightarrow \frac{y}{x}=\frac{\mathrm{BC}}{\mathrm{AC}} \qquad \ldots\ldots(1)$$

In ΔRAC, RC∥QB
By Basic Proportionality Theorem,

$$\frac{\mathrm{QB}}{\mathrm{RC}}=\frac{\mathrm{AB}}{\mathrm{AC}}$$

$$\Rightarrow \frac{y}{z}=\frac{\mathrm{AB}}{\mathrm{AC}} \qquad \ldots\ldots(2)$$

Adding (1) and (2), we get

$$\frac{y}{x}+\frac{y}{z}=\frac{\mathrm{BC}}{\mathrm{AC}}+\frac{\mathrm{AB}}{\mathrm{AC}}$$

$$\Rightarrow y\left(\frac{1}{x}+\frac{1}{z}\right)=\frac{\mathrm{BC}+\mathrm{AB}}{\mathrm{AC}}$$

$$\Rightarrow \frac{1}{x}+\frac{1}{z}=\frac{1}{y}\times\frac{\mathrm{AC}}{\mathrm{AC}}$$

$$\Rightarrow \frac{1}{y}=\frac{1}{x}+\frac{1}{z}$$

$$\Rightarrow \frac{1}{y}=\frac{1}{8}+\frac{1}{6} \qquad [\because x=8 \text{ cm and } y=6 \text{ cm}]$$

$$\Rightarrow \frac{1}{y}=\frac{3+4}{24}=\frac{7}{24}$$

$$\Rightarrow y=\frac{24}{7} \text{ cm}$$

Hence, the correct answer is option (d).

Solution 38

In ΔABC, ∠A = x°, ∠B = $(3x − 2)$° and ∠C = y°.
∴ ∠A + ∠B + ∠C = 180° (By angle sum property)

$$\Rightarrow x^{\circ}+(3x-2)^{\circ}+y^{\circ}=180^{\circ}$$

$$\Rightarrow 4x^{\circ}+y^{\circ}=182^{\circ}\ \ .....(1)$$

Also, ∠C − ∠B = 9°

$$\Rightarrow y^{\circ}-(3x-2)^{\circ}=9^{\circ}$$

$$\Rightarrow y^{\circ}=3x^{\circ}+7^{\circ}\ .....(2)$$

From (1) and (2), we get

$$4x^{\circ}+3x^{\circ}+7^{\circ}=182^{\circ}$$

$$\Rightarrow 7x^{\circ}=175^{\circ}$$

$$\Rightarrow x^{\circ}=25^{\circ}$$

And, $y^{\circ}=3\times 25^{\circ}+7^{\circ}=82^{\circ}$.

Thus, ∠A = 25°, ∠B = (3 × 25 − 2)° = 73° and ∠C = 82°.

Therefore, the sum of the greatest and the smallest angles of the triangle is 82° + 25° = 107°.

Hence, the correct answer is option (a).

Solution 39

$$\sec\theta+\tan\theta=p$$

$$\Rightarrow \sec\theta=p-\tan\theta$$

squaring on both sides, we get

$$\Rightarrow (\sec\theta)^2=(p-\tan\theta)^2$$

$$\Rightarrow \sec^2\theta=p^2+\tan^2\theta-2p\tan\theta$$

$$\Rightarrow 1+\tan^2\theta=p^2+\tan^2\theta-2p\tan\theta \qquad \left(\because \sec^2\theta=1+\tan^2\theta\right)$$

$$\Rightarrow 1-p^2=-2p\tan\theta$$

$$\Rightarrow \tan\theta=\frac{p^2-1}{2p}$$

Hence, the correct answer is option (b).

Solution 40

Given that coordinates of C are (0, −3).
Since the origin is the mid-point of BC.
So, the coordinates of B are (0, 3).
Let coordinates of A be (*x*, *y*).
The distance between two points (x_1, y_1) and (x_2, y_2) is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$\therefore \mathrm{BC} = \sqrt{(0-0)^2 + (3+3)^2} = 6 \text{ units}$$

Since ΔABC is an equilateral triangle.
∴ AB = BC = CA = 6 units

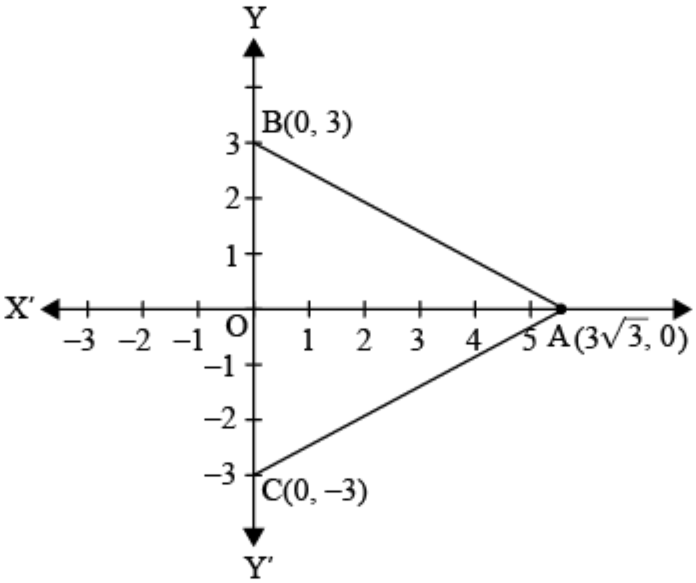
$$\Rightarrow \mathrm{AB} = \sqrt{(x-0)^2 + (y-3)^2}$$
$$\Rightarrow 6^2 = x^2 + y^2 - 6y + 9$$
$$\Rightarrow x^2 + y^2 = 6y + 27 \qquad .....(1)$$
$$\text{And, } \mathrm{AC} = \sqrt{(x-0)^2 + (y+3)^2}$$
$$\Rightarrow 6^2 = x^2 + y^2 + 6y + 9$$
$$\Rightarrow x^2 + y^2 = -6y + 27 \qquad .....(2)$$

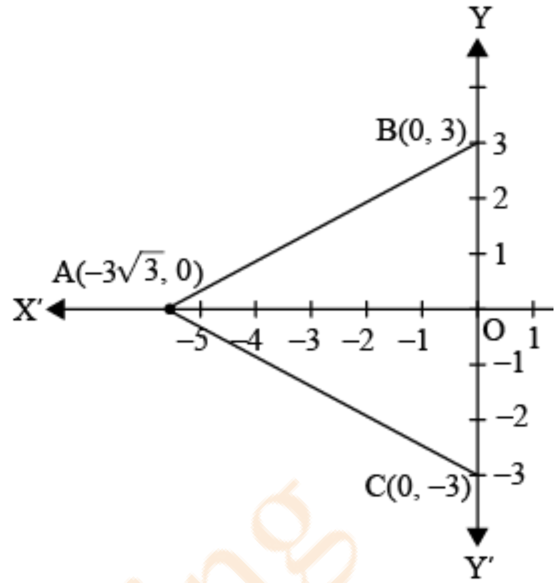
From (1) and (2), we get
$6y + 27 = -6y + 27$
$$\Rightarrow 12y = 0$$
$$\Rightarrow y = 0$$

Substituting the value of *y* in (1), we get
$x^2 = 27$
$$\Rightarrow x = \pm\sqrt{27}$$
$$\Rightarrow x = \pm 3\sqrt{3}$$

Therefore, the coordinates of A are $\left(\pm 3\sqrt{3},\ 0\right)$.

Or





Hence, the correct answer is option (c).

Section C

Solution 41

Given that, the fixed charge be ₹x and the additional charge(per day) be ₹y.

Radhika paid ₹16 for keeping the book for 4 days.

So, Radhika pays a fixed charge for 2 days and an additional charge for 2 days.

Therefore, Radhika pays ₹x for 2 days and ₹$2y$ for 2 days.

$\therefore x + 2y = 16$

Hence, the correct answer is option (d).

Solution 42

Given that, the fixed charge be ₹x and the additional charge(per day) be ₹y.

Amruta paid ₹22 for keeping the book for 6 days.

So, Amruta pays a fixed charge for 2 days and an additional charge for 4 days.

Therefore, Amruta pays ₹x for 2 days and ₹$4y$ for 4 days.

$\therefore x + 4y = 22$

Hence, the correct answer is option (c).

Solution 43

The given situation can be represented algebraically as shown below:

Amruta: $x + 4y = 22$ .....(1)

Radhika: $x + 2y = 16$
$\Rightarrow 2x + 4y = 32$ .....(2)

Where x is the fixed charge and y is the additional charge per day.

Subtracting (1) from (2), we get

$2x + 4y - (x + 4y) = 32 - 22$

$\Rightarrow 2x + 4y - x - 4y = 32 - 22$

$\Rightarrow x = 10$

Thus, the fixed charge for a book is ₹10.

Hence, the correct answer is option (b).

Solution 44

The given situation can be represented algebraically as shown below:

Amruta: $x + 4y = 22$ .....(1)

Radhika: $x + 2y = 16$ .....(2)

Where x is the fixed charge and y is the additional charge.

Subtracting (2) from (1), we get

$x + 4y - x - 2y = 22 - 16$

$\Rightarrow 2y = 6$

$\Rightarrow y = 3$

Thus, the additional charge for each subsequent day for a book is ₹3.

Hence, the correct answer is option (d).

Solution 45

If both of them have kept the book for 2 more days, then they need to pay an additional charge for 2 days i.e., ₹3 ×2 = ₹6.

So, Amruta paid ₹22 + ₹6 i.e., ₹28 and Radhika paid ₹16 + ₹6 i.e., ₹22.

Thus, the total amount paid by both, if both of them have kept the book for 2 more days is ₹28 + ₹22 i.e., ₹50.

Hence, the correct answer is option (c).

Solution 46

In $\triangle$AOB and $\triangle$COD,

$\angle$AOB = $\angle$COD (Vertically opposite angles)

$\angle$BAO = $\angle$DCO (Alternate interior angles)

So, by AA similarity $\triangle$AOB and $\triangle$COD are similar.

Hence, the correct answer is option (a).

Solution 47

$\triangle$AOB and $\triangle$COD are similar by AA criterion.

$$\therefore \frac{\mathrm{AO}}{\mathrm{CO}} = \frac{\mathrm{OB}}{\mathrm{OD}} = \frac{\mathrm{AB}}{\mathrm{CD}} = \frac{5}{10}$$
$$\Rightarrow \frac{\mathrm{AO}}{\mathrm{CO}} = \frac{\mathrm{OB}}{\mathrm{OD}} = \frac{\mathrm{AB}}{\mathrm{CD}} = \frac{1}{2}$$

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\therefore \frac{\mathrm{ar}(\triangle \mathrm{AOB})}{\mathrm{ar}(\triangle \mathrm{COD})} = \left(\frac{\mathrm{AB}}{\mathrm{CD}}\right)^2$$
$$\Rightarrow \frac{\mathrm{ar}(\triangle \mathrm{AOB})}{\mathrm{ar}(\triangle \mathrm{COD})} = \left(\frac{1}{2}\right)^2$$
$$\Rightarrow \frac{\mathrm{ar}(\triangle \mathrm{AOB})}{\mathrm{ar}(\triangle \mathrm{COD})} = \frac{1}{4}$$
$$\Rightarrow \mathrm{ar}\,(\triangle\ \mathrm{AOB}) : \mathrm{ar}\,(\triangle\ \mathrm{COD}) = 1 : 4$$

Hence, the correct answer is option (b).

Solution 48

The corresponding sides of two similar triangles are in the same ratio.

Thus, the ratio of the perimeters of two similar triangles is equal to the ratio of their corresponding sides.

$$\therefore \frac{\text{Perimeter of } \triangle \mathrm{AOB}}{\text{Perimeter of } \triangle \mathrm{COD}} = \frac{\mathrm{AB}}{\mathrm{CD}}$$
$$\Rightarrow \frac{1}{4} = \frac{\mathrm{AB}}{\mathrm{CD}}$$
$$\Rightarrow \mathrm{CD} = 4\,\mathrm{AB}$$

Hence, the correct answer is option (d).

Solution 49

In $\triangle$AOD and $\triangle$BOC, $\frac{\mathrm{AO}}{\mathrm{BC}} = \frac{\mathrm{AD}}{\mathrm{BO}} = \frac{\mathrm{OD}}{\mathrm{OC}}$.

So, $\triangle$AOD ~ $\triangle$BCO.

Hence, the correct answer is option (b).

Solution 50

Given: ar($\triangle$AOB) : ar($\triangle$COD) = 1 : 4

The ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$\therefore \left(\text{ratio of correspoding altitudes}\right)^2 = \frac{1}{4}$

$\Rightarrow \left(\text{ratio of correspoding altitudes}\right) = \sqrt{\frac{1}{4}}$

$\Rightarrow \left(\text{ratio of correspoding altitudes}\right) = \frac{1}{2}$

Hence, the correct answer is option (b).