NDA I 2016_Mathematics

Total Time: 150

Total Marks: 300.0

Solution 1

Cube roots of unity lie on the unit circle $|z|$ = 1 and divide its circumference into three equal parts.
Therefore, the angle between each cube root of unity is $\frac{360^{\circ}}{3} = 120^{\circ}$.
Hence, the correct answer is option C.

Solution 2

$x \leq x^2$ for all positive values of x. Hence, * is reflexive.
For x = 1, y = 2
clearly, $x \leq y^2$ but y is not less than or equal to x^2. Hence, * is not symmetric.
For x = 5, y = 3 and z = 2
clearly, $x \leq y^2$ and $y \leq z^2$ are true but $x \leq z^2$ is not true. Hence * is not transitive.
Hence, the correct answer is option A.

Solution 3

$x^2 - px + 4 > 0$ implies the discriminant is negative.

$\Rightarrow \left(-p\right)^2 - 4.1.4 < 0$

$\Rightarrow p^2 < 4^2$

$\Rightarrow \left|p\right| < 4$

Hence, the correct answer is option A.

Solution 4

Given,

$z = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{-25}$

or $z = \left(\cos\ \frac{\pi}{4} - i\ \sin\ \frac{\pi}{4}\right)^{-25}$

or $z = \left(\cos\ \frac{-25\pi}{4} - i\ \sin\ \frac{-25\pi}{4}\right)$

or $z = \left(\cos\ \frac{25\pi}{4} + i\ \sin\ \frac{25\pi}{4}\right)$

or $z = \left(\cos\ \left(6\pi + \frac{\pi}{4}\right) + i\ \sin\ \left(6\pi + \frac{\pi}{4}\right)\right)$

or $z = \left(\cos\ \frac{\pi}{4} + i\ \sin\ \frac{\pi}{4}\right)$

or $z = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$

$\therefore\ \frac{z-\sqrt{2}}{z-i\sqrt{2}} = \frac{\frac{-1}{\sqrt{2}}+\frac{i}{\sqrt{2}}}{\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}} = \frac{-1+i}{1-i} = -1$

Hence, the fundamental amplitude of $\frac{z-\sqrt{2}}{z-i\sqrt{2}}$ is π.

Hence, the correct answer is option A.

Solution 5

Given, $f\left(x_1\right) - f\left(x_2\right) = f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)$ ...(i)

If $f\left(x\right) = \ln\left(\frac{1-x}{1+x}\right)$, then

$f\left(x_1\right) - f\left(x_2\right) = \ln\left(\frac{1-x_1}{1+x_1}\right) - \ln\left(\frac{1-x_2}{1+x_2}\right) = \ln\left(\frac{1-x_1}{1+x_1} . \frac{1+x_2}{1-x_2}\right)$

$f\left(\frac{x_1-x_2}{1-x_1x_2}\right) = \ln\left(\frac{1-\frac{x_1-x_2}{1-x_1x_2}}{1+\frac{x_1-x_2}{1-x_1x_2}}\right) = \ln\left(\frac{1-x_1x_2-x_1+x_2}{1-x_1x_2+x_1-x_2}\right) = \ln\left(\frac{1-x_1}{1+x_1} . \frac{1+x_2}{1-x_2}\right) = f\left(x_1\right) - f\left(x_2\right)$

Hence, the correct answer is option A.

Solution 6

Given,

$y = \frac{x^2}{1+x^2}$

$y + yx^2 = x^2$

$x = \sqrt{\frac{y}{1-y}}$

For *x* to be real,

$\frac{y}{1-y} > 0$

$\Rightarrow \frac{y}{y-1} < 0$

$\Rightarrow y \in \left[0, 1\right)$

Hence, the correct answer is A.

Solution 7

Let P(*a*,0) and Q(0,*b*). If mid-point of PQ is (3,5), then

$\frac{a}{2} = 3 \Rightarrow a = 6$

$\frac{b}{2} = 5 \Rightarrow b = 10$

Now, area of triangle OPQ = $\frac{1}{2}ab$ = 30 sq units.

Hence, the correct answer is option D.

Solution 8

The radius of the circle, i.e. b, is equal to the distance of the tangent line, i.e. $y = x - \sqrt{2}$, from the centre of the circle, i.e. (0,*b*).

$\Rightarrow b = \left|\frac{0-b-\sqrt{2}}{\sqrt{1+1}}\right|$

$\Rightarrow b = \frac{\left(b+\sqrt{2}\right)}{\sqrt{2}} \quad \left[\because \text{ b is positive}\right]$

$\Rightarrow b = 2 + \sqrt{2}$

Hence, the correct answer is option A.

Solution 9

$$f(\theta) = 4(\sin^2\theta + \cos^4\theta)$$
$$\text{or } f(\theta) = 4(\cos^4\theta - \cos^2\theta + 1)$$
$$\text{or } f(\theta) = 4(\cos^2\theta(\cos^2\theta - 1) + 1)$$
$$\text{or } f(\theta) = 4(-\cos^2\theta\sin^2\theta + 1)$$
$$\text{or } f(\theta) = 4 - 4\cos^2\theta\sin^2\theta$$
$$\text{or } f(\theta) = 4 - \sin^2 2\theta$$
$$\therefore \text{ Maximum value of } f(\theta) = 4, \text{ when } \sin^2 2\theta = 0$$

Hence, the correct answer is option D.

Solution 10

$$f(\theta) = 4(\sin^2\theta + \cos^4\theta)$$
$$\text{or } f(\theta) = 4(\cos^4\theta - \cos^2\theta + 1)$$
$$\text{or } f(\theta) = 4(\cos^2\theta(\cos^2\theta - 1) + 1)$$
$$\text{or } f(\theta) = 4(-\cos^2\theta\sin^2\theta + 1)$$
$$\text{or } f(\theta) = 4 - 4\cos^2\theta\sin^2\theta$$
$$\text{or } f(\theta) = 4 - \sin^2 2\theta$$
$$\therefore \text{ Minimum value of } f(\theta) = 3, \text{ when } \sin^2 2\theta = 1$$

Hence, the correct answer is option D.

Solution 11

$$f(\theta) = 4(\sin^2\theta + \cos^4\theta)$$
$$\text{or } f(\theta) = 4(\cos^4\theta - \cos^2\theta + 1)$$
$$\text{or } f(\theta) = 4(\cos^2\theta(\cos^2\theta - 1) + 1)$$
$$\text{or } f(\theta) = 4(-\cos^2\theta\sin^2\theta + 1)$$
$$\text{or } f(\theta) = 4 - 4\cos^2\theta\sin^2\theta$$
$$\text{or } f(\theta) = 4 - \sin^2 2\theta$$
$$\text{if } f(\theta) = 2 \Rightarrow 4 - \sin^2 2\theta = 2 \Rightarrow \sin^2 2\theta = 2, \text{ which is not possible, hence no solution.}$$
$$\text{if } f(\theta) = \tfrac{7}{2} \Rightarrow 4 - \sin^2 2\theta = \tfrac{7}{2} \Rightarrow \sin^2 2\theta = \tfrac{1}{2} \Rightarrow \sin\ 2\theta = \pm\tfrac{1}{\sqrt{2}}$$
$$\Rightarrow \theta = \pm\tfrac{\pi}{8}, \pm\tfrac{3\pi}{8} \ldots \Rightarrow f(\theta) = \tfrac{7}{2} \text{ has a solution.}$$

Hence, the correct answer is option C.

Solution 12

Given: $f(x) = \begin{Bmatrix} x^2 - 1, & x \geq 0 \\ -x^2 - 1, & x < 0 \end{Bmatrix}$ and $g(x) = \begin{Bmatrix} \frac{3x}{2}, & x > 0 \\ 2x, & x \leq 0 \end{Bmatrix}$

If $x > 0$, then $x^2 - 1 = \frac{3x}{2} \Rightarrow x = 2, \ \frac{-1}{2}$

as x is positive therefore $x = 2 \Rightarrow y = 3$.

If x < 0, then $-x^2 - 1 = 2x \Rightarrow x = -1 \Rightarrow y = -2$

Therefore, the point of intersection of the curves are (2,3) and (−1, −2).
Hence, the correct answer is option C.

Solution 13

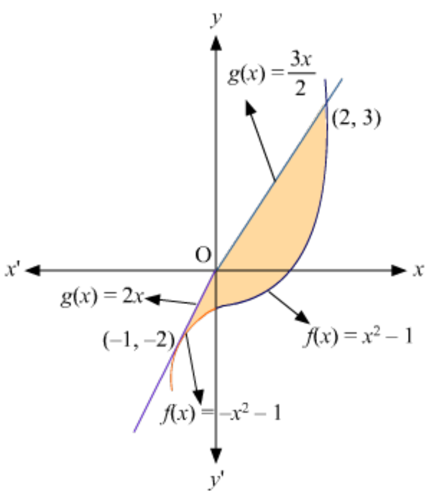
Given: $f(x)=\begin{Bmatrix} x^2-1, & x\geq 0 \\ -x^2-1, & x<0 \end{Bmatrix}$ and $g(x)=\begin{Bmatrix} \frac{3x}{2}, & x>0 \\ 2x, & x\leq 0 \end{Bmatrix}$

If $x>0$, then $x^2-1=\frac{3x}{2} \Rightarrow x=2,\ \frac{-1}{2}$
as x is positive therefore $x = 2 \Rightarrow y = 3$.

If x < 0, then $-x^2-1=2x \Rightarrow x=-1 \Rightarrow y=-2$

Therefore, the point of intersection of the curves are (2,3) and (−1, −2).

The curves are shown below:



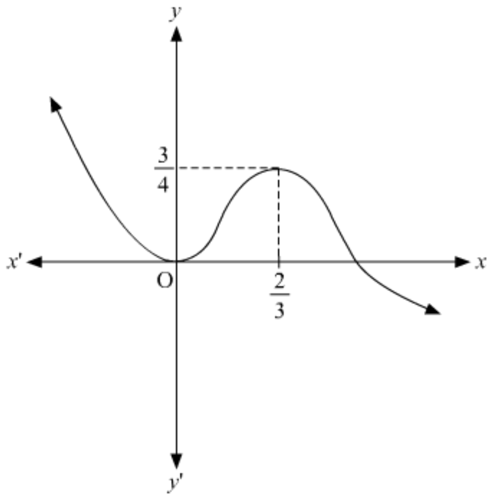
Area of the shaded region will be
$\int_{-1}^{0}\{2x+-(-x^2-1)\}\,dx-\int_{0}^{2}\left\{\frac{3x}{2}-\left(x^2-1\right)\right\}dx$
$=\int_{-1}^{0}\{2x+x^2+1\}\,dx-\int_{0}^{2}\left\{\frac{3x}{2}-x^2+1\right\}dx$
$=\left[x^2+\frac{x^3}{3}+x\right]_{-1}^{0}-\left[\frac{3x^2}{4}-\frac{x^3}{3}+x\right]_{0}^{2}$
$=\frac{1}{3}+\frac{7}{3}=\frac{8}{3}$

Hence, the correct answer is option B.

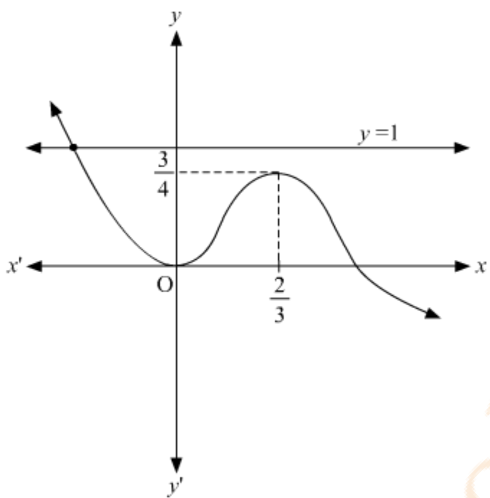
Solution 14

Given $f(x)=y=\frac{27\left(x^{2/3}-x\right)}{4}$
Let $x=t^3$
$\therefore y=\frac{27}{4}\left(t^2-t^3\right)$
$\frac{dy}{dt}=\frac{27}{4}\left(2t-3t^2\right)$
$\frac{dy}{dt}=0 \Rightarrow t=0,\ 2/3$

Hence the graph of the above function 'y' with respect to 't' will be as shown below



Here y =1 will intersect the graph at only one point hence $f(x)$ = 1 will have just one solution.



Hence, the correct answer is A.

Solution 15

Given $f\left(x\right) = y = \frac{27\left(x^{2/3} - x\right)}{4}$

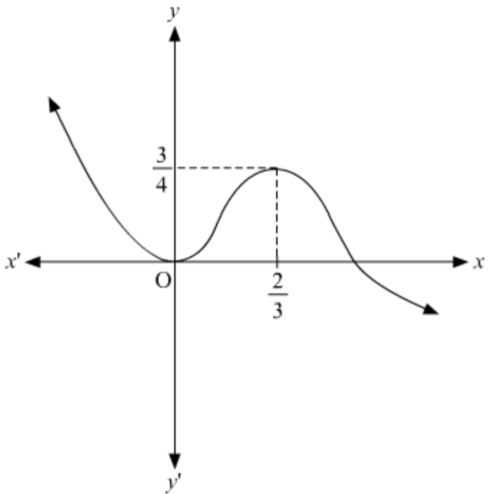
Let $x = t^3$

$\therefore y = \frac{27}{4}\left(t^2 - t^3\right)$

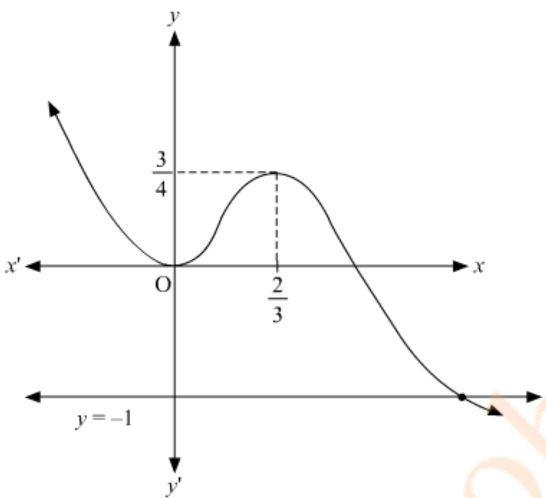
$\frac{dy}{dt} = \frac{27}{4}\left(2t - 3t^2\right)$

$\frac{dy}{dt} = 0 \Rightarrow t = 0,\ 2/3$

Hence the graph of the above function 'y' with respect to 't' will be as shown below



Here y = −1 will intersect the graph at only one point hence f(x) = −1 will have just one solution.



Hence, the correct answer is option A.

Solution 16

$\int_{\frac{1}{3}}^{\frac{1}{2}} \left[\frac{1}{x}\right] \mathrm{d}\,x$

$= \int_{\frac{1}{3}}^{\frac{1}{2}} 2.\,\mathrm{d}\,x \qquad \left[\because \frac{1}{2} < x < \frac{1}{3} \Rightarrow 2 < \frac{1}{x} < 3\right]$

$= 2[x]_{\frac{1}{3}}^{\frac{1}{2}}$

$= \frac{1}{3}$

Hence, the correct answer is option B.

Solution 17

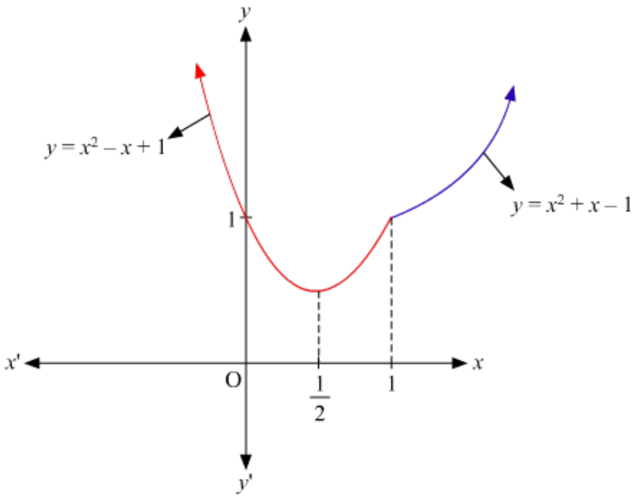
$\int_{\frac{1}{3}}^{1} x\left[\frac{1}{x}\right] \mathrm{d}\,x = \int_{\frac{1}{3}}^{\frac{1}{2}} 2x\,\mathrm{d}\,x + \int_{\frac{1}{2}}^{1} x\,\mathrm{d}\,x = \frac{37}{72}$

Hence, the correct answer is option A.

Solution 18

After removing modulus from $|x - 1|$, we get $f(x) = \left\{\begin{matrix} x^2 + x - 1, & x \geq 1 \\ x^2 - x + 1, & x < 1 \end{matrix}\right\}$

The graph of the function can be represented as it is shown below:

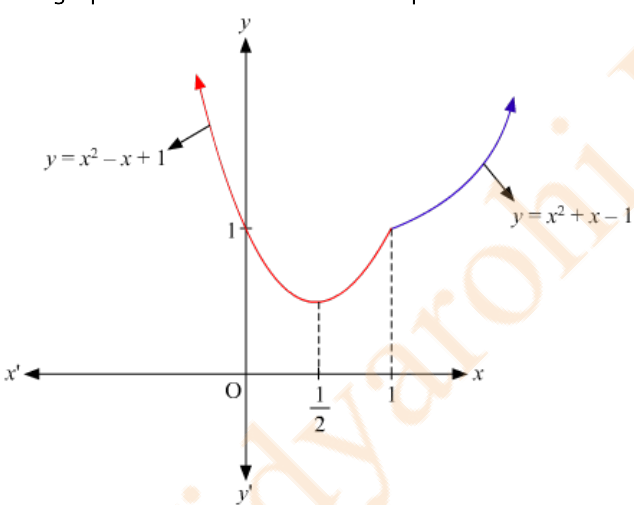


From the graph it's clear that the function is continuous for $x \in$ R.
But at $x = 1$ the graph has a sharp edge, therefore it will not be differentiable at $x = 1$.
Hence, the correct answer is option B.

Solution 19

After removing modulus from $|x - 1|$, we get $f(x) = \begin{Bmatrix} x^2 + x - 1, & x \geq 1 \\ x^2 - x + 1, & x < 1 \end{Bmatrix}$

The graph of the function can be represented as it is shown below:

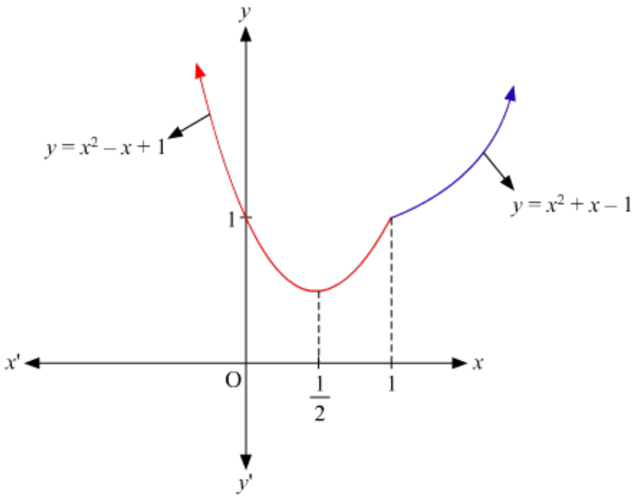


From the graph it's clear that $f(x)$ is decreasing in $\left(-\infty, \frac{1}{2}\right)$ and increasing in $\left(\frac{1}{2}, \infty\right)$.
Hence, the correct answer is option B.

Solution 20

After removing modulus from $|x - 1|$, we get $f(x) = \begin{Bmatrix} x^2 + x - 1, & x \geq 1 \\ x^2 - x + 1, & x < 1 \end{Bmatrix}$

The graph of the function can be represented as it is shown below:

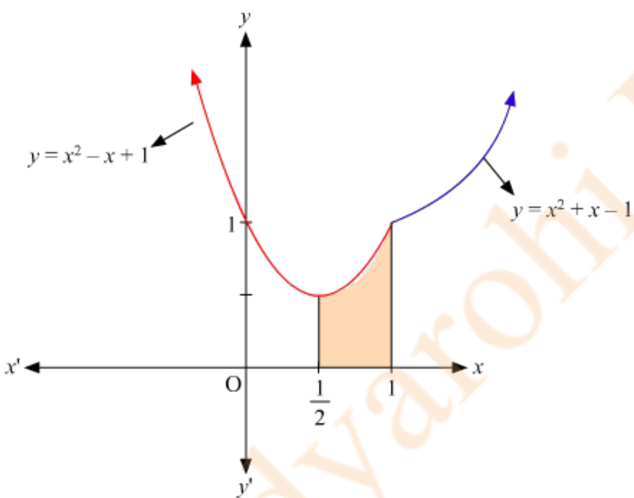


From the graph it's clear that *f*(*x*) has only one local minimum at *x* = 1/2.
Hence, the correct answer is option C.

Solution 21

After removing modulus from |*x* − 1|, we get $f(x) = \begin{Bmatrix} x^2 + x - 1, & x \geq 1 \\ x^2 - x + 1, & x < 1 \end{Bmatrix}$

The graph of the function can be represented as it is shown below:



Here, area of the shaded region

$= \int_{\frac{1}{2}}^{1} \left(x^2 - x + 1\right) \mathrm{d}\,x$

$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_{\frac{1}{2}}^{1}$

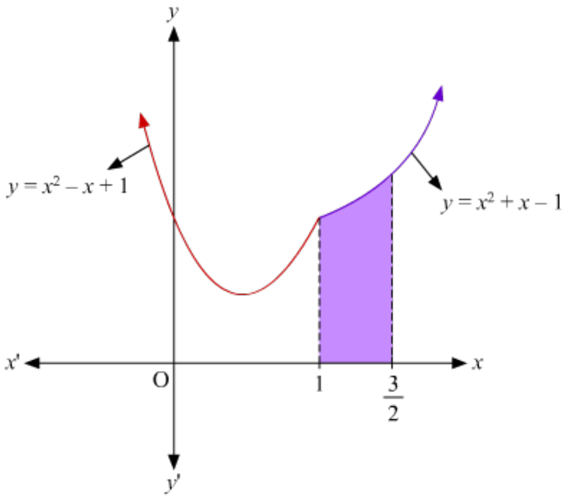
$= \frac{10}{24} = \frac{5}{12}$

Hence, the correct answer is option A.

Solution 22

After removing modulus from |*x* − 1|, we get $f(x) = \begin{Bmatrix} x^2 + x - 1, & x \geq 1 \\ x^2 - x + 1, & x < 1 \end{Bmatrix}$

The graph of the function can be represented as it is shown below:



Here, area of the shaded region

$= \int_1^{\frac{3}{2}} \left(x^2 + x - 1\right) \mathrm{d}\,x$

$= \left[\frac{x^3}{3} + \frac{x^2}{2} - x\right]_1^{\frac{3}{2}}$

$= \frac{11}{12}$

Hence, the correct answer is option A.

Solution 23

Given: $a_n = \int_0^\pi \frac{\sin^2\{(n+1)x\}}{\sin\ 2x}\,\mathrm{d}\,x$

$\Rightarrow a_n = \frac{1}{2}\int_0^\pi \frac{1-\cos\{2(n+1)x\}}{\sin\ 2x}\,\mathrm{d}\,x$

Now,

$a_n - a_{n-1} = \frac{1}{2}\int_0^\pi \frac{[1-\cos\{2(n+1)x\}]-[1-\cos\ 2nx]}{\sin\ 2\mathrm{x}}\,\mathrm{d}\,x$

$a_n - a_{n-1} = \frac{1}{2}\int_0^\pi \frac{\cos\ 2nx-\cos\{2(n+1)x\}}{\sin\ 2\mathrm{x}}\,\mathrm{d}\,x$

$a_n - a_{n-1} = \frac{1}{2}\int_0^\pi \frac{2\ \sin\ (4n+2)x\ .\ \sin\ 2x}{\sin\ 2x}\,\mathrm{d}\,x$

$a_n - a_{n-1} = \int_0^\pi \sin\ \left(4n+2\right)x\ \mathrm{d}\,x$

$a_n - a_{n-1} = \left[\frac{-\cos(4n+2)x}{4n+2}\right]_0^\pi$

$a_n - a_{n-1} = 0$

$a_n = a_{n-1}$

$\Rightarrow a_n = a_{n-1} = a_{n-2} = \ldots = a_2 = a_1 \Rightarrow \{a_{2n}\}$ and $\{a_{2n+1}\}$ are in AP with common difference zero.

Hence, the correct answer is option C.

Solution 24

Given: $a_n = \int_0^\pi \frac{\sin^2\{(n+1)x\}}{\sin\ 2x}\,\mathrm{d}\,x$

$\Rightarrow a_n = \frac{1}{2} \int_0^{\pi} \frac{1-\cos\{2(n+1)x\}}{\sin\ 2x}\, \mathrm{d}\, x$

Now,

$a_n - a_{n-1} = \frac{1}{2} \int_0^{\pi} \frac{[1-\cos\{2(n+1)x\}]-[1-\cos\ 2nx]}{\sin\ 2x}\, \mathrm{d}\, x$

$a_n - a_{n-1} = \frac{1}{2} \int_0^{\pi} \frac{\cos\ 2\text{nx}-\cos\{2(n+1)x\}}{\sin\ 2\text{x}}\, \mathrm{d}\, x$

$a_n - a_{n-1} = \frac{1}{2} \int_0^{\pi} \frac{2\ \sin\ (4n+2)x\ .\ \sin\ 2x}{\sin\ 2x}\, \mathrm{d}\, x$

$a_n - a_{n-1} = \int_0^{\pi} \sin\ \left(4n+2\right)x\ \ \mathrm{d}\, x$

$a_n - a_{n-1} = \left[\frac{-\cos(4n+2)x}{4n+2}\right]_0^{\pi}$

$a_n - a_{n-1} = 0$

$a_n = a_{n-1}$

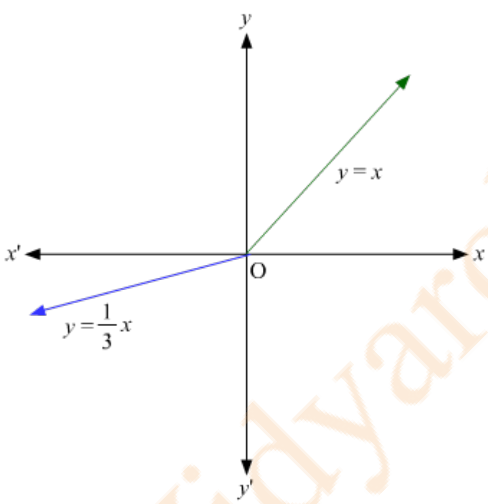
$\Rightarrow a_n = a_{n-1} = a_{n-2} = \ldots = a_2 = a_1 \Rightarrow a_{n-1}$ and a_{n-4} are equal.

Therefore, $a_{n-1} - a_{n-4} = 0$

Hence, the correct answer is option B.

Solution 25

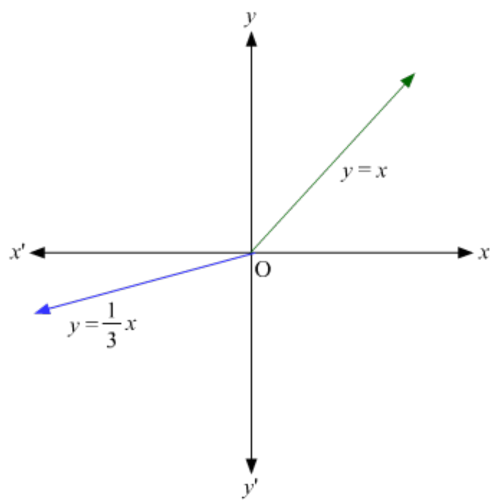
Given: $x + |y| = 2$y
If y > 0, then $x + y = 2\text{y} \Rightarrow y = x$
If $y < 0$, then $x - y = 2y \Rightarrow y = \frac{x}{3}$
We can plot the graph of y as a function of x as shown below



From the graph we can observe that y is defined for all real x, it is continuous at $x = 0$ and is not differentiable at $x = 0$.
Hence, the correct answer is option D.

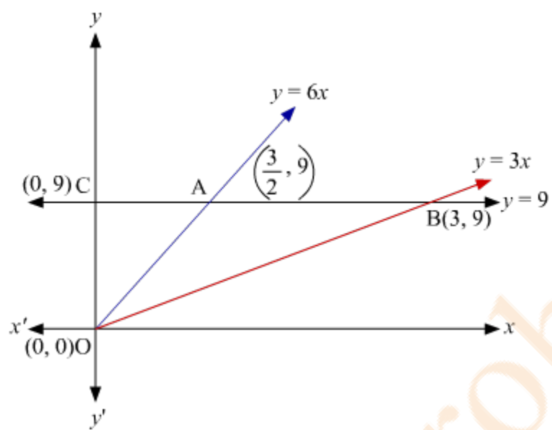
Solution 26

Given: $x + |y| = 2y$
If y > 0, then $x + y = 2\text{y} \Rightarrow y = x$
If $y < 0$, then $x - y = 2y \Rightarrow y = \frac{x}{3}$
We can plot the graph of y as a function of x as shown below



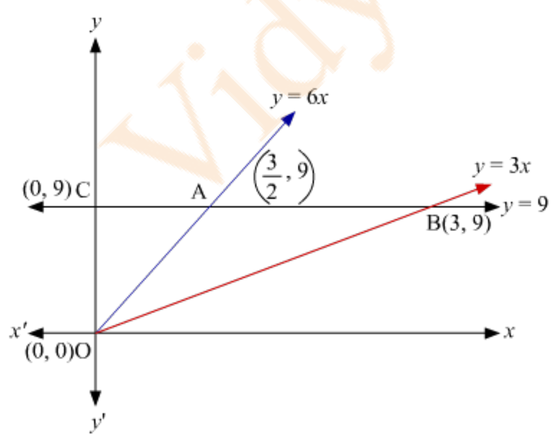
Here if $x < 0$ then $y = \frac{x}{3} \Rightarrow$ derivative of $y = \frac{1}{3}$.
Hence, the correct answer is option D.

Solution 27



Area of triangle ABC = $\frac{1}{2} \times \text{base} \times \text{height} \;=\; \frac{1}{2} \times AB \times OC \;= \frac{1}{2} \times \frac{3}{2} \times 9 = \frac{27}{4}$
Hence, the correct answer is option A.

Solution 28

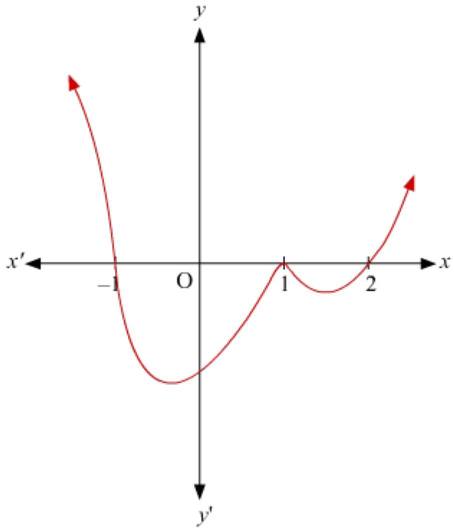


Centroid of the triangle is $\left(\frac{\frac{3}{2}+3+0}{3},\ \frac{9+9+0}{3}\right) = \left(\frac{3}{2},\ 6\right)$

Hence, the correct answer is option B.

Solution 29

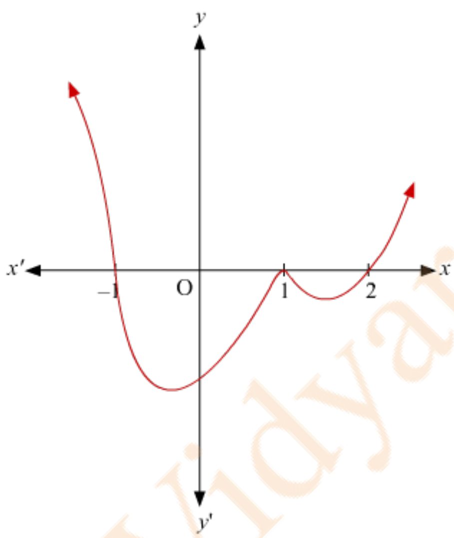
Given: $f(x) = (x-1)^2(x+1)(x-2)^3$
The graph of $f(x)$ is shown below



from the graph we observe that f(x) has two local minima.
Hence, the correct answer is option (c).

Solution 30

Given: $f(x) = (x-1)^2(x+1)(x-2)^3$
The graph of $f(x)$ is shown below



from the graph, we observe that $f(x)$ has one local maxima.
Hence, the correct answer is option B.

Solution 31

$\text{Given}: \; f''(x) = g''(x)$

$\therefore \int f''(x)dx = \int g''(x)dx$

$\Rightarrow f'(x) = g'(x) + c$

$\text{Put } x = 1,$

$f'(1) = g'(1) + c$

$4 = 6 + c \Rightarrow c = -2$

$\Rightarrow f'(x) = g'(x) - 2 \quad \ldots(i)$

$\text{integerating (i), we get}$

$f(x) = g(x) - 2x + k$

$\text{Put } x = 2,$

$f(2) = g(2) - 4 + k$

$3 = 9 - 4 + k$

$\Rightarrow k = -2$

$\Rightarrow f(x) = g(x) - 2x - 2$

$\Rightarrow f(x) - g(x) = -2x - 2$

$\therefore f(4) - g(4) = -10$

Hence, the correct answer is option A.

Solution 32

$$y = \left|x - 1\right| \Rightarrow \left\{ \begin{matrix} x - 1, & x \geq 1 \\ -x + 1 & x < 1 \end{matrix} \right\}$$

$$\text{and, } \left|x\right| = 2 \Rightarrow x = \pm 2$$

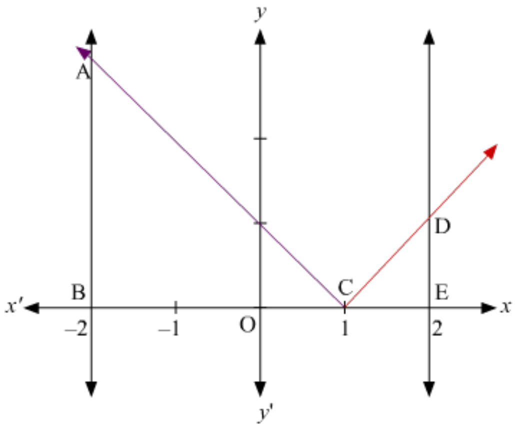
If $x \geq 1$, then x = 2 and y = 2 − 1 = 1. Therefore the point of intersection is (2, 1)
If $x < 1$, then x = −2 and y = −(−2)+1 = 3. Therefore the point of intersection is (−2, 3)
Hence, the correct answer is option C.

Solution 33

$$y = \left|x - 1\right| \Rightarrow \left\{ \begin{matrix} x - 1, & x \geq 1 \\ -x + 1 & x < 1 \end{matrix} \right\}$$

$$\left|x\right| = 2 \Rightarrow x = \pm 2$$

If $x \geq 1$, then x = 2 and y = 2 − 1 = 1. Therefore the point of intersection is (2, 1)
If $x < 1$, then x = −2 and y = −(−2)+1 = 3. Therefore the point of intersection is (−2, 3)
These curves can be represented by the following figure:



The area of the shaded region = Area of triangle ABC + Area of triangle DEC
The area of the shaded region = $\frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times 1 \times 1 = \frac{10}{2} = 5$
Hence, the correct answer is option C.

Solution 34

$$f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\therefore f'(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow f'(0) = \begin{vmatrix} 0 & 1 & 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = -6p^3$$

Hence, the correct answer is option D.

Solution 35

$$f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\therefore f'(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow f''(x) = \begin{vmatrix} 6x & -\sin x & -\cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow f''(0) = \begin{vmatrix} 0 & 0 & -1 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = -\left(6p^2 + p\right)$$

Given $f''(0) = 0 \Rightarrow 6p^2 + p = 0 \Rightarrow p = 0, \frac{-1}{6}$

Hence, the correct answer is option A.

Solution 36

$$\cos\ A+\cos\ B+\cos\ C=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)+\cos\ C$$

$$\cos\ A+\cos\ B+\cos\ C=2\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right)+1-2\sin^2\frac{C}{2}$$

$$\cos\ A+\cos\ B+\cos\ C=2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)-2\sin^2\frac{C}{2}+1$$

$$\cos\ A+\cos\ B+\cos\ C=2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)-\sin\left(\frac{\pi}{2}-\left(\frac{A+B}{2}\right)\right)\right\}+1$$

$$\cos\ A+\cos\ B+\cos\ C=2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right\}+1$$

$$\cos\ A+\cos\ B+\cos\ C=4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}+1\quad\ldots\left(\mathrm{i}\right)$$

$$\text{Given}:\ \cos\ A+\cos\ B+\cos\ C=\sqrt{3}\sin\frac{\pi}{3}=\frac{3}{2}$$

Putting the above value in (i), we get

$$\frac{3}{2}=4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}+1$$

$$\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}=\frac{1}{8}$$

Hence, the correct answer is option C.

Solution 37

$$\cos\ A+\cos\ B+\cos\ C=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)+\cos\ C$$

$$\cos\ A+\cos\ B+\cos\ C=2\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right)+1-2\sin^2\frac{C}{2}$$

$$\cos\ A+\cos\ B+\cos\ C=2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)-2\sin^2\frac{C}{2}+1$$

$$\cos\ A+\cos\ B+\cos\ C=2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)-\sin\left(\frac{\pi}{2}-\left(\frac{A+B}{2}\right)\right)\right\}+1$$

$$\cos\ A+\cos\ B+\cos\ C=2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right\}+1$$

$$\cos\ A+\cos\ B+\cos\ C=4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}+1\quad\ldots\left(\mathrm{i}\right)$$

$$\text{Given}:\ \cos\ A+\cos\ B+\cos\ C=\sqrt{3}\sin\frac{\pi}{3}=\frac{3}{2}$$

Putting the above value in $\left(\mathrm{i}\right)$, we get

$$\frac{3}{2}=4\sin\frac{\mathrm{C}}{2}\sin\frac{\mathrm{A}}{2}\sin\frac{\mathrm{B}}{2}+1$$

$$\sin\frac{\mathrm{C}}{2}\sin\frac{\mathrm{A}}{2}\sin\frac{\mathrm{B}}{2}=\frac{1}{8}$$

Now,

$$\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)=\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)\cos\left(\frac{\pi}{2}-\frac{B}{2}\right)$$

$$\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)=\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}=\frac{1}{8}$$

Hence, the correct answer is option D.

Solution 38

If $\tan\ \alpha$ and $\tan\ \beta$ are roots of equation $x^2+bx+c=0$.
$\Rightarrow\tan\ \alpha+\tan\ \beta=-b$ and $\tan\ \alpha.\tan\ \beta=c$
Now,

$\tan\left(\alpha+\beta\right)=\frac{\tan\ \alpha+\tan\ \beta}{1-\tan\ \alpha.\ \tan\ \beta}$

$\tan\left(\alpha+\beta\right)=\frac{-b}{1-c}$

$\tan\left(\alpha+\beta\right)=\frac{b}{c-1}=b\left(c-1\right)^{-1}$

Hence, the correct answer is option D.

Solution 39

If $\tan\ \alpha$ and $\tan\ \beta$ are roots of equation $x^2+bx+c=0$.
$\Rightarrow \tan\ \alpha+\tan\ \beta=-b$ and $\tan\ \alpha.\tan\ \beta=c$
Now,

$\sin\ \left(\alpha+\beta\right)\ \sec\ \alpha\ \sec\ \beta=\frac{\sin\ \alpha\ \cos\ \beta+\cos\ \alpha\ \sin\ \beta}{\cos\ \alpha\ \cos\ \beta}$

$\sin\ \left(\alpha+\beta\right)\ \sec\ \alpha\ \sec\ \beta=\tan\ \alpha+\tan\ \beta=-b$

Hence, the correct answer is option B.

Solution 40

The centres of the circles are (1, 3) and (4, −1)
The distance between the centres = $\sqrt{3^2+4^2}=5$
Hence, the correct answer is option A.

Solution 41

If two circles intersect then $|r_1-r_2|<c_1c_2<r_1+r_2$
The centres of the circles are (1, 3) and (4, −1)
The distance between the centres= c_1c_2= $\sqrt{3^2+4^2}=5$
r_1 = r and r_2 = $\sqrt{4^2+1^2-8}=3$
$\therefore |r-3|<5<r+3$
$\Rightarrow r>2$ and $-2<r<8$
$\Rightarrow 2<r<8$

Hence, the correct answer is option D.

Solution 42

The point of intersection of $x + y + 1 = 0$ and $3x + 2y + 1 = 0$ is (1, −2).
Therefore the line passing through (1, −2) and parallel to x-axis will be $y = -2$.
Hence, the correct answer is option D.

Solution 43

The point of intersection of $x + y + 1 = 0$ and $3x + 2y + 1 = 0$ is (1, −2).
Therefore the line passing through (1, −2) and parallel to y-axis will be $x = 1$.
Hence, the correct answer is option B.

Solution 44

Given : $k\sin\ x+\cos\ 2x=2k-7$

$\Rightarrow k\sin\ x+1-2\sin^2 x=2k-7$

$\Rightarrow 2\sin^2 x-k\sin\ x+2k-8=0$

$\Rightarrow \sin\ x=\frac{k\pm(k-8)}{2}$

$\Rightarrow \sin\ x=\frac{k-4}{2}$ $\quad\left(\because\ \sin\ x=2\ \text{is not possible}\right)$

as, $-1\le\sin\ x\le1$

$\Rightarrow -1\le\frac{k-4}{2}\le1$

$\Rightarrow 2\le k\le6$

Therefore, the minimum value of k is 2.

Hence, the correct answer is option B.

Solution 45

$$\text{Given}: \ k\sin\ x + \cos\ 2x = 2k - 7$$
$$\Rightarrow k\sin\ x + 1 - 2\sin^2 x = 2k - 7$$
$$\Rightarrow 2\sin^2 x - k\sin\ x + 2k - 8 = 0$$
$$\Rightarrow \sin\ x = \frac{k \pm (k-8)}{2}$$
$$\Rightarrow \sin\ x = \frac{k-4}{2} \qquad \left(\because\ \sin\ x = 2\ \text{is}\ \ \text{not}\ \ \text{possible}\right)$$
$$\text{as},\ -1 \le \sin\ x \le 1$$
$$\Rightarrow -1 \le \frac{k-4}{2} \le 1$$
$$\Rightarrow 2 \le k \le 6$$

Therefore, the maximum value of k is 6.
Hence, the correct answer is option D.

Solution 46

Given: $f(x) = \frac{a^{[x]+x}-1}{[x]+x}$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{a^{0+x}-1}{0+x} = \ln\ a$$

Hence, the correct answer is option B.

Solution 47

Given: $f\left(x\right) = \frac{a^{[x]+x}-1}{[x]+x}$

$$\lim_{x\to 0^-} f\left(x\right) = \lim_{x\to 0^+} \frac{a^{-1+x}-1}{-1+x} = \frac{a^{-1}-1}{-1} = 1 - a^{-1}$$

Hence, the correct answer is option C.

Solution 48

Given: $z^2 = i\bar{z}$
$$\Rightarrow z^3 = i\bar{z}z$$
$$\Rightarrow z^3 = i\left|z\right|^2$$
$$\Rightarrow z^3 - i\left|z\right|^2 = 0$$

If the roots of the above equation are z_1, z_2 and z_3, then
(i) $z_1 + z_2 + z_3 = 0$
(ii) $z_1z_2 + z_2z_3 + z_3z_1 = 0$
(iii) $z_1z_2z_3 = i|z|^2$
Hence, the correct answer is option C.

Solution 49

Given: $z^2 = i\bar{z}$
$$\Rightarrow z^3 = i\bar{z}z$$
$$\Rightarrow z^3 = i\left|z\right|^2$$
$$\Rightarrow z^3 - i\left|z\right|^2 = 0$$

If the roots of the above equation are z_1, z_2 and z_3, then
(i) $z_1 + z_2 + z_3 = 0$ (purely real)
(ii) $z_1z_2 + z_2z_3 + z_3z_1 = 0$ (purely real)
(iii) $z_1z_2z_3 = i|z|^2$ (purely imaginary)
Hence, the correct answer is option C.

Solution 50

$\text{Given}: \log_x y,\ \log_z x \text{ and } \log_y z \text{ are in GP}$

$\Rightarrow \left(\log_z x\right)^2 = \log_x y \times \log_y z$

$\Rightarrow \left(\frac{\log x}{\log z}\right)^2 = \frac{\log y}{\log x} \times \frac{\log z}{\log y}$

$\Rightarrow \left(\frac{\log x}{\log z}\right)^2 = \frac{\log z}{\log x}$

$\Rightarrow (\log x)^3 = (\log z)^3$

$\Rightarrow \log x = \log z$

$\Rightarrow x = z \qquad \ldots(i)$

$\text{Given}: x^3,\ y^3 \text{ and } z^3 \text{ are in AP}$

$\Rightarrow 2y^3 = x^3 + z^3$

$\Rightarrow 2y^3 = 2x^3 \qquad (\because x = z)$

$\Rightarrow y = x \qquad \ldots(ii)$

From (i) and (ii), we get

$x = y = z$

$\Rightarrow x, y, z$ are in AP as well as GP.

Hence, the correct answer is option C.

Solution 51

$\text{Given}: \log_x y,\ \log_z x \text{ and } \log_y z \text{ are in GP}$

$\Rightarrow \left(\log_z x\right)^2 = \log_x y \times \log_y z$

$\Rightarrow \left(\frac{\log x}{\log z}\right)^2 = \frac{\log y}{\log x} \times \frac{\log z}{\log y}$

$\Rightarrow \left(\frac{\log x}{\log z}\right)^2 = \frac{\log z}{\log x}$

$\Rightarrow \left(\log x\right)^3 = \left(\log z\right)^3$

$\Rightarrow \log x = \log z$

$\Rightarrow x = z \qquad \ldots\left(i\right)$

$\text{Given}: x^3,\ y^3 \text{ and } z^3 \text{ are in AP}$

$\Rightarrow 2y^3 = x^3 + z^3$

$\Rightarrow 2y^3 = 2x^3 \qquad \left(\because x = z\right)$

$\Rightarrow y = x \qquad \ldots\left(ii\right)$

From i and ii, we get

$x = y = z$

$\Rightarrow xy,\ yz,\ zx$ is x^2, x^2, x^2, which are terms in AP as well as GP.

Hence, the correct answer is option C.

Solution 52

Given: $|z - 4| = |z - 8|$
Implies z is at equal distance from 4 and 8, which means z lies on the perpendicular bisector of line-

segment joining 4 and 8.
Therefore, $z = 6 + iy$
Now, $2|z| = 3|z - 2|$
$\Rightarrow 2\sqrt{36 + y^2} = 3\sqrt{16 + y^2}$
$\Rightarrow 4\left(36 + y^2\right) = 9\left(16 + y^2\right)$
$\Rightarrow 144 = 144 + 5y^2$
$\Rightarrow y^2 = 0$
$\Rightarrow y = 0$

Therefore, $z = 6 \Rightarrow |z| = 6$
Hence, the correct answer is option A.

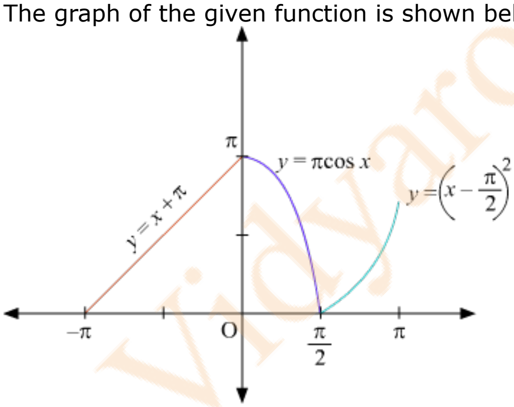
Solution 53

Given: $|z - 4| = |z - 8|$
Implies z is at equal distance from 4 and 8, which means z lies on the perpendicular bisector of line-segment joining 4 and 8.
Therefore, $z = 6 + iy$
Now, $2|z| = 3|z - 2|$
$\Rightarrow 2\sqrt{36 + y^2} = 3\sqrt{16 + y^2}$
$\Rightarrow 4\left(36 + y^2\right) = 9\left(16 + y^2\right)$
$\Rightarrow 144 = 144 + 5y^2$
$\Rightarrow y^2 = 0$
$\Rightarrow y = 0$

Therefore, $z = 6 \Rightarrow \left|\frac{z-6}{z+6}\right| = 0$
Hence, the correct answer is option D.

Solution 54

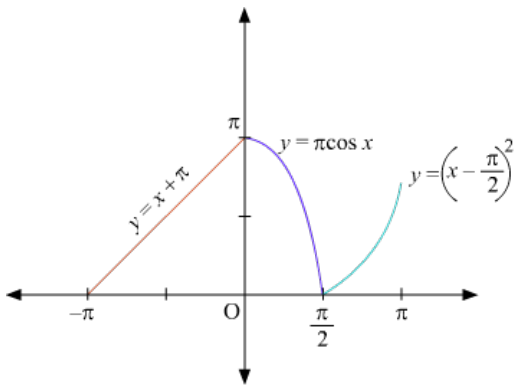
The graph of the given function is shown below:



From the graph, we can observe that $f(x)$ is continuous at $x = 0$ and at $x = \frac{\pi}{2}$.
Hence, the correct answer is option C.

Solution 55

The graph of the given function is shown below:



From the graph, we can observe that at $x = 0$ and at $x = \frac{\pi}{2}$ the graph shows sharp edges therefore, $f(x)$ is not differentiable at $x = 0$ and at $x = \frac{\pi}{2}$.
Hence, the correct answer is option D.

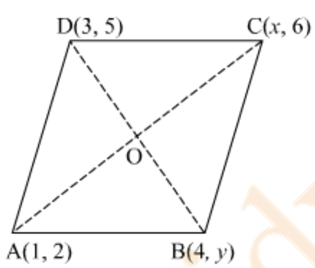
Solution 56

If α and β are roots of equation $x^2 + bx + c = 0$, then $\alpha + \beta = -b$ and $\alpha\beta = c$.
It's given that $b > 0$ and $c < 0$.
Therefore, $\alpha + \beta < 0$ and $\alpha\beta < 0$.
$\Rightarrow \beta < -\alpha$, henec statement 1 is correct.
It's given that $\alpha < \beta$ and we know $\alpha < -\beta$, implies $|\alpha| > \beta$, hence statement 2 is also correct.
Hence, the correct answer is option C.

Solution 57

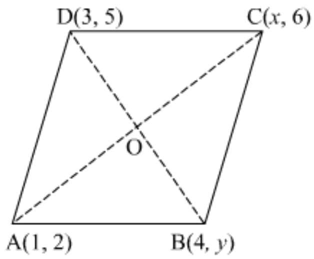
If α and β are roots of equation $x^2 + bx + c = 0$, then $\alpha + \beta = -b$ and $\alpha\beta = c$.
It's given that $b > 0$ and $c < 0$.
Therefore, $\alpha + \beta < 0$ and $\alpha\beta < 0$.
$\Rightarrow \alpha + \beta + \alpha\beta < 0$, hence statement 1 is incorrect.
also, $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta) > 0$, hence the statement 2 is correct.
Hence, the correct answer is option B.

Solution 58



As diagonals of a parallelogram bisect each other, we get
Coordinates of point O as mid-point of AC = $\left(\frac{1+x}{2}, \frac{6+2}{2}\right) = \left(\frac{1+x}{2}, 4\right)$
Coordinates of point O as mid-point of BD = $\left(\frac{3+4}{2}, \frac{5+y}{2}\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$
On comparison, we get
$\frac{1+x}{2} = \frac{7}{2}$ and $\frac{5+y}{2} = 4 \Rightarrow x = 6$ and $y = 3$
$\therefore AC^2 - BD^2 = (5^2 + 4^2) - (1^2 + 2^2) = 36$
Hence, the correct answer is option C.

Solution 59



As diagonals of a parallelogram bisect each other, we get

Coordinates of point O as mid-point of AC = $\left(\frac{1+x}{2}, \frac{6+2}{2}\right) = \left(\frac{1+x}{2}, 4\right)$

Coordinates of point O as mid-point of BD = $\left(\frac{3+4}{2}, \frac{5+y}{2}\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$

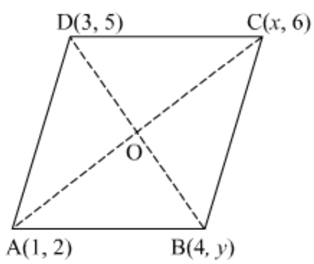
On comparison, we get

$\frac{1+x}{2} = \frac{7}{2} \text{ and } \frac{5+y}{2} = 4 \Rightarrow x = 6 \text{ and } y = 3$

∴Coordinates of point O are $\left(\frac{7}{2}, 4\right)$

Hence, the correct answer is option A.

Solution 60



As diagonals of a parallelogram bisect each other, we get

Coordinates of point O as mid-point of AC = $\left(\frac{1+x}{2}, \frac{6+2}{2}\right) = \left(\frac{1+x}{2}, 4\right)$

Coordinates of point O as mid-point of BD = $\left(\frac{3+4}{2}, \frac{5+y}{2}\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$

On comparison, we get

$\frac{1+x}{2} = \frac{7}{2} \text{ and } \frac{5+y}{2} = 4 \Rightarrow x = 6 \text{ and } y = 3$

The equation of line AB is $x - 3y + 5 = 0$.

The perpendicular distance from D(3, 5) to AB = $\frac{|3-3\times5+5|}{\sqrt{1^2+(-3)^2}} = \frac{7}{\sqrt{10}}$

and AB = $\sqrt{3^2+1^2} = \sqrt{10}$

Therefore, the area of the parallelogram ABCD = $\frac{7}{\sqrt{10}} \times \sqrt{10} = 7$

Hence, the correct answer is option D.

Solution 61

Given : $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3) \qquad \ldots(i)$

differentiating $f(x)$ with respect to x, we get

$f'(x) = 3x^2 + 2x f'(1) + f''(2) \qquad \ldots(\text{ii})$

differentiating (ii) with respect to x, we get

$f''(x) = 6x + 2f'(1) \qquad \ldots(iii)$

differentiating (iii) with respect to x, we get

$f'''(x) = 6 \qquad \ldots(iv)$

put $x = 2$ in (iii) and $x = 1$ in (ii), we get

$f''(2) = 12 + 2f'(1)$

and, $f'(1) = 3 + 2f'(1) + f''(2)$

solving the above two equations , we get

$f'(1) = -5$ and $f''(2) = 2 \qquad \ldots(v)$

from (i), (iv) and (v), we get

$f(x) = x^3 - 5x^2 + 2x + 6$

$\therefore\ f(1) = 4$

Hence, the correct answer is option D.

Solution 62

Given : $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3) \qquad \ldots(i)$

differentiating $f(x)$ with respect to x, we get

$f'(x) = 3x^2 + 2x f'(1) + f''(2) \qquad \ldots(ii)$

differentiating (ii) with respect to x, we get

$f''(x) = 6x + 2f'(1) \qquad \ldots(iii)$

differentiating (iii) with respect to x, we get

$f'''(x) = 6 \qquad \ldots(iv)$

put $x = 2$ in (iii) and $x = 1$ in (ii), we get

$f''(2) = 12 + 2f'(1)$

and, $f'(1) = 3 + 2f'(1) + f''(2)$

solving the above two equations , we get

$f'(1) = -5$

Hence, the correct answer is option B.

Solution 63

Given : $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3) \quad \ldots(i)$

differentiating $f(x)$ with respect to x, we get

$f'(x) = 3x^2 + 2x f'(1) + f''(2) \quad \ldots(ii)$

differentiating (ii) with respect to x, we get

$f''(x) = 6x + 2f'(1) \quad \ldots(iii)$

differentiating (iii) with respect to x, we get

$f'''(x) = 6$

$\Rightarrow f'''(10) = 6$

Hence, the correct answer is option C.

Solution 64

Given : $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3) \quad \ldots(i)$

differentiating $f(x)$ with respect to x, we get

$f'(x) = 3x^2 + 2x f'(1) + f''(2) \quad \ldots(ii)$

differentiating (ii) with respect to x, we get

$f''(x) = 6x + 2f'(1) \quad \ldots(iii)$

differentiating (iii) with respect to x, we get

$f'''(x) = 6 \quad \ldots(iv)$

put $x = 2$ in (iii) and $x = 1$ in (ii), we get

$f''(2) = 12 + 2f'(1) \quad \ldots(v)$

and, $f'(1) = 3 + 2f'(1) + f''(2)$

solving the above two equations , we get

$f'(1) = -5$ and $f''(2) = 2 \quad \ldots(vi)$

from (i), (iv) and (vi), we get

$f(x) = x^3 - 5x^2 + 2x + 6$

$\therefore\ f(0) = 6,\ f(1) = 4$ and $f(2) = -2$

$\Rightarrow f(2) = f(1) - f(0)$, hence statement 1 is correct.

from (v), we get $f''(2) - 2f'(1) = 12$, hence statement 2 is also correct.

Hence, the correct answer is option C.

Solution 65

The equation of a plane passing the intersection of planes $2x - y + 3z = 2$ and $x + y - z = 1$ is
$(2x - y + 3z - 2) + \lambda(x + y - z - 1) = 0$
$\Rightarrow (2 + \lambda)x + (\lambda - 1)y + (3 - \lambda)z - (2 + \lambda) = 0 \quad \ldots(i)$
as the plane passes through (1, 0, 1), it will satisfy (i)
$\Rightarrow 2 + \lambda + 3 - \lambda - 2 - \lambda = 0$
$\Rightarrow \lambda = 3$
hence, the equation of the plane P is $5x + 2y = 5$.
DR's of normal to the plane P is 5, 2, 0.
Sum of the product of DR's of normal and DR's of the line of intersection of the planes must be zero.
Only option (a) i.e., (2, −5, −3) satisfies the above condition e.g. $5 \times 2 + 2 \times (-5) + 0 \times (-3) = 0$
Hence, the correct answer is option A.

Solution 66

The equation of a plane passing the intersection of planes $2x - y + 3z = 2$ and $x + y - z = 1$ is
$(2x - y + 3z - 2) + \lambda(x + y - z - 1) = 0$

$\Rightarrow (2 + \lambda)x + (\lambda - 1)y + (3 - \lambda)z - (2 + \lambda) = 0$ ...(i)
as the plane passes through (1, 0, 1), it will satisfy (i)
$\Rightarrow 2 + \lambda + 3 - \lambda - 2 - \lambda = 0$
$\Rightarrow \lambda = 3$
therefore, the equation of the plane P is $5x + 2y = 5$.
Hence, the correct answer is option B.

Solution 67

The equation of a plane passing the intersection of planes $2x - y + 3z = 2$ and $x + y - z = 1$ is
$(2x - y + 3z - 2) + \lambda(x + y - z - 1) = 0$
$\Rightarrow (2 + \lambda)x + (\lambda - 1)y + (3 - \lambda)z - (2 + \lambda) = 0$ ...(i)
as the plane passes through (1, 0, 1), it will satisfy (i)
$\Rightarrow 2 + \lambda + 3 - \lambda - 2 - \lambda = 0$
$\Rightarrow \lambda = 3$
hence, the equation of the plane P is $5x + 2y = 5$.
If plane P touches sphere $x^2 + y^2 + z^2 = r^2$, then r is the perpendicular distance from (0, 0, 0) to $5x + 2y = 5$.
$\Rightarrow r = \frac{5}{\sqrt{5^2+2^2}} = \frac{5}{\sqrt{29}}$
Hence, the correct answer is option C.

Solution 68

Given: $f(x) = |x^2 - 5x + 6|$

$$\Rightarrow f(x) = \begin{Bmatrix} x^2 - 5x + 6, & x < -2 \\ -x^2 + 5x - 6, & -2 \le x \le 3 \\ x^2 - 5x + 6, & x > 3 \end{Bmatrix}$$

$$\Rightarrow f'(x) = \begin{Bmatrix} 2x - 5, & x < -2 \\ -2x + 5, & -2 \le x \le 3 \\ 2x - 5, & x > 3 \end{Bmatrix}$$

$\Rightarrow f'(4) = 8 - 5 = 3$
Hence, the correct answer is option C.

Solution 69

Given: $\mathrm{f(x) = |x^2 - 5x + 6|}$

$$\Rightarrow f(x) = \begin{Bmatrix} x^2 - 5x + 6, & x < -2 \\ -x^2 + 5x - 6, & -2 \le x \le 3 \\ x^2 - 5x + 6, & x > 3 \end{Bmatrix}$$

$$\Rightarrow f'(x) = \begin{Bmatrix} 2x - 5, & x < -2 \\ -2x + 5, & -2 \le x \le 3 \\ 2x - 5, & x > 3 \end{Bmatrix}$$

$$\Rightarrow f''(x) = \begin{Bmatrix} 2, & x < -2 \\ -2, & -2 \le x \le 3 \\ 2, & x > 3 \end{Bmatrix}$$

$\Rightarrow \mathrm{f''(2.5)} = -2$
Hence, the correct answer is option B.

Solution 70

Given: $f(x) = [x]$ and $g(x) = |x|$
$\Rightarrow (g \circ f)(x) = |[x]|$ and $(f \circ g)(x) = [|x|]$
$\therefore (g \circ f)\left(\frac{-5}{3}\right) - (f \circ g)\left(\frac{-5}{3}\right) = \left|\left[\frac{-5}{3}\right]\right| - \left[\left|\frac{-5}{3}\right|\right] = |-2| - \left[\frac{5}{3}\right] = 2 - 1 = 1$
Hence, the correct answer is option C.

Solution 71

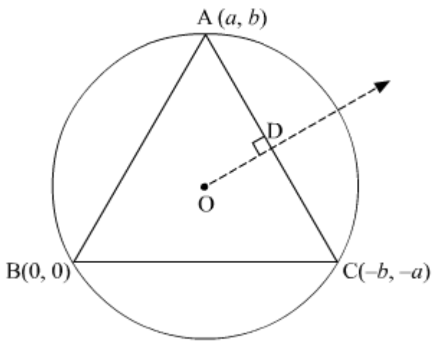
Given: $f(x) = [x]$ and $g(x) = |x|$

$$\Rightarrow \left(f\ o\ f\right)\left(x\right) = [[x]] \text{ and } \left(g\ o\ g\right)\left(x\right) = ||x||$$

$$\therefore \left(f\ o\ f\right)\left(\frac{-9}{5}\right) + \left(g\ o\ g\right)(-2) = \left[\left[\frac{-9}{5}\right]\right] + ||-2|| = -2 + 2 = 0$$

Hence, the correct answer is option B.

Solution 72



Centre of the circle lies on the perpendicular bisector of any chord.

Coordinates of point mid-point of AC i.e., D are $\left(\frac{a-b}{2}, \frac{b-a}{2}\right)$

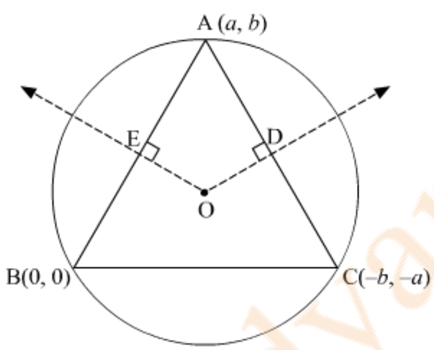
Slope of line AC = $\frac{b+a}{a+b} = 1$

Therefore, the slope of line OD = −1

⇒The equation of line OD is $y = -1(x) \Rightarrow x + y = 0$

Hence, the correct answeris option A.

Solution 73



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Coordinates of point mid-point of AC i.e., D are $\left(\frac{a-b}{2}, \frac{b-a}{2}\right)$

Slope of line AC = $\frac{b+a}{a+b} = 1$

Therefore, the slope of line OD = −1

⇒The equation of line OD is $y = -1(x) \Rightarrow x + y = 0$

Similarly, the equation of line OE is $\left(y - \frac{b}{2}\right) = \frac{-a}{b}\left(x - \frac{a}{2}\right) \Rightarrow ax + by = \frac{a^2+b^2}{2}$

Solving the equations of line OD and OE simultaneously, we get coordinates of the point O i.e., the $\left(\frac{a^2+b^2}{2(a-b)}, \frac{-\left(a^2+b^2\right)}{2(a-b)}\right)$

∴equation of the circle is $x^2 + y^2 - \frac{a^2+b^2}{(a-b)}x + \frac{a^2+b^2}{(a-b)}y = 0$

⇒ x-intercept = $\frac{a^2+b^2}{(a-b)}$ and y-intercept = $\frac{a^2+b^2}{(a-b)}$

∴Sum of the squares of the intercepts = $2\left(\frac{a^2+b^2}{a-b}\right)^2$

Hence, the correct answer is option B.

Solution 74

$$\left|\hat{a}+\hat{b}\right|^2 = \left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 + 2\left|\hat{a}\right|\left|\hat{b}\right|\cos\ \theta$$

$$\left|\hat{a}+\hat{b}\right|^2 = 1+1+2\cos\ \theta$$

$$\left|\hat{a}+\hat{b}\right|^2 = 2\left(1+\cos\ \theta\right)$$

$$\left|\hat{a}+\hat{b}\right|^2 = 2\left(2\cos^2\frac{\theta}{2}\right)$$

$$\left|\hat{a}+\hat{b}\right| = 2\cos\frac{\theta}{2}$$

$$\cos\left(\frac{\theta}{2}\right) = \frac{\left|\hat{a}+\hat{b}\right|}{2}$$

Hence, the correct answer is option B.

Solution 75

$$\left|\hat{a}-b\right|^2 = \left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 - 2\left|\hat{a}\right|\left|\hat{b}\right|\cos\ \theta$$

$$\left|\hat{a}-\hat{b}\right|^2 = 1+1-2\cos\ \theta$$

$$\left|\hat{a}-\hat{b}\right|^2 = 2\left(1-\cos\ \theta\right)$$

$$\left|\hat{a}-\hat{b}\right|^2 = 2\left(2\sin^2\frac{\theta}{2}\right)$$

$$\left|\hat{a}-\hat{b}\right| = 2\sin\frac{\theta}{2}$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{\left|\hat{a}-\hat{b}\right|}{2}$$

Hence, the correct answer is option A.

Solution 76

By properties of inverse we know $\tan^{-1}(\tan\theta) = \theta$ for all $\theta \in \left(\frac{-\pi}{2},\ \frac{\pi}{2}\right)$, hence statement 1 is incorrect.

Now,

$$\sin^{-1}\left(\frac{1}{3}\right) - \sin^{-1}\left(\frac{1}{5}\right)$$

$$= \sin^{-1}\left(\frac{1}{3}\sqrt{1-\left(\frac{1}{5}\right)^2} - \frac{1}{5}\sqrt{1-\left(\frac{1}{3}\right)^2}\right)$$

$$= \sin^{-1}\left(\frac{1}{3}\cdot\frac{\sqrt{24}}{5} - \frac{1}{5}\cdot\frac{\sqrt{8}}{3}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{24}}{15} - \frac{\sqrt{8}}{15}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{24}}{15} - \frac{\sqrt{8}}{15}\right)$$

$$= \sin^{-1}\left(\frac{2\sqrt{2}\left(\sqrt{3}-1\right)}{15}\right),\ \text{hence statement 2 is correct.}$$

Hence, the correct answer is option B.

Solution 77

$\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}\left(\frac{x+\frac{1}{x}}{1-x.\frac{1}{x}}\right) = \tan^{-1}\infty = \frac{\pi}{2}$, hence statement 1 is incorrect.

$\sin^{-1} x + \cos^{-1} y = \frac{\pi}{2}$

$\Rightarrow \sin^{-1} x = \frac{\pi}{2} - \cos^{-1} y$

$\Rightarrow \sin^{-1} x = \sin^{-1} y$

$\Rightarrow x = y,\ \text{hence statement 2 is also incorrect}$

Hence, the correct answer is option D.

Solution 78

$y = cx + c^2 - 3c^{3/2} + 2 \quad \ldots(i)$

$\Rightarrow y' = c$

substituting 'c' in (i), we get

$y = y'x + (y')^2 - 3(y')^{3/2} + 2$

$\Rightarrow \left(y - y'x - (y')^2 - 2\right)^2 = 9(y')^3$

from the above differential equation we observe that order is 1 and degree is 4.
Hence, the correct answer is option D.

Solution 79

$\int_{-2}^{2} x \, dx - \int_{-2}^{2} [x] \, dx$

$= \left[\frac{x^2}{2}\right]_{-2}^{2} - \int_{-2}^{-1} -2 \, dx - \int_{-1}^{0} -1 \, dx - \int_{0}^{1} 0 \, dx - \int_{1}^{2} 1 \, dx$

$= 2$

Hence, the correct answer is option (c).

Alternate method:

$\int_{-2}^{2} x \, dx - \int_{-2}^{2} [x] \, dx = \int_{-2}^{2} \{x\} \, dx$

We know, for any integer r and k, $\int_{r}^{r+k} \{x\} \, dx = \left|\frac{k}{2}\right|$

Therefore, $\int_{-2}^{2} \{x\} \, dx = \int_{-2}^{-2+4} \{x\} \, dx = \frac{4}{2} = 2$

Solution 80

$\text{Given}: \int_{0}^{5} \{1 + f(x)\} \, dx = 7$

$\Rightarrow \int_{0}^{5} f(x) \, dx = 7 - 5 = 2 \quad \ldots(i)$

$\text{Given}: \int_{-2}^{5} f(x) \, dx = 4$

$\Rightarrow \int_{-2}^{0} f(x) \, dx + \int_{0}^{5} f(x) \, dx = 4$

$\Rightarrow \int_{-2}^{0} f(x) \, dx + 2 = 4 \qquad [\text{from } (i)]$

$\Rightarrow \int_{-2}^{0} f(x) \, dx = 2$

Hence, the correct answer is option B.

Solution 81

$\text{Given}: \lim_{x \to 0} \phi(x) = a^2$

$\Rightarrow \lim_{\frac{x}{a} \to 0} \phi\left(\frac{x}{a}\right) = a^2$

$\text{Therefore}, \lim_{x \to 0} \phi\left(\frac{x}{a}\right) = a^2$

Hence, the correct answer is option A.

Solution 82

$\lim\limits_{x \to 0} e^{-1/x^2} = \lim\limits_{x \to 0} \frac{1}{e^{1/x^2}} = \frac{1}{e^{\infty}} = 0$

Hence, the correct answer is option A.

Solution 83

$\text{adj}(A^{-1}) - (\text{adj } A)^{-1}$
$= (\text{adj } A)^{-1} - (\text{adj } A)^{-1} \quad [\because \text{adj}(A^{-1}) = (\text{adj } A)^{-1}]$
$= 0$
Hence, the correct answer is option B.

Solution 84

To convert a decimal number from decimal system to binary system we multiply the decimal by 2 then the fractional part by 2 and so on till we obtain 1. Taking the integral part of each result in order gives us the binary equivalent.
Here,
$0.3125 \times 2 = 0.625$
$0.625 \times 2 = 1.25$
$0.25 \times 2 = 0.5$
$0.5 \times 2 = 1$
Therefore the binary equivalent is 0.0101
Hence, the correct answer is option C.

Solution 85

For natural numbers n, m and p.
n is always a factor of $n \Rightarrow n\text{R}n \Rightarrow \text{R}$ is reflexive
If n is a factor of m, then m is not necessarily a factor of $n \Rightarrow n\text{R}m \not\Rightarrow m\text{R}n \Rightarrow \text{R}$ is not symmetric.
If n is a factor of m and m is a factor of p, then n is a factor of $p \Rightarrow n\text{R}m$ and $m\text{R}p \Rightarrow n\text{R}p \Rightarrow \text{R}$ is transitive.
Hence, the correct answer is option C.

Solution 86

$|\cos x|$ is a periodic function with period π.

$$\therefore \int_0^{4\pi} |\cos x| \, dx$$
$$= 4\int_0^{\pi} |\cos x| \, dx$$
$$= 4\left\{\int_0^{\pi/2} \cos x \, dx - \int_{\pi/2}^{\pi} \cos x \, dx\right\}$$
$$= 4\left\{[\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi}\right\}$$
$$= 4\left\{\left(1 - 0\right) - \left(0 - 1\right)\right\}$$
$$= 8$$

Hence, the correct answer is option D.

Solution 87

Let A, B and C be set of natural numbers less than or equal to 1000 divisible by 10, 15 and 25 respectively.
Number of numbers less than 1000 divisible by 10 i.e., $n(A) = 100$
Number of numbers less than 1000 divisible by 15 i.e., $n(B) = 66$
Number of numbers less than 1000 divisible by 25 i.e., $n(C) = 40$
Number of numbers less than 1000 divisible by 30 i.e., $n(A \cap B) = 33$
Number of numbers less than 1000 divisible by 50 i.e., $n(A \cap C) = 20$
Number of numbers less than 1000 divisible by 75 i.e., $n(B \cap C) = 13$
Number of numbers less than 1000 divisible by 150 i.e., $n(A \cap B \cap C) = 6$
$\Rightarrow n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
$\Rightarrow n(A \cup B \cup C) = 100 + 66 + 40 - (33 + 20 + 13) + 6 = 146$
So, there are 146 natural numbers less than or equal to 1000 that are divisible by 10 or 15 or 25.
Therefore, the number of natural numbers less than or equal to 1000 that is not divisible by 10 or 15 or 25 = 1000 − 146 = 854

Hence, the correct answer is option B.

Solution 88

If (a, 2b) is the mid-point of line segment joining (10, −6) and (k, 4), then
$a = \frac{10+k}{2}$ and $2b = \frac{-6+4}{2} \Rightarrow b = \frac{-1}{2}$
Replacing a and b in given equation a − 2b = 7, we get
$\frac{10+k}{2} - \left(-1\right) = 7$

$\Rightarrow k = 2$

Hence, the correct answer is option A.

Solution 89

Statement 1:
If ABC is an equilateral triangle then ∠A, ∠B and ∠C are 60°.
∴ 3 tan (A + B) tan C = 3 tan 120°tan 60° = −9, hence statement 1 is incorrect.
Statement 2:
Given: ∠A = 78°, ∠B = 66° ⇒ ∠C = 36 °
∴ tan$\left(\frac{A}{2} + C\right)$ = tan 75°

(tan 75°) is less than (tan 78°) i.e., tan C
⇒ tan$\left(\frac{A}{2} + C\right)$ < tan C, hence statement 2 is correct.
Statement 3:
tan$\left(\frac{A+B}{2}\right)$ sin $\left(\frac{C}{2}\right)$
= tan$\left(\frac{\pi}{2} - \frac{C}{2}\right)$ sin $\left(\frac{C}{2}\right)$
= cot $\left(\frac{C}{2}\right)$ sin $\left(\frac{C}{2}\right)$
= cos $\left(\frac{C}{2}\right)$, hence statement 3 is incorrect.
Hence, the correct answer is option B.

Solution 90

Given: A = (cos 12° − cos 36°)(sin 96° + sin 24°) and B = (sin 60° − sin 12°)(cos 48° − cos 72°)
⇒ A = (2 sin 24° sin 12°)(2 sin 60° cos 36°) and B = (2 cos 36° sin 24°)(2 sin 60° sin 12°)
⇒ A = B ⇒ $\frac{A}{B}$ = 1
Hence, the correct answer is option C.

Solution 91

Mean = $\frac{10+9+21+16+24}{5} = 16$
⇒ Mean Deviation = $\sum_{i=1}^{n} \frac{|x_i - \overline{x}|}{n} = \frac{6+7+5+0+8}{5} = 5.2$
Hence, the correct answer is option A.

Solution 92

Sum on the three faces of dice is less than 5 for all the permutations of (1,1,1), (1,1,2), (1,1,3) and (1,2,2).
Permutations of 1,1,1 is 1
Permutations of 1,1,2 is $\frac{3!}{2!} = 3$
Permutations of 1,1,3 is $\frac{3!}{2!} = 3$
Permutations of 1,2,2 is $\frac{3!}{2!} = 3$
Hence, the total number of ways of getting the sum less than 5 = 1 + 3 + 3 + 3 = 10
Therefore, the number of ways of getting the sum atleast 5 = Total outcomes in rolling 3 dice − 10 = 6^3 − 10 = 206
⇒ Probability(Sum ≥ 5) = 206/216 = 53/54
Hence, the correct answer is option B.

Solution 93

Given: P(A) = 1/3 and P(B) = 3/4
As A and B are independent events, implies P($A∩B$) = P(A)P(B) = 1/4
The probability that exactly one of the two events A or B occur = P(A) + P(B) − 2P($A∩B$) = 1/3 + 3/4 − 2/4 = 7/12.
Hence, the correct answer is option D.

Solution 94

$\text{Probability} = \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$\text{Total outcomes when a coin is tossed 3 times} = 2^3 = 8$
$\text{Favourable outcomes are outcomes of getting head and tail alternately} \Rightarrow \text{HTH and THT}$
$\text{Hence, favourable outcomes} = 2$
$\therefore \text{Required Probability} = \frac{2}{8} = \frac{1}{4}$

Hence, the correct answer is option B.

Solution 95

$\text{Var}\left(X\right) = \frac{1}{n}\sum_{\text{i}=1}^{\text{n}} {x_i}^2 - \left\{\frac{1}{n}\sum_{i=1}^{n} x_i\right\}^2$

$\text{Given}: \sum x_i = 1000, \sum {x_i}^2 = 84000 \text{ and } n = 20$

$\therefore \text{Var}\left(X\right) = \frac{1}{20} \times 84000 - \left(\frac{1}{20} \times 1000\right)^2 = 4200 - 2500 = 1700$

Hence, the correct answer is option C.

Solution 96

Number of queen of spade = 1
Therefore, the probability of getting queen of spade = 1/52.
Hence, the correct answer is option A.

Solution 97

Total number of outcomes = 6^2 = 36
Number of outcomes having the sum less than 4 i.e. (1, 1), (1, 2) and (2, 1) = 3

Number of outcomes having the sum greater than or equal to 4 = 36 − 3 = 33

Thus, required probability = $\frac{33}{36} = \frac{11}{12}$
Hence, the correct answer is option C.

Solution 98

Let n number of missiles be fired. p = probability that the missile hits the target = 0.3
q = probability that the missile misses the target = 0.7
Let x is a random variable that defines the number of times missile hits the target
Now, P(x = r) = ${}^nC_r\, p^r\, q^{n-r} = {}^nC_r\, (0.3)^r\, (0.7)^{n-r}$
According to the question,
P(x ≥ 1) ≥ 0.8
⇒ 1 − P(x = 0) ≥ 0.8
⇒ 1 − ${}^nC_0\, (0.3)^0\, (0.7)^{n-0}$ ≥ 0.8
⇒ 1 − $(0.7)^n$ ≥ 0.8
⇒ $(0.7)^n$ ≤ 0.2
n ≥ 5, Minimum value of n = 5.
Hence, the correct answer is option A.

Solution 99

$P(A|(A \cup B)) = \frac{P(A\cap(A\cup B))}{P(A\cup B)}$

$= \frac{P(A)}{P(A \cup B)} \qquad [\because P(A \cap (A \cup B)) = P(A)]$

$= \frac{P(A)}{P\left(A\right) + P\left(\overline{A} \cap B\right)} \qquad \left[\because P\left(\overline{A} \cap B\right) = P\left(A \cup B\right) - P\left(A\right)\right]$

$= \frac{0.2}{0.2+0.3} = \frac{2}{5}$

Hence, the correct answer is opiton B.

Solution 100

There are 31 days in December, implies 4 weeks and 3 extra days.
Each of the 4 weeks will have exactly one Sunday, therefore 4 weeks will have 4 Sundays.
Now, the remaining 3 days can be
Sun, Mon, Tue

or Mon, Tue, Wed

or Tue, Wed, Thu

or Wed, Thu, Fri

or Thu, Fri, Sat

or Fri, Sat, Sun

or Sat, Sun, Mon

Total number of outcomes = 7

Favourable number of outcomes i.e. Sun, Mon, Tue; Fri, Sat, Sun; Sat, Sun, Mon = 3

Thus, required probability = $\frac{3}{7}$

Hence, the correct answer is option C.

Solution 101

$m = \left(\left(\frac{y}{z}\right)^{\log(yz)} \left(\frac{z}{x}\right)^{\log(zx)} \left(\frac{x}{y}\right)^{\log(xy)}\right)^{1/3}$

$m = \left(x^{\log(xy)-\log(zx)} y^{\log(yz)-\log(xy)} z^{\log(zx)-\log(yz)}\right)^{1/3}$

$m = \left(x^{\log\left(\frac{y}{z}\right)} y^{\log\left(\frac{z}{x}\right)} z^{\log\left(\frac{x}{y}\right)}\right)^{1/3}$

$\log\ m = \frac{1}{3}\left(\log\left(\frac{y}{z}\right)\log\ x + \log\left(\frac{z}{x}\right)\log\ y + \log\left(\frac{x}{y}\right)\log\ z\right)$

$3\log\ m = (\log\ y - \log\ z)\log\ x + (\log\ z - \log\ x)\log\ y + (\log\ x - \log\ y)\log\ z$

$3\log\ m = \log\ y\ \log\ x - \log\ z\ \log\ x + \log\ z\ \log\ y - \log\ x\ \log\ y + \log\ x\ \log\ z - \log\ y\ \log\ z$

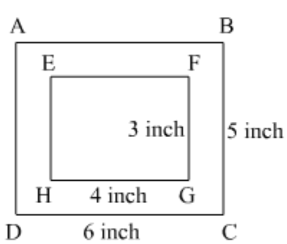
$3\log\ m = 0$

$\log\ m = 0$

$\therefore m = 1$

Hence, the correct answer is option A.

Solution 102



$\text{Required probability} = \frac{\text{Area of rectangle EFGH}}{\text{Aera of rectangle ABCD}}$

$\therefore$ Required probability $= \frac{4\times3}{6\times5} = \frac{2}{5}$
Hence, the correct answer is option D.

Solution 103

Mean of $x_1,\ x_2,\ x_3, \ldots, x_n$ is given as $\overline{X}$.
$\therefore x_1 + x_2 + \ldots + x_n = n\,\overline{X}$ $\quad\ldots(i)$
Now, x_2 is replaced by λ
$\therefore$ New Mean $= \frac{(x_1+x_2+\ldots+x_n)-x_2+\lambda}{n}$
$\Rightarrow$ New Mean $= \frac{n\overline{X}-x_2+\lambda}{n}$ $\quad$ [from (i)]
Hence, the correct answer is option D.

Solution 104

Data in order: 1, 2, 3, 5, 5, 5, 6, 6, 8, 9
Therefore,
Median = y is $\frac{5+5}{2}$ = 5
Mode = z = 5
Mean = x = $\frac{1+2+3+5+5+5+6+6+8+9}{10}$ = 5
⇒ $x = y = z$
Hence, the correct answer is option D.

Solution 105

Statement 1 and 2 are properties of the histogram.
Hence, the correct answer is option C.

Solution 106

Here, n = 100
p = probability of getting a tail in a toss of a coin = 1/2
q = probability of getting a head in a toss of a coin = 1/2

Probability of getting tail an odd number of times
$= \sum_{r=1}^{50} {}^{n}C_{2r-1}(p)^{2r-1}(q)^{n-(2r-1)}$
$= \sum_{r=1}^{50} {}^{100}C_{2r-1}\left(\frac{1}{2}\right)^{2r-1}\left(\frac{1}{2}\right)^{100-(2r-1)}$
$= \sum_{r=1}^{50} {}^{100}C_{2r-1}\left(\frac{1}{2}\right)^{100}$
$= \left(\frac{1}{2}\right)^{100}\left({}^{100}C_1 + {}^{100}C_3 + {}^{100}C_5 + \ldots + {}^{100}C_{99}\right)$

$= \left(\frac{1}{2}\right)^{100}\left(2^{99}\right)$ $\quad \left[\because\ {}^{n}C_1 + {}^{n}C_3 + {}^{n}C_5 + \ldots = 2^{n-1}\right]$
$= \frac{1}{2}$
Hence, the correct answer is option A.

Solution 107

Each ticket can be given to anyone out of the 10 employee

The required number of ways in which tickets can be distributed = 10 × 10 × 10 = 1000.
Hence, the correct answer is option D.

Solution 108

Let the roots of the equation $(l-m)x^2 + lx + 1 = 0$ be α and 2α.
Sum of roots = 3α = $\frac{-l}{l-m}$
Product of roots = $2\alpha^2$ = $\frac{1}{l-m}$

solving both the equations, we get
$2l^2 - 9l + 9m = 0$
for l to be real the discriminant of the above equation must be greater than or equal to 0.

$\Rightarrow 81 - 72m \geq 0$
$\Rightarrow m \leq 9/8$
so, the greatest value of m is 9/8
Hence, the correct answer is option B.

Solution 109

The ten-thousandths place can be filled in 9 ways except 0. Thousandths, hundredths and tenths place can be filled in 9, 8, 7 ways respectively when the repetition is not allowed.
The required number of ways = 7×8×9×9 = 4536.
Hence, the correct answer is option B.

Solution 110

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$, then vector orthogonal to these vectors is

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 3 & -1 \end{vmatrix} = -4\hat{i} + 3\hat{j} + \hat{k}$$

Therefore, the unit vector along $\vec{a} \times \vec{b}$ is $\frac{-4\hat{i}+3\hat{j}+\hat{k}}{\sqrt{(-4)^2+3^2+1^2}} = \frac{-4\hat{i}+3\hat{j}+\hat{k}}{\sqrt{26}}$

Hence, the correct answer is option B.

Solution 111

In an equilateral triangle orthocentre, circumcentre, centroid and incentre coincide. Therefore, the orthocentre = centroid = $\vec{a} + \vec{b} + \vec{c} = 0.$
Hence, the correct answer is option A.

Solution 112

Area of parallelogram

$$= \left| \frac{1}{2} \left(3\hat{i} + \hat{j} - 2\hat{k}\right) \times \left(\hat{i} - 3\hat{j} + 4\hat{k}\right) \right|$$

$$= \left| \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} \right|$$

$$= \frac{1}{2} \left| -2\hat{i} - 14\hat{j} - 10\hat{k} \right|$$

$$= \frac{1}{2} \times \sqrt{(-2)^2 + (-14)^2 + (-10)^2}$$

$$= 5\sqrt{3} \text{ square units}$$

Hence, the correct answer is option C.

Solution 113

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$\therefore A^2 = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = -2 \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = -2A$, hence statement 1 is incorrect.
Now, $A^3 = A.\,A^2 = A(-2A) = -2A^2 = -2(-2A) = 4A$, hence statement 2 is correct.
Hence, the correct answer is option B.

Solution 114

Statement 1:

$$\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$$

$C_1 \to C_1 - 8C_3$

$$\begin{vmatrix} 1 & 1 & 5 \\ 7 & 7 & 9 \\ 5 & 5 & 3 \end{vmatrix} = 0, \text{ because } \mathrm{c}_1 \text{ and } \mathrm{c}_2 \text{ are the same.}$$

Hence, statement 1 is correct.

Satement 2:

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

$C_3 \to C_3 + C_2$

$$\begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} = \left(a+b+c\right) \begin{vmatrix} 1 & a & a \\ 1 & b & b \\ 1 & c & c \end{vmatrix} = 0$$

Hence, statement 2 is correct.

Statement 3:

It's a skew-symmetric matrix of order 3. The determinant of a skew-symmetric matrix of odd order is zero.

Hence, statement 3 is also correct.

Hence, the correct answer is option D.

Solution 115

Given lines are $l_1 \equiv y - \sqrt{3}x - 5 = 0$, with slope, m_1 = $\sqrt{3}$ and $l_2 \equiv \sqrt{3}y - x + 6 = 0$, with slope, m_2 = $\frac{1}{\sqrt{3}}$.

The acute angle between l_1 and l_2 = $\tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \tan^{-1}\left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}\right| = \tan^{-1}\frac{1}{\sqrt{3}} = 30^\circ$

Hence, the correct answer is option A.

Solution 116

The system of equations has a unique solution if

$$\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} \neq 0$$

$\Rightarrow k^3 - 3k + 2 \neq 0$

$\Rightarrow \left(k-1\right)^2\left(k+2\right) \neq 0$

$\Rightarrow k \neq 1 \text{ and } k \neq -2$

Hence, the correct answer is option A.

Solution 117

The number of different messages represented by three 0's and two 1's is the number of arrangements of 0, 0, 0, 1, 1.

Number of arrangements of 00011 = $\frac{5!}{3!\,2!} = 10$

Hence, the correct answer is option A.

Solution 118

$\log_a(ab) = x$

$\Rightarrow \frac{\log\ a + \log\ b}{\log\ a} = x$

$\Rightarrow 1 + \frac{\log\ b}{\log\ a} = x$

$\Rightarrow \frac{\log\ a}{\log\ b} = \frac{1}{x-1}$

$\Rightarrow 1 + \frac{\log\ a}{\log\ b} = 1 + \frac{1}{x-1}$

$\Rightarrow \frac{\log\ b + \log\ a}{\log\ b} = \frac{x}{x-1}$

$\Rightarrow \frac{\log\ ba}{\log\ b} = \frac{x}{x-1}$

$\Rightarrow \log_b\left(ab\right) = \frac{x}{x-1}$

Hence, the correct answer is option D.

Solution 119

$y = \frac{\ln\ x}{\ln\ 10} + \frac{\ln\ 10}{\ln\ x} + 1 + 1$

$\therefore \frac{dy}{dx} = \frac{1}{x \ln\ 10} - \frac{\ln\ 10}{(\ln\ x)^2} \cdot \frac{1}{x}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=10} = \frac{1}{10\ \ln\ 10} - \frac{\ln\ 10}{(\ln\ 10)^2} \cdot \frac{1}{10} = 0$

Hence, the correct answer is option D.

Solution 120

Let $\omega_1 = \omega$ and $\omega_2 = \omega^2$

$$\begin{aligned}\Rightarrow \left(\omega_1 - \omega_2\right)^2 &= \left(\omega - \omega^2\right)^2 \\ &= \omega^2 + \omega^4 - 2\omega^3 \\ &= \omega^2 + \omega - 2 \\ &= -1 - 2\ = -3\end{aligned}$$

Hence, the correct answer is option D.