## NDA II 2016_Mathematics

**Total Time: 150**

**Total Marks: 300.0**

### Section A

**Solution 1**

The number of different numbers of the form $\frac{p}{q}$ are :

If $p$ = 1, $q$ = 1,2,3,4,5,6
$p$ = 2, $q$ = 1,3,5,
$p$ = 3, $q$ = 1,2,4,5,
$p$ = 4, $q$ = 1,3,5
$p$ = 5, $q$ = 1,2,3,4,6
$p$ = 6, $q$ = 1,5

Total different numbers = 6 + 3 + 4 + 3 + 5 + 2 = 23
So, the cardinality of the set S is 23.

Hence, the correct answer is option B.

**Solution 2**

Given: $c > 0$ and $4a + c < 2b$ then,

For $x = 0$, $f(0) = a(0) + b(0) + c = c > 0$

For $x = 2$, $f(2) = a(2)^2 - b(2) + c = 4a - 2b + c < 0$
So, $f(0) > 0$ and $f(2) < 0$

Thus, there is root in the interval (0, 2).
Hence, the correct answer is option A.

**Solution 3**

$A = \{x^2 + 6x - 7 < 0\}$
$A = \{(x + 7)(x - 1) < 0\}$
$A = \{-7 < x < 1\}$
$B = \{x^2 + 9x + 14 > 0\}$
$B = \{(x + 7)(x + 2) > 0\}$
$B = \{x < -7 \text{ or } x > -2\}$
$A \cap B = (x \in R : -2 < x < 1)$
$A/B = (x \in R : -7 < x < -2)$
Thus, both 1 & 2 are correct
Hence, the correct answer is option C.

**Solution 4**

$$\det \text{ of } \left[\left(xA\right)^{-1}\right] = x^n \frac{1}{det\,(A)}$$

$$\det \text{ of } \left[\left(2A\right)^{-1}\right] = 2^3 \times \frac{1}{5} = \frac{8}{5}$$

$n$: order of matrix A
$n$ = 3
Hence, the correct answer is option C.

**Solution 5**

$\omega^{100} = \left(\omega^3\right)^{33} * \omega = \omega \quad \left(\omega^3 = 1\right)$

$\omega^{200} = \left(\omega^3\right)^{66} * \omega^2 = \omega^2 \quad \left(\omega^3 = 1\right)$

$\omega^{300} = \left(\omega^3\right)^{100} = 1 \quad \left(\omega^3 = 1\right)$

$\omega^{100} + \omega^{200} + \omega^{300} = \omega + \omega^2 + 1 = 0 \quad (\because \text{As the sum of cube roots of unity is always zero})$

Hence, the correct answer is option D.

**Solution 6**

Let $z = x + iy$

$\Rightarrow \frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy} = \frac{[(x-1)+iy][(x+1)-iy]}{[(x+1)+iy][(x+1)-iy]}$

$\Rightarrow \frac{z-1}{z+1} = \frac{x^2+y^2-1+2iy}{x^2+y^2+2x+1}$

$\Rightarrow \text{Re}\left(\frac{z-1}{z+1}\right) = \frac{x^2+y^2-1}{x^2+y^2+2x+1} = 0$

$x^2 + y^2 - 1 = 0$

Now, $z\bar{z} = \left(x + iy\right)\left(x - iy\right)$

$z\bar{z} = x^2 + y^2 - 1 = 0$

$z\bar{z} = x^2 + y^2 = 1 = |z|^2$

$\Rightarrow |z|^2 = 1$

Hence, the correct answer is option D.

**Solution 7**

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

$$= \begin{bmatrix} ax + hy + gz & hx + by + fz & gx + fy + cz \end{bmatrix}$$

Hence, the correct answer is option D.

**Solution 8**

If there are *n* distinct points in a plane, out of which *m* are collinear then the number of the triangles formed $= {}^nC_3 - {}^mC_3$

There are 15 points in a plane out of which *n* points are in same straight line, so number of triangles are = ${}^nC_3 - {}^mC_3$

${}^{15}C_3 - {}^nC_3 = 445$

$\Rightarrow \frac{15}{12!\times3!} - \frac{n!}{(n-3)!\times3!} = 445$

$455 - {}^nC_3 = 445$

$\Rightarrow {}^nC_3 = 10$

$\Rightarrow n = 5$

Hence, the correct answer is option C.

**Solution 9**

Given:

$z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{107} + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{107}$

$= (\cos 30^\circ + \sin 30^\circ)^{107} + (\cos 30^\circ - \sin 30^\circ)^{107}$

$= [\cos(30 \times 107)^\circ + \sin(30 \times 107)^\circ] + [\cos(30 \times 107)^\circ - \sin(30 \times 107)^\circ]$

(Using DeMoivre's Theorem)

$= 2\cos(30 \times 107)^\circ$

Clearly, *z* is purely real.

Hence, the correct answer is option A.

**Solution 10**

$x^2 - 2kx + k^2 - 4 = 0$

$\left(x - k\right)^2 - 4 = 0$

$\left(x - k - 2\right)\left(x - k + 2\right) = 0$

so the two roots of this quadratic are :

$x = k + 2$ & $x = k - 2$

now these two roots lie between $\left(-3,\ 5\right)$

$\Rightarrow k + 2 < 5$ and $k - 2 > -3$

$\Rightarrow k < 3$ and $k > -1$

$\Rightarrow -1 < k < 3$

Hence, the correct answer is option D.

**Solution 11**

$z^2 + |z| = 0 \qquad .....(i)$

$\Rightarrow z^2 = -|z|$

$\Rightarrow |z^2| = |-|z|| = |-1|\,|z|$

$\Rightarrow |z^2| = |z|$

$\Rightarrow |z|^2 = |z|$
$\Rightarrow |z|^2 - |z| = 0$
$\Rightarrow |z|(|z| - 1) = 0$
$\Rightarrow |z| = 0 \text{ or } |z| = 1$

Putting $|z| = 0$ in (*i*), we have
$z^2 = 0 \Rightarrow z = 0$

Putting $|z| = 1$ in (*i*), we have
$z^2 + 1 = 0 \Rightarrow z = \pm i$

Therefore, the given equation has three distinct solutions.

Hence, the correct answer is option C.

**Solution 12**

Let *a* be the first term and *r* be the common ratio of the G.P.
Now let 27, 8 and 12 are the $p^{th}$ , $q^{th}$ and the $t^{th}$ term of this G.P.

$$27 = a(r)^{p-1} \qquad .....(1)$$
$$8 = a(r)^{q-1} \qquad .....(2)$$
$$12 = a(r)^{t-1} \qquad .....(3)$$

Now, $27 \times 8^2 = 12^3$

Using (1), (2) & (3)

$$a(r)^{p-1} \times \left(a(r)^{q-1}\right)^2 = \left(a(r)^{t-1}\right)^3$$
$$(r)^{p-1} \times \left((r)^{q-1}\right)^2 = \left((r)^{t-1}\right)^3$$
$$p - 1 + 2q - 2 = 3t - 3$$
$$p + 2q - 3t = 0$$

Thus, we see that the given equation can have infinite solutions.
Hence, the correct answer is option D.

**Solution 13**

R = {(1,3)(1,5)(2,3)(2,5)(3,5)(4,5)}
Domain = {1, 2, 3, 4}
Range = { 3, 5}
$R^{-1}$ = {(3,1)(5,1)(3,2)(5,2)(5,3)(5,4)}
Domain = {3, 5}
Range = {1, 2, 3, 4}
$RoR^{-1}$ = {(3,3)(3,5)(5,3)(5,5)}

Hence, the correct answer is option C.

**Solution 14**

A number is only divisible by 3 when the sum of its digits is divisible by 3 too.

We have five digits 0, 1, 2, 3, 4 and we have to form a five digit number without repetition so all of them need to be used

but the sum is 0 + 1 + 2 + 3 + 4 = 10, which is not divisible by 3.
So no such number can be formed.
Hence, the correct answer is option D.

**Solution 15**

Expanding the above summation

$$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$$
$$= \left({}^{47}C_3 + {}^{47}C_4\right) + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$$
$$= \left({}^{48}C_4 + {}^{48}C_3\right) + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 \qquad \left(\text{as } {}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}\right)$$
$$= \left({}^{49}C_4 + {}^{49}C_3\right) + {}^{50}C_3 + {}^{51}C_3$$
$$= \left({}^{50}C_4 + {}^{50}C_3\right) + {}^{51}C_3$$
$$= {}^{51}C_3 + {}^{51}C_4$$
$$= {}^{52}C_4$$

Hence, the correct answer is option A.

**Solution 16**

$S_n = \frac{n}{2}(a+l)$

$\Rightarrow a+x+y+z+b = \frac{5}{2}(a+b)$

$\Rightarrow a+b+15 = \frac{5}{2}(a+b) \qquad (x+y+z=15)$

$\Rightarrow \mathrm{a+b=10}$

$\text{Now for the H.P.}$

$\frac{1}{a}+\frac{1}{p}+\frac{1}{q}+\frac{1}{r}+\frac{1}{b} = \frac{5}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{5}{3} = \frac{5}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\Rightarrow \frac{3(a+b)}{ab} = \frac{10}{3}$

$\Rightarrow \frac{3\times10}{ab} = \frac{10}{3}$

$\Rightarrow ab = 9$

Hence, the correct answer is option B.

**Solution 17**

$S_n = \frac{n}{2}(a+l)$

$\Rightarrow a+x+y+z+b = \frac{5}{2}(a+b)$

$\Rightarrow a+b+15 = \frac{5}{2}(a+b) \qquad (x+y+z=15)$

$\Rightarrow \mathrm{a+b=10}$

$\text{Now for the H.P.}$

$\frac{1}{a}+\frac{1}{p}+\frac{1}{q}+\frac{1}{r}+\frac{1}{b} = \frac{5}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{5}{3} = \frac{5}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\Rightarrow \frac{3(a+b)}{ab} = \frac{10}{3}$

$\Rightarrow \frac{3\times10}{ab} = \frac{10}{3}$

$\Rightarrow ab = 9$

$a + b = 10$ .........(1)
$ab = 9$ .........(2)
Now there are two possibilities
$a = 1$ and $b = 9$ or $a = 9$ and $b = 1$
when $a = 1$ and $b = 9$
$a + 4d = 9$
$1 + 4d = 9$
$d = 2$
for $a = 1$ and $d = 2$
$x = 3$, $y = 5$ and $z = 7$
When $a = 9$ and $b = 1$
$a + 4d = 1$
$9 + 4d = 1$
$d = -2$
for $a = 9$ and $d = -2$
$x = 7$, $y = 5$ and $z = 3$
In both cases $xyz = 7 \times 5 \times 3$
$= 105$
Hence, the correct answer is option B.

**Solution 18**

$a, x, y, z, b$ be in AP, where $x + y + z = 15$. Also, $a, p, q, r, b$ be in HP, where $p^{-1}+q^{-1}+r^{-1} = 5/3$.
So, $ab = 9$ and $a + b = 10$.
We can either have a = 1 and b = 9 or a = 9 and b = 1.
Now $a, p, q, r, b$ are in H.P
With $a = 1$ and $b = 9$,
So 1, $p, q, r$ and 9 are in H.P

$\frac{1}{1}, \frac{1}{p}, \frac{1}{q}, \frac{1}{r}\ \&\ \frac{1}{9}\ \text{are in A.P.}$

$\Rightarrow 1+(5-1)d = 9$

$\Rightarrow \frac{1}{1+4d} = 9$

$\Rightarrow d = \frac{-2}{9}$

$\text{And } \frac{1}{p} = 1-\frac{2}{9} = \frac{7}{9}$

$p = \frac{9}{7}$

$\frac{1}{q} = \frac{7}{9}-\frac{2}{9} = \frac{5}{9}$

$q = \frac{9}{5}$

$\frac{1}{r} = \frac{5}{9}-\frac{2}{9} = \frac{3}{9}$

$r = \frac{9}{3}$

$p\times q\times r = \frac{9}{7}\times\frac{9}{5}\times\frac{9}{3} = \frac{243}{35}$

Hence, the correct answer is option C.

**Solution 19**

$a + 5d = 2$ and $d > 1$

Now the product of $a_1$, $a_4$ and $a_5$ need to be greatest

$a \times \left(a+3d\right) \times \left(a+4d\right) \ = \text{greatest}$

$a+\ 5d=2\ \Rightarrow a=2-5d$

$\text{Product }(p)=a\times\left(a+3d\right)\times\left(a+4d\right)=\left(2-5d\right)\times\left(2-5d+3d\right)\times\left(2-5d+4d\right)$

$\text{Product }(p)=-10d^3+34d^2-32d+8$

$\frac{dp}{dx}=15d^2-34d-16=0$

$\left(5d-8\right)\left(3d-2\right)=0$

$d=\frac{8}{5}\ \text{or}\ \frac{2}{3}$

$\frac{2}{3}\ \text{is not posssible as}\ x\ >1$

$\text{so,}\ d=\frac{8}{5}$

Hence, the correct answer is option A.

**Solution 20**

$a + 5d = 2$ and $d > 1$

Now the product of $a_1$, $a_4$ and $a_5$ need to be greatest

$a \times \left(a+3d\right) \times \left(a+4d\right) \ = \text{greatest}$

$a+\ 5d=2\ \Rightarrow a=2-5d$

$\text{Product }(p)=a\times\left(a+3d\right)\times\left(a+4d\right)=\left(2-5d\right)\times\left(2-5d+3d\right)\times\left(2-5d+4d\right)$

$\text{Product }(p)=-10d^3+34d^2-32d+8$

$\frac{dp}{dx}=15d^2-34d-16=0$

$\left(5d-8\right)\left(3d-2\right)=0$

$d=\frac{8}{5}\ \text{or}\ \frac{2}{3}$

$\frac{2}{3}\ \text{is not posssible as}\ x\ >1$

$\text{so,}\ d=\frac{8}{5}$

$a = 2 - 5d$

$a=2-5\left(\frac{8}{5}\right)=-6$

Hence, the correct answer is optin B.

**Solution 21**

Given:

$$ax^3+bx^2+cx+d=\begin{vmatrix}x+1 & 2x & 3x\\ 2x+3 & x+1 & x\\ 2-x & 3x+4 & 5x-1\end{vmatrix}$$

Differentiating both sides, we get

$$3ax^2+2bx+c=\begin{vmatrix}1 & 2 & 3\\ 2x+3 & x+1 & x\\ 2-x & 3x+4 & 5x-1\end{vmatrix}+\begin{vmatrix}x+1 & 2x & 3x\\ 2 & 1 & 1\\ 2-x & 3x+4 & 5x-1\end{vmatrix}+\begin{vmatrix}x+1 & 2x & 3x\\ 2x+3 & x+1 & x\\ -1 & 3 & 5\end{vmatrix}$$

To find the value of $c$, put $x = 0$ in the above equation. We get,

$$c=\begin{vmatrix}1 & 2 & 3\\ 3 & 1 & 0\\ 2 & 4 & -1\end{vmatrix}+\begin{vmatrix}1 & 0 & 0\\ 2 & 1 & 1\\ 2 & 4 & -1\end{vmatrix}+\begin{vmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ -1 & 3 & 5\end{vmatrix}$$

$\Rightarrow c=\left[1\left(-1\right)-2\left(-3\right)+3\left(10\right)\right]+\left[1\left(-5\right)\right]+\left[1\left(5\right)\right]$

$\Rightarrow c=35-5+5=35$

Hence, the correct answer is option C.

**Solution 22**

Given:

$$ax^3+bx^2+cx+d=\begin{vmatrix}x+1 & 2x & 3x\\ 2x+3 & x+1 & x\\ 2-x & 3x+4 & 5x-1\end{vmatrix}$$

Put $x = 1$ in the above equation.

$$a+b+c+d=\begin{vmatrix}2 & 2 & 3\\ 5 & 2 & 1\\ 1 & 7 & 4\end{vmatrix}$$

$=2\left(1\right)-2\left(19\right)+3\left(33\right)$

$=63$

Hence, the correct answer is option B.

**Solution 23**

Sum of all the interior angles of a triangle = $(n - 2) \times 180°$

sum of the A.P.

$S_n = \frac{n}{2}[2a+(n-1)d] \ = \ (n-2) \times 180^\circ$

$a \ = \ 120^\circ, \ d \ = \ 5^\circ$

$\Rightarrow \frac{n}{2}[2 \times 120^\circ + (n-1) \times 5^\circ] = \ (n-2) \times 180^\circ$

$\Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow (n-9)(n-16) = 0$

$n = 9 \text{ or } 16$

$\text{when } n = \ 16$

$\mathrm{T}_{16} \ = \ 120^\circ \ + \ (16-1) \times 5^\circ \ = \ 195^\circ \text{ which is not possible}$

$So \, n = 9 \text{ is the only correct answer}$

Hence, the correct answer is option (A)

**Solution 24**

Sum of all the interior angles of a triangle = ($n$ − 2) × 180º
sum of the A.P.

$S_n = \frac{n}{2}[2a+(n-1)d] \ = \ (n-2) \times 180^\circ$

$a \ = \ 120^\circ, \ d \ = \ 5^\circ$

$\Rightarrow \frac{n}{2}[2 \times 120^\circ + (n-1) \times 5^\circ] = \ (n-2) \times 180^\circ$

$\Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow (n-9)(n-16) = 0$

$n = 9 \text{ or } 16$

$\text{when } n = \ 16$

$\mathrm{T}_{16} \ = \ 120^\circ \ + \ (16-1) \times 5^\circ \ = \ 195^\circ \text{ which is not possible}$

$\text{So, } n = 9 \text{ is the only correct answer.}$

For $n$ = 9
$T_9$ = 120º + (9 − 1) × 5º
= 160º
So the largest possible angle is 160º

Hence, the correct answer is option A.

**Solution 25**

$m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow m \cos\,\theta \ = \ \begin{bmatrix} \cos\,\theta & 0 \\ 0 & \cos\,\theta \end{bmatrix}$

$n = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\Rightarrow n \sin\,\theta = \begin{bmatrix} 0 & \sin\,\theta \\ -\sin\,\theta & 0 \end{bmatrix}$

$m \cos\,\theta \ - \ n \sin\,\theta \ = \ \begin{bmatrix} \cos\,\theta & 0 \\ 0 & \cos\,\theta \end{bmatrix} - \begin{bmatrix} 0 & \sin\,\theta \\ -\sin\,\theta & 0 \end{bmatrix}$

$= \begin{bmatrix} \cos\,\theta & -\sin\,\theta \\ \sin\,\theta & \cos\,\theta \end{bmatrix}$

$|m \cos\,\theta \ - \ n \sin\,\theta| = \cos^2\,\theta \ + \ \sin^2\,\theta \ = \ 1$

Hence, the correct answer is option C.

**Solution 26**

$f(\theta) = \begin{bmatrix} \cos\,\theta & -\sin\,\theta & 0 \\ \sin\,\theta & \cos\,\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$f(\phi) = \begin{bmatrix} \cos\,\varphi & -\sin\,\phi & 0 \\ \sin\,\phi & \cos\,\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$f(\theta) \times f(\phi) = \ \begin{bmatrix} \cos\,\theta & -\sin\,\theta & 0 \\ \sin\,\theta & \cos\,\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\,\varphi & -\sin\,\phi & 0 \\ \sin\,\phi & \cos\,\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$f(\theta)\times f(\phi)=\begin{bmatrix}\cos\theta\times\cos\varphi+(-\sin\theta)\times\sin\phi+0\times0 & \cos\theta\times(-\sin\varphi)+(-\sin\theta)\times\cos\phi+0\times0 & \cos\theta\times0+(-\sin\theta)\times0+0\times1\\ \sin\theta\times\cos\varphi+\cos\theta\times\sin\phi+0\times0 & \sin\theta\times(-\sin\varphi)+\cos\theta\times\cos\phi+0\times0 & \sin\theta\times0+\cos\theta\times0+0\times1\\ 0\times\cos\varphi+0\times\sin\phi+1\times0 & 0\times(-\sin\varphi)+0\times\cos\phi+1\times0 & 0\times0+0\times0+1\times1\end{bmatrix}$$

$$f(\theta)\times f(\phi)=\begin{bmatrix}\cos\theta\cos\varphi-\sin\theta\sin\phi & -\cos\theta\sin\varphi-\sin\theta\cos\phi & 0\\ \sin\theta\cos\varphi+\cos\theta\sin\phi & -\sin\theta\sin+\cos\theta\cos\phi & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(\theta)\times f(\phi)=\begin{bmatrix}\cos(\theta+\phi) & -\sin(\theta+\phi) & 0\\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0\\ 0 & 0 & 1\end{bmatrix}=f(\theta+\phi)$$

So, statement 1 is correct.

$|f(\theta+\phi)|=\cos^2(\theta+\phi)+\sin^2(\theta+\phi)=1$
So, statement 2 is correct as well.

$f(x)=\cos^2x+\sin^2x=1$ which is a constant funtion, neither even nor odd
So, statement 3 is incorrect.

Hence, the correct answer is option A.

**Solution 27**

The given system of equations can be written as
$x+y+z=8$
$x-y+2z=6$
$3x-y+5z=k$

$$\text{or, }\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 3 & -1 & 5\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}8\\ 6\\ \text{k}\end{bmatrix}$$

or, AX = B, where

$$\text{A}=\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 3 & -1 & 5\end{bmatrix},\ \text{X}=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\ \&\ \text{B}=\begin{bmatrix}8\\ 6\\ \text{k}\end{bmatrix}$$

Now, |A| = 1(−3) − 1(−1) + 1(2) = 0

So, the given system of equations is inconsistent or it has infinitely many solutions according as (adj A)B ≠ O or, (adj A)B = O respectively.

$$\text{Now, (adj A)}=\begin{bmatrix}-3 & 1 & 2\\ -6 & 2 & 4\\ 3 & -1 & -2\end{bmatrix}^T=\begin{bmatrix}-3 & -6 & 3\\ 1 & 2 & -1\\ 2 & 4 & -2\end{bmatrix}$$

1. For $k=15$,

$$\text{(adj A)B}=\begin{bmatrix}-3 & -6 & 3\\ 1 & 2 & -1\\ 2 & 4 & -2\end{bmatrix}\begin{bmatrix}8\\ 6\\ 15\end{bmatrix}=\begin{bmatrix}-15\\ 5\\ 10\end{bmatrix}\neq\text{O}$$

Therefore, the given system of equations is inconsistent for $k=15$.

So, Statement 1 is true.

2. For $k=20$,

$$\text{(adj A)B}=\begin{bmatrix}-3 & -6 & 3\\ 1 & 2 & -1\\ 2 & 4 & -2\end{bmatrix}\begin{bmatrix}8\\ 6\\ 20\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}=\text{O}$$

Therefore, the given system of equations has infinitely many solutions for $k=20$.

So, Statement 2 is true.

3. For $k=25$,

$$\text{(adj A)B}=\begin{bmatrix}-3 & -6 & 3\\ 1 & 2 & -1\\ 2 & 4 & -2\end{bmatrix}\begin{bmatrix}8\\ 6\\ 25\end{bmatrix}=\begin{bmatrix}15\\ -5\\ -10\end{bmatrix}\neq\text{O}$$

Therefore, the given system of equations is inconsistent for $k=25$.

So, Statement 3 is false.

Hence, the correct answer is option A.

**Solution 28**

$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

$|A| = 3 - \left(-2\right) = 5$ & $|B| = -4 - \left(-3\right) = -1$

$A^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$ & $B^{-1} = -1\begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow A^{-1}B^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix} \times \left(-1\right)\begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 5 & 7 \\ -5 & 8 \end{bmatrix}$

$\Rightarrow AB\left(A^{-1}B^{-1}\right) = \begin{bmatrix} 3 & 5 \\ 1 & 0 \end{bmatrix}\frac{1}{5}\begin{bmatrix} 5 & 7 \\ -5 & 8 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -10 & 61 \\ 5 & 7 \end{bmatrix} \neq 1$

Also, $|AB| = 0 - 5 = -5$

$\left(AB\right)^{-1} = \frac{-1}{5}\begin{bmatrix} 0 & -5 \\ -1 & 3 \end{bmatrix} \neq A^{-1}B^{-1}$

Hence, the correct answer is option D.

**Solution 29**

$x^{\ln\left(\frac{y}{z}\right)} \cdot y^{\ln(xz)^2} \cdot z^{\ln\left(\frac{x}{y}\right)} = y^{4 \ln y}$

Taking log on both sides,

$\Rightarrow \ln\left[x^{\ln\left(\frac{y}{z}\right)}\right] + \ln\left[y^{\ln(xz)^2}\right] + \ln\left[z^{\ln\left(\frac{x}{y}\right)}\right] = \ln\left[y^{4 \ln y}\right]$

$\Rightarrow \left[\ln\left(\frac{y}{z}\right) \ln x\right] + \left[2 \ln (xz) \ln y\right] + \left[\ln\left(\frac{x}{y}\right) \ln z\right] = 4[\ln y]^2$

$\Rightarrow \ln x\left[\ln y - \ln z\right] + 2 \ln y\left[\ln x + \ln z\right] + \ln z\left[\ln x - \ln y\right] = 4[\ln y]^2$

$\Rightarrow 3 \ln x + \ln z = 4 \ln y$

$\Rightarrow \frac{\ln x + \ln x + \ln x + \ln z}{4} = \ln y$

So, ln y is the arithmetic mean of ln x, ln x, ln x & ln z

Hence, the correct answer is option B.

**Solution 30**

Dividing 235 by 2

| | Given decimal number | Remainder |
|---|---|---|
| | 235 | 1 |
| 2 | 117 | 1 |
| 2 | 58 | 0 |
| 2 | 29 | 1 |
| 2 | 14 | 0 |
| 2 | 7 | 1 |
| 2 | 3 | 1 |
| | 1 | |

So $(235)_{10} = (11101011)_2$

Hence, the correct answer is option B.

**Solution 31**

For real roots, $D \geq 0$

$\Rightarrow \left(1 - 2a^2\right)^2 - 4\left(1 - 2a^2\right) \geq 0$

$\Rightarrow 4a^4 + 4a^2 - 3 \geq 0$

$\Rightarrow \left(2a^2 - 1\right)\left(2a^2 + 3\right) \geq 0$

$\Rightarrow a^2 \geq \frac{1}{2}$

or $a^2 \leq \frac{-3}{2}$ which is not possible

Hence, the correct answer is option D.

**Solution 32**

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}<1$

$\Rightarrow \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}<1$

$\Rightarrow \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}<1$

Now comparing with the quadratic

$\Rightarrow \alpha+\beta=\frac{-b}{a}=\left(1-2a^2\right)$

$\Rightarrow \alpha\beta=\frac{c}{a}=\left(1-2a^2\right)$

substituting these values

$\Rightarrow \frac{4a^4-1}{4a^4-4a^2+1}<1$

$\Rightarrow 4a^4-1<4a^4-4a^2+1$

$\Rightarrow 4a^2<2$

$\Rightarrow a^2<\frac{1}{2}$

Hence, the correct answer is option A.

**Solution 33**

$1+\omega+\omega^2=0$ & $\omega^3=1$

$\Rightarrow \sqrt{\frac{1+\omega^2}{1+\omega}}=\sqrt{\frac{-\omega}{-\omega^2}}=\sqrt{\frac{1}{\omega}}=\sqrt{\frac{1}{\omega}\times\frac{\omega^2}{\omega^2}}=\sqrt{\frac{\omega^2}{\omega^3}}=\sqrt{\frac{\omega^2}{1}}=\omega$

Hence, the correct answer is option B.

**Solution 34**

Number of students who failed in Physics = (100 − 70)% = 30%
Number of students who failed in Chemistry = (100 − 80)% = 20%
Number of students who failed in Mathematics = (100 − 75)% = 25%
Number of students who failed in Biology = (100 − 85)% = 15%

Clearly,
Number of students who failed in all the four subjects = 15%

Hence, the correct answer is option C.

**Solution 35**

For infinite solutions

$\left(adj\ A\right)B=0$

where, $A=\begin{bmatrix}2&3&5\\7&3&-2\\2&3&\lambda\end{bmatrix}$ and $B=\begin{bmatrix}9\\8\\\mu\end{bmatrix}$

$\left(\text{adj A}\right)=\begin{bmatrix}3\lambda+6&15-3\lambda&-21\\-(7\lambda+4)&2\lambda-10&39\\15&0&-15\end{bmatrix}$

For infinite solution, $\left(\text{adj A}\right)B=0$

$\Rightarrow \begin{bmatrix}3\lambda+6&15-3\lambda&-21\\-(7\lambda+4)&2\lambda-10&39\\15&0&-15\end{bmatrix}\begin{bmatrix}9\\8\\\mu\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$\mu=9$ and $\lambda=5$

Hence, the correct answer is option B.

**Solution 36**

For unique solution
$|A|\neq 0$

where, $|A|=\begin{vmatrix}2&3&5\\7&3&-2\\2&3&\lambda\end{vmatrix}\neq 0$

$\Rightarrow |A|=2(3\lambda+6)-3(7\lambda+4)+5(21-6)\neq 0$

$\Rightarrow \lambda\neq 5$

$\mu$ can take any real value.

Hence, the correct answer is option C.

**Solution 37**

We have to form 4 digit numbers using 0,1,2,3,4,5,6,7,8,9

Last digit can be filled in = 5 ways (1, 3, 5, 7, 9) odd number
First digit can be filled in = 8 ways (excluding 0 and one odd number taken for last place)

Now 2 digits are already taken for first and last place and 8 digits are left.

2nd place can be filled in = 8 ways
3rd place can be filled in = 7 ways

Total = 8 × 8 × 7 × 5 = 2240 numbers
Hence, the correct answer is option C.

**Solution 38**

LHS forms a G.P. with $a = 1$ and $r = \frac{1}{2}$

Using sum of G.P.

$\Rightarrow \frac{1-\frac{1}{2^n}}{1-\frac{1}{2}} < 2 - \frac{1}{100}$

$\Rightarrow 2 - \frac{1}{2^{n-1}} < 2 - \frac{1}{1000}$

$\Rightarrow 2^{n-1} < 1000$

$2^9 = 512$ & $2^{10} = 1024$

$\Rightarrow n - 1 = 9$

$\Rightarrow n = 10$

Hence, the correct answer is option C.

**Solution 39**

For the equation $2x^2 + 3x - \alpha = 0$

Roots are $-2$ and $\beta$

Sum of roots is $-2 + \beta = \frac{-3}{2} \Rightarrow \beta = \frac{1}{2}$

Product of roots is $-2\left(\beta\right) = \frac{-\alpha}{2} \Rightarrow \alpha = 2$

Hence, the correct answer is option C.

**Solution 40**

For the equation $2x^2 + 3x - \alpha = 0$

Roots are $-2$ and $\beta$

Sum of roots is $-2 + \beta = \frac{-3}{2} \Rightarrow \beta = \frac{1}{2}$

$\beta = \frac{1}{2}$

$\Rightarrow \beta,\ 2,\ 2m$ form a G.P.

$\Rightarrow \frac{2}{\beta} = \frac{2m}{2}$

$\Rightarrow m = \frac{2}{\beta} = 2 \times \frac{2}{1}$

$\Rightarrow m = 4$

$\Rightarrow \beta\sqrt{m} = \frac{1}{2} \times \sqrt{4} = 1$

Hence, the correct answer is option A.

**Solution 41**

Given: sin*A* + 2sin2*A* + sin3*A*
= sin*A* + sin3*A* + 2sin2*A*
We know $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$
So, 2sin2*A*cos*A* + 2sin 2*A*
= 2sin2*A*(cos*A* + 1)

$= 2\sin\ 2A\left(2\cos^2\frac{A}{2}\right) \qquad \left(\because \cos 2x = 2\cos^2 x - 1\right)$

$= 8\sin A\cos A\cos^2\frac{A}{2} \qquad (\because \sin 2x = 2\sin x\cos x)$

Hence, the correct answer is the option C.

**Solution 42**

$x = \sin\ 70^\circ.\sin\ 50^\circ\ \ and\ y = \cos\ 60^\circ.\cos\ 80^\circ$

$\Rightarrow xy = \cos\ 60^\circ.\sin\ 70^\circ.\sin\ 50^\circ.\cos\ 80^\circ$

$\Rightarrow xy = \frac{1}{2}.\sin\ \left(90 - 20\right)\sin\ \left(90 - 40\right).\cos\ 80^\circ \qquad \because\ \sin\ \left(90 - x\right) = \cos\ x$

$\Rightarrow xy = \frac{1}{2}\cos\ 20^\circ.\cos\ 40^\circ.\cos\ 80^\circ$

$\Rightarrow xy = \frac{1}{2}\cos\ 20^\circ.\left[\frac{1}{2}\left(\cos\left(40^\circ + 80^\circ\right) + \cos\left(80^\circ - 40^\circ\right)\right)\right]$

$\Rightarrow xy = \frac{1}{2}\cos\ 20^\circ.\left[\frac{1}{2}\left(\cos 120^\circ + \cos 40^\circ\right)\right]$

$\Rightarrow xy = \frac{1}{2}\cos 20^\circ\left[\frac{1}{2} \times \left(\frac{-1}{2}\right) + \frac{1}{2}\cos 40^\circ\right]$

$\Rightarrow xy = \frac{1}{2}\cos 20^\circ\left[\frac{-1}{4} + \frac{1}{2}\cos 40^\circ\right]$

$\Rightarrow xy = \frac{-1}{8}\cos 20^\circ + \frac{1}{8}\cos 20^\circ\cos 40^\circ$

$\Rightarrow xy = \frac{-1}{8}\cos 20^\circ + \frac{1}{8}\left[\frac{1}{2}\left(\cos 60^\circ + \cos 20^\circ\right)\right]$

$\Rightarrow xy = \frac{-1}{8}\cos 20^\circ + \frac{1}{16} + \frac{1}{8}\cos 20^\circ = \frac{1}{16}$

Hence, the correct answer is the option A.

**Solution 43**

Given,

$\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 = 4 \qquad \ldots(i)$

the RHS $= 4$ is only possible when each term is 1 , i.e each term is equal to its maximum value.

$\Rightarrow \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = \sin\theta_4 = 1$

$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \theta_4 = 90^\circ$

Now,

$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4 = [\cos\ 90^\circ] \times 4 = 0$

Hence, the correct answer is the option A.

**Solution 44**

To find value of $\left[1+\cos\frac{\pi}{8}\right]\left[1+\cos\frac{3\pi}{8}\right]\left[1+\cos\frac{5\pi}{8}\right]\left[1+\cos\frac{7\pi}{8}\right]$

$\cos\frac{7\pi}{8} = \cos\left[\pi - \frac{\pi}{8}\right] = -\cos\frac{\pi}{8}$

and $\cos\frac{5\pi}{8} = \cos\left[\pi - \frac{3\pi}{8}\right] = -\cos\frac{3\pi}{8}$

So, $\left[1+\cos\frac{\pi}{8}\right]\left[1+\cos\frac{3\pi}{8}\right]\left[1+\cos\frac{5\pi}{8}\right]\left[1+\cos\frac{7\pi}{8}\right] = \left[1+\cos\frac{\pi}{8}\right]\left[1+\cos\frac{3\pi}{8}\right]\left[1-\cos\frac{3\pi}{8}\right]\left[1-\cos\frac{\pi}{8}\right]$

$= \left[1-\cos^2\frac{\pi}{8}\right]\left[1-\cos^2\frac{3\pi}{8}\right] = \sin^2\frac{\pi}{8}.\sin^2\frac{3\pi}{8}$

$= \frac{1}{4}\left[2\ \sin^2\frac{\pi}{8}.2\ \sin^2\frac{3\pi}{8}\right]$

$= \frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)\right] \qquad \left(\because\ 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right)$

$= \frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)\right] = \frac{1}{8}$

Hence, the correct answer is the option D.

**Solution 45**

Given, $z = x\cos\theta + y\sin\theta$

$z^2 = x^2\cos^2\theta + y^2\sin^2\theta + 2xy\ \sin\theta\cos\theta$

$\Rightarrow 2xy\ \sin\theta\cos\theta = z^2 - x^2\cos^2\theta - y^2\sin^2\theta$

Let $M = \left(x\sin\theta - y\cos\theta\right)^2$

$\Rightarrow M = x^2\sin^2\theta + y^2\cos^2\theta - 2xy\ \sin\theta\cos\theta$

$\Rightarrow M = x^2\sin^2\theta + y^2\cos^2\theta - \left[z^2 - x^2\cos^2\theta - y^2\sin^2\theta\right]$

$\Rightarrow M = x^2\left[\sin^2\theta + \cos^2\theta\right] + y^2\left[\sin^2\theta + \cos^2\theta\right] - z^2$

$\Rightarrow M = x^2 + y^2 - z^2$

Hence, the correct answer is the option A.

**Solution 46**

$\cos\left(2\cos^{-1}(0.8)\right)$

$= 2\cos^2\left(\cos^{-1}(0.8)\right) - 1 \qquad \because\ \cos\ 2A = 2\cos^2 A - 1$

$= 2\left[\cos\left(\cos^{-1}(0.8)\right)\cos\left(\cos^{-1}(0.8)\right)\right] - 1$

$= 2\left[(0.8)(0.8)\right] - 1$

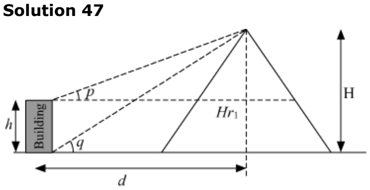
$= 2(0.8)^2 - 1$

$= 1.28 - 1$

$= 0.28$

Hence, the correct answer is the option D.

**Solution 47**



Let height of hill $= H$

Horizontal distance between building & hill $= d$

$\tan q = \frac{H}{d} \Rightarrow d = \frac{H}{\tan q} \Rightarrow H\cot q \qquad \ldots\ldots(\text{i})$

$\tan p = \frac{(H-h)}{d} \Rightarrow d = (H-h)\cot p \qquad \ldots\ldots(\text{ii})$

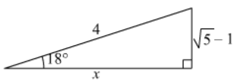
From $(\text{i})$ and $(\text{ii})$

$H\cot q = (H-h)\cot p$

$H = \frac{h\cot p}{\cot p - \cot q}$

Hence, the correct answer is the option B.

**Solution 48**




$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

$x^2 = 4^2 - \left(\sqrt{5}-1\right)^2$

$x^2 = 16 - 5 - 1 + 2\sqrt{5}$

$\Rightarrow x = \sqrt{10 + 2\sqrt{5}}$

$\Rightarrow \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$

$\Rightarrow 2\cos^2 9 - 1 = \frac{\sqrt{10+2\sqrt{5}}}{4} \qquad \because 2\cos^2\theta - 1 = \cos 2\theta$

$\cos^2 9 = \frac{4+\sqrt{10+2\sqrt{5}}}{8}$

$\Rightarrow \cos^2(90^\circ - 81^\circ) = \frac{4+\sqrt{10+2\sqrt{5}}}{8}$

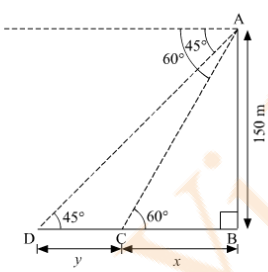
$\Rightarrow \sin^2 81 = \frac{4+\sqrt{10+2\sqrt{5}}}{8}$

$\Rightarrow \sin^2 81^\circ = \frac{8+2\sqrt{10+2\sqrt{5}}}{16} \qquad \ldots\ldots(i)$

On squaring option $A\left[\frac{\sqrt{\left(3+\sqrt{5}\right)}+\sqrt{5-\sqrt{5}}}{4}\right]$ we get $(i)$ as the result.

Hence, the correct answer is the option A.

**Solution 49**




In $\triangle$ ABC,

$\tan 60^\circ = \frac{150}{x} \Rightarrow x = \frac{150}{\sqrt{3}}$

In $\triangle$ ABC,

$\tan 45^\circ = \frac{150}{x+y}$

$\Rightarrow x + y = 150$

$\Rightarrow y = 150 - x = 150 - \frac{150}{\sqrt{3}}$

$\Rightarrow y = 150\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$

Speed $\left(\text{m/hr}\right) = \frac{\text{Distance}}{\text{Time}} = \frac{150\left(\sqrt{3}-1\right)}{\sqrt{3}} x \frac{60}{2} = 4500\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$

Hence, the correct answer is the option B.

**Solution 50**

Let $N=\frac{1-\tan 2^{\circ}\cot 62^{\circ}}{\tan 152^{\circ}-cot\,88^{\circ}}=\frac{1-\tan 2^{\circ}\cot(90-28)^{\circ}}{\tan(180-28)^{\circ}-cot\,(90-2)^{\circ}}$

$\Rightarrow N=\frac{1-\tan 2^{\circ}\tan 28^{\circ}}{\tan(-28)^{\circ}-\tan 2^{\circ}}=-\left[\frac{1-\tan 2^{\circ}\tan 28^{\circ}}{\tan 2^{\circ}+\tan 28^{\circ}}\right]$

$\Rightarrow N=-\frac{1}{\tan(2+28)^{\circ}}=-\frac{1}{\tan 30^{\circ}}=-\sqrt{3}$

Hence, the correct answer is the option B.

**Solution 51**

$Let\ ABC\ is\ an\ equilateral\ triangle\ with\ A\left(0,0\right)\ and\ B\left(3,\sqrt{3}\right)\ and\ C\left(x,y\right)$

$AB=\sqrt{(3-0)^2+\left(\sqrt{3}-0\right)^2}=\sqrt{9+3}=\sqrt{12}$

$Upon\ taking\ option\ C\ \left(0,2\sqrt{3}\right)$

$CA=\sqrt{0^2+\left(2\sqrt{3}\right)^2}=\sqrt{12}$

$CB=\sqrt{(0-3)^2+\left(2\sqrt{3}-\sqrt{3}\right)^2}=\sqrt{12}$

If the third point was C $\left(3,-\sqrt{3}\right)$ then,

$CA=\sqrt{(3-0)^2+\left(-\sqrt{3}-0\right)^2}=\sqrt{9+3}=\sqrt{12}$

$CB=\sqrt{(3-3)^2+\left(-\sqrt{3}-\sqrt{3}\right)^2}=\sqrt{0+\left(-2\sqrt{3}\right)^2}=\sqrt{12}$

Thus, both the points $\left(0,2\sqrt{3}\right)$ and $\left(3,-\sqrt{3}\right)$ can be the coordinate of the third vertex.

$\therefore$ Both options **A** and **B** are correct

Hence, the correct answer is the option **C.**

**Solution 52**

Equation of the line segment joining $\left(1,1\right)\ and\ \left(2,3\right)$

$(y-1)=\frac{3-1}{2-1}(x-1)$

$y-1=2(x-1)$

$2x-y-1=0\quad\Rightarrow\ y=2x-1$

Slope $=2$

and so the slope of the perpendicular to this line segment will be $-\frac{1}{2}$

Since the perpendicular is bisecting it , so it will pass through its mid $-$ point

Coordiantes of midpoint of given line will be : $\left(\frac{2+1}{2},\frac{3+1}{2}\right)$ or $\left(\frac{3}{2},2\right)$

So, the equation of perpendicular bisector is :

$\left(y-2\right)=-\frac{1}{2}\left(x-\frac{3}{2}\right)$

$\Rightarrow 4y-8=-2x+3$

$\Rightarrow 2x+4y-11=0$

Hence, the correct answer is the option A.

**Solution 53**

The intersection of the lines $x-y=4$ and $2x+3y+7=0$

Solving the above two equations we get $x=1$ and $y=-3$

$\therefore$ Center of circle is $C\left(1,-3\right)$

The radius will be distance between center and the point $\left(2,4\right)$

$r=\sqrt{(2-1)^2+(4+3)^2}=\sqrt{1+49}=5\sqrt{2}\ units$

Hence, the correct answer is the option D.

**Solution 54**

Suppose equation of hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Latus Rectum $=\ 8=\frac{2b^2}{a}\Rightarrow b^2=4a \qquad \ldots\left(i\right)$

Also, $b^2=a^2\left(e^2-1\right)$

$\Rightarrow 4a=a^2\left[\left(\frac{3}{\sqrt{5}}\right)^2-1\right]$

$\Rightarrow 4=a\left[\frac{9}{5}-1\right]$

$\Rightarrow 4=a\left[\frac{4}{5}\right]\Rightarrow a=5$

$\therefore\ \ a^2=25$

Using $a=5$ in $\left(\mathrm{i}\right)$ we get,

$b^2=4\times 5=20$

Equation of Hyperbola will be $\frac{x^2}{25}-\frac{y^2}{20}=1$

Hence, the correct answer is the option A.

**Solution 55**

$|x+y|=2$

$\Rightarrow x+y=\pm 2$

$\Rightarrow\ x+y-2=0 \quad$ and $\quad x+y+2=0$

Clearly the two equations representing two distinct lines are parallel.

Accoring to the question $(a,a)$ lies between the two parallel lines and must also lie on line $x=y$ as the coordiante is $(a,a)$

Intersection point of line x = y with the two parallel lines and putting $x=y=a$ gives the range of $-1\le a\le 1$, however $(a,a)$ lies between the two parallel lines and should not lie on it , therefore the range of $a$ is

$-1<a<1$ which can also be written as $|\boldsymbol{a}|<\mathbf{1}.$

Hence, the correct answer is the option C.

**Solution 56**

The intersecting lines are $x+2y=5$ and $3x+7y=17$

On solving for $x$ & $y$ for the two intersecting lines we get, $x=1$ & $y=2$

Equation of perpendicular line is $3x+4y=10.$

$\Rightarrow y=\frac{-3}{4}x+\frac{10}{4}$

Slope $=\frac{-3}{4}$

Then slope of line that will be perpendicular to $3x+4y=10$ will have slope $\frac{4}{3}$.

Equation of line passing through $\left(1,2\right)$ and having slope $\frac{4}{3}$ will be

$\left(y-2\right)=\frac{4}{3}\left(x-1\right)$

$3y-6=4x-4$

$4x-3y+2=0$

Hence, the correct answer is the option D.

**Solution 57**

Distance of a point $\left(x_1,y_1\right)$ from a line $ax+by+c=0$ is given by

$d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

Here, $d=\left|\frac{8a+6b+1}{\sqrt{8^2+6^2}}\right|=1$

$\Rightarrow |8a+6b+1|=10$

$\Rightarrow 8a+6b+1=\pm 10$

$\Rightarrow\ 8a+6b+1=10\quad \&\quad 8a+6b+1=-10$

$\Rightarrow\ 8a+6b-9=0\quad \&\quad 8a+6b+11=0$

Conditions 2 and 3 are correct.

Hence, the correct answer is the option B.

**Solution 58**

Ellipse is $9x^2+16y^2=144 \quad \ldots\left(i\right)$

and line $3x+4y=12 \Rightarrow x=\frac{12-4y}{3}$

Using $x=\frac{12-4y}{3}$ in $\left(i\right)$ to find the point of intersection

$\Rightarrow 9\left(\frac{12-4y}{3}\right)^2+16y^2=144$

$\Rightarrow (12-4y)^2+16y^2=144$

$\Rightarrow 144+16y^2-96y+16y^2=144$

$\Rightarrow 32y^2-96y=0$

$\Rightarrow 32y\left(y-3\right)=0$

$y=0$ *or* $y=3$

*For* $y=0$ ; $x=4$ so the point will be $\left(4,0\right)$

*For* $y=3$ ; $x=0$ so the point will be $\left(0,3\right)$

$\therefore$ Length of chord $=\sqrt{(0-3)^2+(4-0)^2}=\sqrt{9+16}=\sqrt{25}=5$ units

Hence, the correct answer is the option A.

**Solution 59**

The given line passes through $\left(-3,5\right)$ and $\left(2,0\right)$.

Equation of a line passing through two points is

$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)\left(x-x_1\right)$

$\Rightarrow \left(y-5\right)=\left(\frac{0-5}{2+3}\right)\left(x+3\right)$

$\Rightarrow y=-x+2 \quad \ldots\ldots\left(\text{i}\right)$

Slope $=\ m=-1$

and slope of perpendicular line will be $\frac{-1}{m}=1$

So equation of line passing through $\left(3,3\right)$ having slope equal to 1 is

$\left(y-3\right)=1\left(x-3\right)$

$\Rightarrow y=x$

From equation $\left(\text{i}\right)$ we get,

$y=-x+2$

$x=-x+2$

$x=1$ and $y=1$

Hence, the correct option is answer D.

**Solution 60**

Here, $b^2=a^2\left(e^2-1\right)$

$a=b$ for rectangular hyperbola

$\Rightarrow b^2=b^2\left(e^2-1\right)$

$\Rightarrow e^2-1=1$

$\Rightarrow e^2=2$

$\Rightarrow e=\pm\sqrt{2}$

$e > 1$ so, $e=\sqrt{2}$.

Hence, the correct answer is the option A.

**Solution 61**

Let $Q\left(x_1,\ y_1,\ z_1\right)$ be the image of the point $P$,

The direction ratios of $PQ$ are $(3,\ -2,\ 2)$

The equation of line $PQ$ is $\frac{x+2}{3}=\frac{y-1}{-2}=\frac{z+5}{2}=t$

Coordinate of any point on the line $PQ$ is $(3t-2,\ -2t+1,\ 2t-5)$

Let $Q$ $(3t-2,\ -2t+1,\ 2t-5)$ be such a point

Given : $P\left(-2,\ 1,\ -5\right)$

Let M be the midpoint of $PQ$, $M = \left(\frac{3t}{2} - 2,\ -t+1,\ t-5\right)$

Since $M$ lies on the plane $3x - 2y + 2z + 1 = 0$

So, $3\left(\frac{3t}{2} - 2\right) - 2(-t+1) + 2(t-5) + 1 = 0$

$\Rightarrow \frac{17}{2}t - 17 = 0$

$\Rightarrow t = 2$

So, coordinates of $Q$ are $\left(4, -3, -1\right)$

Also the midpoint of $PQ$ is $M$, $= \left(1, -1, -3\right)$

$\therefore\ PQ = \sqrt{(-2-4)^2 + (1+3)^2 + ((-5+1)^2} = \sqrt{68}$

$\Rightarrow PQ = 2\sqrt{17}$ which is greater than 8.

Thus, all the three statements are correct.
Hence, the correct answer is the option D.

**Solution 62**

Let $Q\left(x_1,\ y_1,\ z_1\right)$ be the image of the point $P$,

The direction ratios of $PQ$ are $(3,\ -2,\ 2)$

The equation of line $PQ$ is $\frac{x+2}{3} = \frac{y-1}{-2} = \frac{z+5}{2} = t$

Coordinate of any point on the line $PQ$ is $(3t-2,\ -2t+1,\ 2t-5)$

Let $Q\ (3t-2,\ -2t+1,\ 2t-5)$ be such a point

Given : $P\left(-2,\ 1,\ -5\right)$

Let M be the midpoint of $PQ$, $M = \left(\frac{3t}{2} - 2,\ -t+1,\ t-5\right)$

Since $M$ lies on the plane $3x - 2y + 2z + 1 = 0$

So, $3\left(\frac{3t}{2} - 2\right) - 2(-t+1) + 2(t-5) + 1 = 0$

$\Rightarrow \frac{17}{2}t - 17 = 0$

$\Rightarrow t = 2$

So, coordinates of $Q$ are $\left(4, -3, -1\right)$

Also the midpoint of $PQ$ is $M$, $= \left(1, -1, -3\right)$

So, statement I is correct

$Let\ Q\left(x_1,\ y_1,\ z_1\right)\ be\ the\ image\ of\ the\ point\ P,$

$The\ direction\ ratios\ of\ PQ\ are\ 3,\ -2,\ 2 \qquad \ldots\left(i\right)$

$The\ equation\ of\ line\ PQ\ is\ \frac{x+2}{3} = \frac{y-1}{-2} = \frac{z+5}{2} = t$

$Direction\ \cos ines\ of\ PQ\ is\ \left(\frac{3}{\sqrt{3^2+(-2)^2+2^2}}; \frac{-2}{\sqrt{3^2+(-2)^2+2^2}}; \frac{2}{\sqrt{3^2+(-2)^2+2^2}}\right)$

$= \left(\frac{3}{\sqrt{17}}; \frac{-2}{\sqrt{17}}; \frac{2}{\sqrt{17}}\right)$

$Now,\ sum\ of\ the\ square\ of\ the\ DC's\ will\ be\ \left(\frac{3}{\sqrt{17}}\right)^2 + \left(\frac{-2}{\sqrt{17}}\right)^2 + \left(\frac{2}{\sqrt{17}}\right)^2$

$= \frac{9}{17} + \frac{4}{17} + \frac{4}{17} = \frac{17}{17} = 1$

So, statement II is correct as well.

Hence, the correct answer is the option C.

**Solution 63**

Let the direction ratios of the required line be proportional to *a*, *b*, *c*.
Also, it passes through (5, –6, 7). So, it equation is
$\frac{x-5}{a} = \frac{y+6}{b} = \frac{z-7}{c} \qquad \ldots..(1)$
Since (1) is parallel to the planes *x* + *y* + *z* = 1 and 2*x* – *y* – 2*z* = 3.

$\therefore\ a\left(1\right)+b\left(1\right)+c\left(1\right)=0\ \ and\ a\left(2\right)+b\left(-1\right)+c\left(-2\right)=0$

By cross multiplying, we get

$\frac{a}{1(-2)-1(-1)}=\frac{b}{1(2)-1(-2)}=\frac{c}{1(-1)-1(2)}$

$\Rightarrow \frac{a}{-1}=\frac{b}{4}=\frac{c}{-3}=\lambda\ \left(\text{say}\right)$

$\Rightarrow a=-\lambda\ ,\ b=4\lambda,\ c=-3\lambda$

So, the direction ratios of the line are (−1, 4, −3) or (1, −4, 3).
Hence, the correct answer is the option C.

**Solution 64**

Let the direction ratios of the required line be proportional to *a*, *b*, *c*.
Also, it passes through (5, −6, 7). So, it equation is

$\frac{x-5}{a}=\frac{y+6}{b}=\frac{z-7}{c}$ $\qquad .....(1)$

Since (1) is parallel to the planes $x + y + z = 1$ and $2x - y - 2z = 3$.

$\therefore\ a\left(1\right)+b\left(1\right)+c\left(1\right)=0\ \ and\ a\left(2\right)+b\left(-1\right)+c\left(-2\right)=0$

By cross multiplying, we get

$\frac{a}{1(-2)-1(-1)}=\frac{b}{1(2)-1(-2)}=\frac{c}{1(-1)-1(2)}$

$\Rightarrow \frac{a}{-1}=\frac{b}{4}=\frac{c}{-3}=\lambda\ \left(\text{say}\right)$

$\Rightarrow a=-\lambda\ ,\ b=4\lambda,\ c=-3\lambda$

Substituting the values of *a*, *b* and *c*, in (1), we get the equation of line as

$\frac{x-5}{-1}=\frac{y+6}{4}=\frac{z-7}{-3}$

Hence, the correct answer is the option A.

**Solution 65**

Let $\vec{d}=x\hat{i}+y\hat{j}+z\hat{k}$

Since $\vec{c}$ is parallel to $\vec{a}$

$\vec{c}=\lambda\,\vec{a}$

Now, $\vec{b}=\vec{c}+\vec{d}=\lambda\,\vec{a}+\vec{d}$

$=\lambda\left(\hat{i}+\hat{j}\right)+x\hat{i}+y\hat{j}+z\hat{k}$

$3\hat{i}+4\hat{k}=(\lambda+x)\hat{i}+(\lambda+y)\hat{j}+z\hat{k}$

On comparing we get

$z=4\ ,\ \lambda+y=0,\ \lambda+x=3$

$\Rightarrow \lambda=-y$ $\qquad .....(i)$

$\Rightarrow x-y=3$ $\qquad .....(ii)$

$Now\ \vec{d}\perp\vec{a}$

So, $\cos\theta=0$

$\vec{d}\cdot\vec{a}=0$

$\Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(\hat{i}+\hat{j}\right)=0$

$\Rightarrow x+y=0$ $\qquad .....(iii)$

Solving $\left(ii\right)$ and $\left(iii\right)$

$2x=3$

$\Rightarrow x=\frac{3}{2}\ ,\ y=-\frac{3}{2}$

$\Rightarrow\ \vec{c}=\lambda\left(\vec{a}\right)=\frac{3}{2}\left(\hat{i}+\hat{j}\right)$

Hence, the correct answer is the option A.

**Solution 66**

Since $\vec{c}$ is parallel to $\vec{a}$

$\vec{c}=\lambda\,\vec{a}$

Now, $\vec{b}=\vec{c}+\vec{d}=\lambda\,\vec{a}+\vec{d}$

$=\lambda\left(\hat{i}+\hat{j}\right)+x\hat{i}+y\hat{j}+z\hat{k}$

$3\hat{i}+4\hat{k}=(\lambda+x)\hat{i}+(\lambda+y)\hat{j}+z\hat{k}$

On comparing we get

$z=4\ ,\ \lambda+y=0,\ \lambda+x=3$

$\Rightarrow \lambda=-y$ $\qquad .....(i)$

$\Rightarrow x-y=3$ $\qquad .....(ii)$

$Now\ \vec{d} \perp \vec{a}$
So, $\cos\theta = 0$
$\vec{d}\cdot\vec{a} = 0$
$\Rightarrow \left(x\hat{i} + y\hat{j} + z\hat{k}\right)\cdot\left(\hat{i}+\hat{j}\right) = 0$
$\Rightarrow x + y = 0 \qquad \ldots\ldots(iii)$
Solving $\left(ii\right)$ and $\left(iii\right)$
$2x = 3$
$\Rightarrow x = \frac{3}{2}\ ,\ y = -\frac{3}{2}$
$\Rightarrow\ \vec{c} = \lambda\left(\vec{a}\right) = \frac{3}{2}\left(\hat{i}+\hat{j}\right)$

Since $z = 4$ and $x = \frac{3}{2},\ y = -\frac{3}{2}$

Neither equation 1 that is $y - x = 4$ is correct nor equation 2, $2z - 3 = 0$.

Hence, the correct answer is the option D.

**Solution 67**

We have $\vec{a} + \vec{b} + \vec{c} = \vec{0}$
$So\left|\vec{a} + \vec{b} + \vec{c}\right| = 0$

$\Rightarrow \left|\vec{a} + \vec{b} + \vec{c}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right)$

$\Rightarrow 0 = (10)^2 + (6)^2 + (14)^2 + 2\left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right)$

$\Rightarrow 0 = 100 + 36 + 196 + 2\left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right)$

$\Rightarrow -\frac{332}{2} = \left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right)$

$\Rightarrow -166 = \left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right)$

Hence, the correct answer is the option B.

**Solution 68**

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$
$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$
$\Rightarrow \left|\vec{a} + \vec{b}\right| = \left|-\vec{c}\right| = \left|\vec{c}\right|$

$\Rightarrow \left|\vec{a} + \vec{b}\right|^2 = \left|\vec{c}\right|^2$

$\Rightarrow \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\left(\vec{a}\cdot\vec{b}\right) = \left|\vec{c}\right|^2$

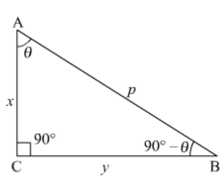
$\Rightarrow (10)^2 + (6)^2 + 2\left(\vec{a}\cdot\vec{b}\right) = (14)^2$

$\Rightarrow 2\left(\vec{a}\cdot\vec{b}\right) = 60$

$\Rightarrow \vec{a}\cdot\vec{b} = 30$
$\Rightarrow \cos\theta = \frac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|} = \frac{30}{10x6} = \frac{1}{2}$
$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta = 60^\circ$

Hence, the correct answer is the option C.

**Solution 69**



$\overrightarrow{AB} \cdot \overrightarrow{AC} + \overrightarrow{BC} \cdot \overrightarrow{BA} + \overrightarrow{CA} \cdot \overrightarrow{CB}$
$= (AB \cdot AC \ \cos\ \theta) + (BC \cdot BA \ \cos\ (90 - \theta)) + (CA \cdot CB \ \cos\ 90)$
$= (p \cdot x \ \ \cos\ \theta\ ) + (y \cdot p\ \sin\ \theta\ ) + 0$
$= p\,[x\ \cos\ \theta\ +\ y\ \sin\theta]$
By projection formula
$p = x\ \cos\ \theta\ +\ y\ \ \cos\ \ (90 - \theta)$
$= x\ \cos\ \theta\ +\ y\ \sin\ \theta$
$\therefore p\,[x\ \cos\ \theta\ +\ y\ \sin\theta] = p \times p = p^2$

Hence, the correct answer is the option B.

**Solution 70**

Let the point $P$ is $(1, -1, 2)$ and point $Q$ is $(2, -1, 3)$
$\Rightarrow$ Position vector of $P$ with respect to $Q$ is
$\overrightarrow{r} = (1-2)\hat{i} + (-1+1)\hat{j} + (2-3)\hat{k}$
$\Rightarrow \overrightarrow{r} = -\hat{i} + 0\hat{j} - \hat{k}\ \ and\ \overrightarrow{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$
$\Rightarrow$ Moment $= \overrightarrow{r}\ x\ \overrightarrow{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$
$= \hat{i}\,(0+2) - \hat{j}\,(4+3) + \hat{k}\,(-2+0) = 2\hat{i} - 7\hat{j} - 2\hat{k}$

Hence, the correct answer is the option C.

**Solution 71**

We know that
$|x| = \begin{cases} x\ , & x \ge 0 \\ -x\ , & x < 0 \end{cases}$
For Domain, $|x| - x > 0$
Case 1 : $x \ge 0\ \Rightarrow x - x = 0$ (This will make denominator zero and hence $f(x)$ undefined)
*Case* 2 : $x < 0 \Rightarrow -x - x = -2x$ (This will make the term inside root positive for some negative value of $x$)
$\therefore\ \ -2x > 0$
$\Rightarrow x < 0$
So, $x \in (-\infty, 0)$

Hence, the correct answer is the option A.

**Solution 72**

For $x \ge 0$ , i.e (RHL)
$\lim\limits_{x\to1^+} f\left(x\right) = \lim\limits_{x\to1^+}\left(2+x\right) = 3$
For $x < 0$ , i.e (LHL)
$\lim\limits_{x\to1^-} f\left(x\right) = \lim\limits_{x\to1^-}\left(2-x\right) = 1$
So, $\lim\limits_{x\to1} f\left(x\right)$ does not exist.
At $x = 0$
RHL : $\lim\limits_{h\to0^+} f\left(0+h\right) = \lim\limits_{h\to0^+}\left(2+h\right) = 2$
LHL : $\lim\limits_{h\to0^-} f\left(0-h\right) = \lim\limits_{h\to0^-}\left(2-h\right) = 2$
$f\left(0\right) = 2 + 0 = 2$
$\Rightarrow f\left(x\right)$ is continuous at $x = 0$
Differentiability at $x = 0$
LHD : $\lim\limits_{h\to0^-} \frac{f(0-h)-f(0)}{-h} = \lim\limits_{h\to0^-} \frac{2+h-2}{-h} = \frac{-h}{h} = -1$
RHD : $\lim\limits_{h\to0^+} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h\to0^+} \frac{2+h-2}{h} = \frac{h}{h} = 1$
Since, LHD $\neq$ RHD
So, $f\left(x\right)$ is not differentiable at $x = 0$

Hence, the correct answer is the option D.

**Solution 73**

For $x \geq 0$

$f\left(x\right) = \frac{x+x}{x} = 2, \lim\limits_{x \to 0^+} f\left(x\right) = 2$

$For\ x < 0$

$f\left(x\right) = \frac{x-x}{x} = 0, \lim\limits_{x \to 0^-} f\left(x\right) = 0$

$f\left(x\right)$ is not defined for $x = 0$.

$\Rightarrow$ function $f\left(x\right)$ is discontinuous at $x = 0$, since $\lim\limits_{x \to 0^+} f\left(x\right) \neq \lim\limits_{x \to 0^-} f\left(x\right)$

*f*(*x*) is continuous on A, where A = **R**\[0].
Hence, the correct answer is the option A.

**Solution 74**

$f(x) = x^3 \sin x$

$f'(x) = 3x^2 \sin x + x^3 \cos x$

$f'(x) = 0$

$\Rightarrow 3x^2 \sin x + x^3 \cos x = 0$

$\Rightarrow x^2 (3 \sin x + x \cos x) = 0$

$\Rightarrow x = 0\ ,\ 3 \sin x + x \cos x = 0 \qquad \ldots (i)$

Put $x = 0$ in $(i)$

$f''(x) = 6x \sin x + 3x^2 \cos x + 3x^2 \cos x + x^3 (-\sin x)$

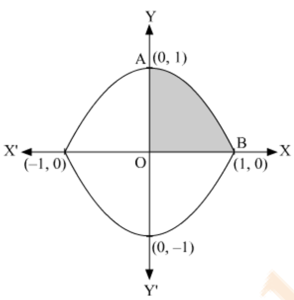
$f''(0) = 0$

So, neither maximum nor min at $x = 0$

Hence, the correct answer is the option C.

**Solution 75**

$$|y| = \begin{cases} y = 1 - x^2 & y > 0 \\ y = x^2 - 1 & y < 0 \\ x = \pm 1 & y = 0 \end{cases}$$




Now,
Area under the curve = 4 × Area under the region OABO (By Symmetry)

$= 4 \times \int_0^1 \left(1 - x^2\right) dx$

$= 4 \times \left[x - \frac{x^3}{3}\right]_0^1$

$= 4 \times \left(1 - \frac{1}{3}\right)$

$= 4 \times \frac{2}{3} = \frac{8}{3}$ sq units

Hence, the correct answer is the option B.

**Solution 76**

For $-1 \le x \le 2$

$f(x) = 3x^2 + 12x - 1$

$f'(x) = 6x + 12$

If we take any point in the interval $[-1, 2]$ then

$f'(1) = 6 \times 1 + 12 = 18 > 0$

$\Rightarrow f(x)$ is increasing in the interval $[-1, 2]$

For $2 < x \le 3$

$f(x) = 37 - x$

$f'(x) = -1 < 0$

$\Rightarrow f(x)$ is decreasing in the interval $(2, 3]$.

Hence, the correct answer is the option C.

**Solution 77**

For continuity at $x = 2$

$$\text{RHL} : \lim_{x \to 2^+} f\left(x\right) = \lim_{x \to 2^+} (37 - x) = 35$$

$$\text{LHL} : \lim_{x \to 2^-} f\left(x\right) = \lim_{x \to 2^-} \left(3x^2 + 12x - 1\right) = 3(2)^2 + 12 \times 2 - 1 = 35$$

$$f\left(2\right) = 3 \times 4 + 12 \times 2 - 1 = 12 + 24 - 1 = 35$$

So, RHL = LHL

$\Rightarrow f\left(x\right)$ is continuos at $x = 2$

For differentiability at $x = 2$

$$\text{LHD} : \lim_{x \to 2^-} = \frac{f(x) - f(2)}{x - 2} = \lim_{h \to 0} \frac{f(2-h) - f(2)}{2 - h - 2}$$

$$= \lim_{h \to 0} \frac{3(2-h)^2 + 12(2-h) - 1 - (12 + 24 - 1)}{-h} = \lim_{h \to 0} \frac{3h^2 - 24h}{-h} = \lim_{h \to 0} (24 - 3h) = 24$$

$$\text{RHD} : \lim_{x \to 2^+} = \frac{f(x) - f(2)}{x - 2} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{2 + h - 2}$$

$$= \lim_{h \to 0} \frac{37 - 2 - h - 35}{h} = -1$$

LHD $\neq$ RHD

$\Rightarrow f\left(x\right)$ is not differentiable at $x = 2$

At $x = 2$,

$$f\left(2\right) = 3(2)^2 + 12\,(2) - 1 = 12 + 24 - 1 = 35$$

When $x = 1$

$$f\left(1\right) = 3(1)^2 + 12\,(1) - 1 = 14$$

$\Rightarrow f\,(2) > f\,(1)$

So, $f\left(x\right)$ attains greatest value at $x = 2$.

Thus, statement 1 and 2 are correct.
Hence, the correct answer is the option A.

**Solution 78**

$f\,(x) = [|x| - |x - 1|]^2$

For $x<0$,

$$f\,(x) = [(-x) - \{-(x-1)\}]^2$$
$$= (-1)^2 = 1$$

For $0 \le x < 1$,

$$f\,(x) = [x - \{-(x-1)\}]^2$$
$$= (2x - 1)^2$$

For $x \ge 1$,

$$f\,(x) = [x - (x-1)]^2$$
$$= (1)^2 = 1$$

So,

$$f\left(x\right)=\begin{cases}1 & x<0\\(2x-1)^2 & 0\le x<1\\1 & 1\le x\end{cases}$$

Now, when $x > 1$, $f(x) = 1$
$\Rightarrow f'(x) = 0$

Hence, correct answer is option A.

**Solution 79**

$$f(x)=[|x|-|x-1|]^2$$

$$\text{For } x<0,$$
$$f(x)=[(-x)-\{-(x-1)\}]^2$$
$$=(-1)^2=1$$

$$\text{For } 0\le x<1,$$
$$f(x)=[x-\{-(x-1)\}]^2$$
$$=(2x-1)^2$$

$$\text{For } x\ge 1,$$
$$f(x)=[x-(x-1)]^2$$
$$=(1)^2=1$$

So,

$$f\left(x\right)=\begin{cases}1 & x<0\\(2x-1)^2 & 0\le x<1\\1 & 1\le x\end{cases}$$

Now, when $0 < x < 1$,
$f(x) = (2x - 1)^2$
$\Rightarrow f'(x) = 2(2x - 1) \times 2$
$= 8x - 4$

Hence, the correct answer is option D.

**Solution 80**

$$f(x)=[|x|-|x-1|]^2$$

$$f\left(x\right)=\begin{cases}1 & x<0\\(2x-1)^2 & 0\le x<1\\1 & 1\le x\end{cases}$$

$$\Rightarrow f'\left(x\right)=\begin{cases}0 & x<0\\4(2x-1) & 0\le x<1\\0 & 1\le x\end{cases}$$

$$\Rightarrow f''\left(x\right)=\begin{cases}0 & x<0\\8 & 0\le x<1\\0 & 1\le x\end{cases}$$

1. $f(-2) = 1$ and $f(5) = 1$
   So, $f(-2) = f(5)$ is correct.

2. $f''(-2) = 0$, $f''(0{\cdot}5) = 8$ and $f''(3) = 0$
   $\therefore f''(-2) + f''(0{\cdot}5) + f''(3) = 8$
   So, $f''(-2) + f''(0{\cdot}5) + f''(3) = 4$ is incorrect.

Hence, the correct answer is option A.

**Solution 81**

We know that greatest integer function is discontinuous at integers and $\sin x$ is continuous in it's complete domain.
So, at $x = 0$, $f(x)$ is discontinuous and $g(x)$ is continuous.

Hence, correct answer is option C.

**Solution 82**

$fog(x) = f(g(x)) = [\sin x]$

$gof(x) = g(f(x)) = \sin [x]$

$\lim_{x \to 0^+} fog\left(x\right) = [\sin\ x] = 0$

$\lim_{x \to 0^-} fog\left(x\right) = [\sin\ x] = -1$

$\text{LHL} \neq \text{RHL}, \text{ limit does not exist.}$

$\lim_{x \to 0^+} gof\left(x\right) = \sin\ [x] = \sin\ 0 = 0$

$\lim_{x \to 0^-} fog\left(x\right) = \sin\ [x] = \sin\ (-1) = -\sin\ 1$

$\text{LHL} \neq \text{RHL}, \text{ limit does not exist.}$

$\lim_{x \to 0^+} fog\left(x\right) = \lim_{x \to 0^+} gof\left(x\right) = 0$

Hence, the correct answer is option D.

**Solution 83**

1. $f(x) = [x]$

$f(f(x)) = [f(x)] = [[x]] = [x] = f(x)$
which is correct.

2. $g(g(x)) = \sin(\sin x)$

$(gog)(x) = g(x)$

$\sin(\sin x) = \sin x$

which is true for multiple values of $x$, such as 0, π, 2π, etc.

3. $(go(fog))(x) = (g([\sin x])$

Since $\sin x$ lies between $[-1, 1]$, $[\sin x]$ can take only three values ie $-1, 0, 1$.

Hence $(go(fog))(x)$ can take only 3 values $g(-1)$, $g(0)$ and $g(1)$.

Hence, correct answer is option D.

**Solution 84**

$f(x) = \frac{e^x - 1}{x}, \text{ for } x > 0$

$\Rightarrow f'(x) = \frac{xe^x - (e^x - 1)}{x^2} = \frac{e^x(x-1)+1}{x^2}, \text{ which is always positive}$

$\text{So, if } f'(x) > 0 \text{ then } f(x) \text{ is strictly increasing in } (0, x).$

Hence, the correct answer is option B.

**Solution 85**

At $x = 0$, $f(0) = 0$.

$\text{RHL} = \lim_{h \to 0} \frac{e^{(0+h)} - 1}{0+h} = \frac{e^h - 1}{h} = 1$

$\text{RHL} \neq f(0)$

$\text{So, } f(x) \text{ is not right continuous at } x = 0$

$\text{Now for } x = 1$

$\text{RHL} = \lim_{h \to 0^+} \frac{e^{(1+h)} - 1}{1+h} = \frac{e^1 - 1}{1} = e - 1$

$\text{LHL} = \lim_{h \to 0^-} \frac{e^{(1-h)} - 1}{1-h} = \frac{e^1 - 1}{1} = e - 1$

$f(1) = e - 1$

$\text{RHL} = \text{LHL} = f(1)$

$f(x) \text{ is contiuous at } x = 1$

Hence, the correct answer is option D.

**Solution 86**

Line is $y = 3x - 3$, now all the points given lie on the parabola
So, distance of these points from the line is

$\text{For } (0, 2): \text{ distance } = \left|\frac{3(0)-(2)-3}{\sqrt{(3)^2+(-1)^2}}\right| = \frac{5}{\sqrt{10}}$

$\text{For } (-2, -8): \text{ distance } = \left|\frac{3(-2)-(-8)-3}{\sqrt{(3)^2+(-1)^2}}\right| = \frac{1}{\sqrt{10}}$

$\text{For } (-7, 2): \text{ distance } = \left|\frac{3(-7)-(2)-3}{\sqrt{(3)^2+(-1)^2}}\right| = \frac{26}{\sqrt{10}}$

$\text{For } (1, 10): \text{ distance } = \left|\frac{3(1)-(10)-3}{\sqrt{(3)^2+(-1)^2}}\right| = \frac{10}{\sqrt{10}}$

So, (−2, −8) is closest

Hence, the correct answer is option B.

**Solution 87**

The points given in the previous question were (0, 2), (−2, −8), (−7, 2) and (1, 10).
The distance of these points from the line is

$$\text{For } (0,2): \text{ distance } = \left|\frac{3(0)-(2)-3}{\sqrt{(3)^2+(-1)^2}}\right| = \frac{5}{\sqrt{10}}$$

$$\text{For } (-2,-8): \text{ distance } = \left|\frac{3(-2)-(-8)-3}{\sqrt{(3)^2+(-1)^2}}\right| = \frac{1}{\sqrt{10}}$$

$$\text{For } (-7,2): \text{ distance } = \left|\frac{3(-7)-(2)-3}{\sqrt{(3)^2+(-1)^2}}\right| = \frac{26}{\sqrt{10}}$$

$$\text{For } (1,10): \text{ distance } = \left|\frac{3(1)-(10)-3}{\sqrt{(3)^2+(-1)^2}}\right| = \frac{10}{\sqrt{10}}$$

Shortest distance is thus $\frac{1}{\sqrt{10}}$ from the point (−2, −8)

Hence, the correct asnwer is option C.

**Solution 88**

$f(x) = \begin{cases} -2, & -3 \le x \le 0 \\ x-2, & 0 < x \le 3 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$

1. At $x = 0$

For LHD:
$g(x) = -2 + |-2| = -2 + 2 = 0$
$\Rightarrow g(x) = 0$

$$\begin{aligned} \text{LHD} &= \lim_{x\to 0^-} \frac{g(x)-g(0)}{x-0} \\ &= \lim_{h\to 0} \frac{g(-h)-g(0)}{-h} \\ &= \lim_{h\to 0} \frac{0-0}{-h} = 0 \end{aligned}$$

For RHD:
$g(x) = (|x| - 2) + |x - 2|$
$g(x) = x - 2 - (x - 2)$ (As $x$ is just greater than zero.)
$\Rightarrow g(x) = 0$

$$\begin{aligned} \text{RHD} &= \lim_{x\to 0^+} \frac{g(x)-g(0)}{x-0} = \lim_{h\to 0} \frac{g(h)-g(0)}{h} \\ &= \lim_{h\to 0} 0 = 0 \end{aligned}$$

As LHD = RHD,
So, $g(x)$ is differentiable at $x = 0$.

Therefore, statement 1 is correct.

2. At $x = 2$

For LHD:
$g(x) = |x| - 2 + |x - 2|$
$= x - 2 - (x - 2) = 0$

$$\text{LHD} = \lim_{x\to 2^-} \frac{g(x)-g(2)}{x-2} = \lim_{x\to 2^-} \frac{0}{x-2} = 0$$

For RHD:
$g(x) = |x| - 2 + |x - 2|$
$= x - 2 + x - 2 = 2x - 4$

$$\begin{aligned} \text{RHD} &= \lim_{x\to 2^+} \frac{g(x)-g(2)}{x-2} = \lim_{x\to 2^+} \frac{2x-4-2(2)+4}{x-2} \\ &= \lim_{x\to 2^+} \frac{2(x-2)}{x-2} = 2 \end{aligned}$$

As LHD ≠ RHD,
So, $g(x)$ is not differentiable at $x = 2$.

Therefore, statement 2 is incorrect.

Hence, the correct answer is option A.

**Solution 89**

For $x = -2$
$g(x) = -2 + |-2| = -2 + 2$
$\Rightarrow g(x) = 0$
$\Rightarrow$ differential coefficient at $x = -2$ is given as :

$g'\left(x\right) = \lim_{h \to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h \to 0} \frac{0-0}{4} = 0.$

**Solution 90**

1. At $x$ = 0

For LHL : $g(x)$ = −2 + |−2| = 0
For RHL : $g(x)$ = |$x$| −2 + |$x$ − 2| = $x$ − 2 − ($x$ − 2) = 0
For ($x$ = 0) : $g(x)$ = −2 + | −2| = 0

Clearly, LHL = RHL = $g(0)$ = 0
⇒ $g(x)$ is continuous at $x$ = 0.
Therefore, statement 1 is correct.

2. At $x$ = 2

$\text{For LHL} \;:\; g\left(x\right) = |x| - 2 + |x-2| = x - 2 - (x-2) = 0$
$\text{For RHL} \;:\; g\left(x\right) = |x| - 2 + |x-2| = x - 2 + x - 2 = 2x - 4$
For ($x$ = 2): $g(x)$ = |$x$| − 2 + |$x$ − 2| = |2| − 2 + |2 − 2| = 0

$\text{LHL} = \lim_{x \to 2^-} g\left(x\right) = 0$

$\text{RHL} = \lim_{x \to 2^+} g\left(x\right) = \lim_{x \to 2} 2x - 4 = 2\left(2\right) - 4 = 0$

Clearly, LHL = RHL = $g(2)$ = 0
⇒ $g(x)$ is continuous at $x$ = 2.
Therefore, statement 2 is correct.

3. At $x$ = −1

$\text{For LHL} \;:\; g\left(x\right) = -2 + |-2| = 0$
$\text{For RHL} \;:\; g\left(x\right) = -2 + |-2| = 0$
For ($x$ = −1) : $g(x)$ = −2 + |−2| = 0

Clearly, LHL = RHL = $g(-1)$ = 0
⇒ $g(x)$ is differentiable at $x$ = −1.
Therefore, statement 3 is correct.

Hence, the correct answer is option D.

**Solution 91**

$I = \int_{-1}^{1} f\left(x\right) dx$

$I = \int_{-1}^{1} (1) f\left(x\right) dx$

$\text{Integrating by parts}$

$I = f\left(x\right) \int_{-1}^{1} (1)\text{dx} - \int_{-1}^{1} f'\left(x\right) \text{ x dx}$

$I = \text{f}\left(1\right) + f\left(-1\right) - I_1 \qquad \left(\text{I}_1 = \int_{-1}^{1} \text{f'}\left(x\right) \text{x dx}\right)$

$I_1 = \int_{-1}^{1} f'\left(x\right) x \text{ dx} = \int_{-1}^{1} \frac{1}{x^2} f'\left(\frac{1}{x}\right) \text{ dx} \qquad \left(f'\left(x\right) x = -\frac{1}{x^2} f'\left(\frac{1}{x}\right) \quad \text{from equation}\right)$

$\text{let } \frac{1}{x} = t \Rightarrow \frac{-1}{x^2}\text{dx} = \text{dt}$

$I_1 = \int_{-1}^{1} f'\left(t\right) \text{dt} = \left[f\left(1\right) - f\left(-1\right)\right]$

$I = f\left(1\right) - f\left(-1\right) - \left[f\left(1\right) - f\left(-1\right)\right] = 2f\left(-1\right)$

Hence, the correct answer is option C.

**Solution 92**

$\text{I} = \int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}} dx$

$I = \int \frac{x^4-1}{x^3\sqrt{x^2+1+\frac{1}{x^2}}} dx$

$I = \int \frac{\left(x-\frac{1}{x^3}\right)}{\sqrt{x^2+1+\frac{1}{x^2}}} dx$

$Now,\ let\ x^2 + 1 + \frac{1}{x^2} = t \ \Rightarrow 2x - \frac{2}{x^3} = \frac{dt}{dx} \Rightarrow \left(x - \frac{1}{x^3}\right) dx = \frac{dt}{2}$

$I = \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \sqrt{t} \ + \ c = \sqrt{x^2 + 1 + \frac{1}{x^2}} \ + c$

Hence the correct answer is option C.

**Solution 93**

Taking log on both sides

$y\sqrt{1-x^2}+x\sqrt{1-y^2}=\log\left(ce^x\right)$

$\Rightarrow y\sqrt{1-x^2}+x\sqrt{1-y^2}=\log \text{c} + \text{x} \log e$

$\Rightarrow y\sqrt{1-x^2}+x\sqrt{1-y^2}=\log \text{c} + x \qquad \left(\log \text{e} = 1\right)$

$\text{Differentiation w.r.t.} x$

$\frac{\text{dy}}{\text{dx}}\sqrt{1-x^2}+y\frac{1}{2\sqrt{1-x^2}}\left(-2x\right)+\sqrt{1-y^2}+x\frac{1}{2\sqrt{1-y^2}}\left(-2y\right).\frac{\text{dy}}{\text{dx}}-1=0$

$\Rightarrow \frac{\text{dy}}{\text{dx}}\sqrt{1-x^2}-\frac{xy}{\sqrt{1-x^2}}+\sqrt{1-y^2}-\frac{xy}{\sqrt{1-y^2}}\frac{\text{dy}}{\text{dx}}-1=0$

Degree = 1, order = 1
Hence, the correct answer is option A.

**Solution 94**

$\text{slope}=\frac{\text{dy}}{\text{dx}}=\frac{2y}{x}$

$\Rightarrow \frac{\text{dy}}{\text{y}}=2\frac{\text{dx}}{\text{x}}$

$\Rightarrow \int\frac{\text{dy}}{y}=2\int\frac{\text{dx}}{x}$

$\Rightarrow \log y = 2 \log x + \text{c}$

$\text{Curve passes through} \left(1,1\right)$

$\Rightarrow \log 1 = 2 \log 1 + \text{c} \Rightarrow \text{c}=0$

$\Rightarrow \log y = 2 \log x$

$\Rightarrow \log y = \log x^2$

$\Rightarrow y = x^2$

Thus, the given equation represents a parabola.
Hence, the correct answer is option B.

**Solution 95**

$xdy=ydx + y^2dy$

$\Rightarrow xdy - ydx = y^2dy$

$\frac{\Rightarrow xdy-ydx}{y^2} = dy$

$\frac{\Rightarrow ydx-xdy}{y^2} = -dy$

$\Rightarrow d\left(\frac{x}{y}\right)=-dy$

$\text{integrating both sides}$

$\int \text{d}\left(\frac{x}{y}\right)=-\int \text{dy}$

$\Rightarrow \frac{x}{y}=-\text{y} + \text{c}$

$\text{now y}\left(1\right) = 1$

$\Rightarrow \frac{1}{1}=-1+\text{c}\Rightarrow \text{c}=2$

$\Rightarrow \frac{x}{y}+y-2=0$

$y\left(-3\right)\Rightarrow \frac{-3}{y}+y-2=0$

$\Rightarrow -3+y^2-2y=0$

$\Rightarrow y^2-2y-3=0$

$\Rightarrow \left(y-3\right)\left(y+1\right)=0$

$\Rightarrow y=3 \text{ or } y=-1$

But $y > 0$
So, $y = 3$ is the only correct answer

Hence, the correct answer is option A.

**Solution 96**

$\frac{dx}{dy} + \int y dx = x^3$

Differentiating w.r.t. $x$

$\Rightarrow \frac{d}{dx}\left(\frac{dx}{dy}\right) + y = 3x^2$

$\Rightarrow \frac{d}{dx}\left(\frac{1}{\frac{dy}{dx}}\right) + y = 3x^2$

$\Rightarrow \left(\frac{1}{-\left(\frac{dy}{dx}\right)^2}\right)\left(\frac{d^2y}{dx^2}\right) + y = 3x^2$

$\Rightarrow -\left(\frac{d^2y}{dx^2}\right) + y\left(\frac{dy}{dx}\right)^2 = 3x^2\left(\frac{dy}{dx}\right)^2$

$\Rightarrow y\left(\frac{dy}{dx}\right)^2 = 3x^2\left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)$

Which is a differential equation of order 2

Hence, the correct answer is option B.

**Solution 97**

let the equation of straight line be $\Rightarrow y = mx + c$

$\Rightarrow \frac{dy}{dx} = m \qquad \ldots\ldots(1)$

distance of this line from origin,

$\frac{m(0)-(0)+c}{\sqrt{1+m^2}} = 1$

$\Rightarrow \frac{c}{\sqrt{1+m^2}} = 1$

$\Rightarrow c = \sqrt{1+m^2}$

$\Rightarrow c^2 = 1 + m^2 \qquad \ldots\ldots(2)$

$\Rightarrow \left(y - mx\right)^2 = 1 + m^2$

$\Rightarrow \left(y - \left(\frac{dy}{dx}\right)x\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

Hence, the correct answer is option C.

**Solution 98**

$I = \int e^{\sin x} \frac{x \cos^3 x - \sin x}{\cos^2 x} dx$

$I = \int e^{\sin x}\left(x \cos x - \sec x \tan x\right) dx$

$I = \int e^{\sin x}\left(x \cos x - \sec x \tan x + 1 - 1\right) dx$

$I = \int e^{\sin x}\left(x \cos x - \sec x \tan x + 1 - \frac{1}{\cos x}\cos x\right) dx$

$I = \int e^{\sin x}\left(x \cos x - \sec x \tan x + 1 - \sec x \cos x\right) dx$

$I = \int\left(e^{\sin x} + xe^{\sin x} \cos x\right) dx - \int\left(e^{\sin x} \sec x \tan x + e^{\sin x} \cos x \sec x\right) dx$

$I = xe^{\sin x} - e^{\sin x} \sec x + c$

$I = e^{\sin x}\left(x - \sec x\right) + c$

Hence the correct answer is option B.

**Solution 99**

$I = \int_0^{\frac{\pi}{2}} \frac{dx}{3 \cos x + 5}$

$I = \int_0^{\frac{\pi}{2}} \frac{dx}{3\left[\frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}\right] + 5}$

$I = \int_0^{\frac{\pi}{2}} \frac{\left(1+\tan^2 \frac{x}{2}\right) dx}{3 - 3 \tan^2 \frac{x}{2} + 5 + 5 \tan^2 \frac{x}{2}}$

$I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2} dx}{2 \tan^2 \frac{x}{2} + 8}$

$I = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2} dx}{\tan^2 \frac{x}{2} + 2^2}$

Let, $\tan \frac{x}{2} = y \quad \Rightarrow \frac{1}{2}\sec^2 \frac{x}{2} dx = dy$

$I = \int_0^1 \frac{dy}{y^2+2^2}$

$I = \frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right)\Big|_0^1 = \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right) - \frac{1}{2}\tan^{-1}\left(\frac{0}{2}\right) = \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)$

So, $\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right) = k \cot^{-1}(2)$ (Given)

$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right) = k \tan^{-1}\left(\frac{1}{2}\right)$ $\left(\tan^{-1}(x) = \cot^{-1}\left(\frac{1}{x}\right)\right)$

$k = \frac{1}{2}$

Hence, the correct answer is option B.

**Solution 100**

$I = \int_1^3 |1-x^4| \, dx$

$|x| = x,\ x \geq 0$ and $-x,\ x < 0$

$I = \int_1^3 -\left(1-x^4\right) dx$ (When $x > 0$)

$I = \int_1^3 \left(x^4 - 1\right) dx$

$I = \left[\frac{x^5}{5} - x\right] = \left(\frac{3^5}{5} - 3\right) - \left(\frac{1^5}{5} - 1\right) = \frac{232}{5}$

Hence, the correct answer is option D.

**Solution 101**

Die is thrown thrice, so total possible outcomes = 6 × 6 × 6 = 216

Favourable outcomes resulting in sum zero are

(1,−1,0) which can be arranged in 3! ways so, 6 cases are there
(2,−2,0) which can be arranged in 3! ways so, 6 cases are there
(3,−2,−1)which can be arranged in 3! ways so, 6 cases are these
(2,−1,−1) which can be arranged in $\frac{3!}{2!}$ ways
(−2,1,1) these can be arranged in $\frac{3!}{2!}$ ways
(0,0,0) can be arranged in 1 way only
So, total no. of favourable outcomes = 6 + 6 + 6 + 3 + 3 + 1 = 25 ways
Probability = $\frac{25}{216}$
Hence, the correct answer is option D.

**Solution 102**

P(rainy day) = 25% = 0.25
P(not a rainy day) = 1 − 0.25 = 0.75

P(atleast one rainy day) = 1 − (no rainy day in 7 days)
= 1 − $(0.75)^7$
= $1 - \left(\frac{3}{4}\right)^7$
Hence , the correct answer is option D.

**Solution 103**

$P(A) = P(B) = \frac{70}{100} = \frac{7}{10}$

A and B are independent

$\Rightarrow P(A \cap B) = P(A)P(B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{7}{10} + \frac{7}{10} - \frac{7}{10} \times \frac{7}{10} = 0.91$

Hence, the correct answer is option B.

**Solution 104**

The student can be successful if he passes I, II or I, III or all the three

P(passing I) = $m$ , P(passing II) = $n$ , P(passing III) = $\frac{1}{2}$ ,

P(successful) = (passing I & II and failing in III) + (passing I & III and failing in II ) + (passing I,II & III)

P(successful) = $m \times n \times \frac{1}{2} + m \times \frac{1}{2} \times (1-n) + m \times n \times \frac{1}{2} = \frac{1}{2}$
$\Rightarrow \frac{mn}{2} + \frac{m}{2} - \frac{mn}{2} + \frac{mn}{2} = \frac{1}{2}$
$\Rightarrow \frac{mn}{2} + \frac{m}{2} = \frac{1}{2}$
$\Rightarrow mn + m = 1$
$\Rightarrow m(n+1) = 1$

Hence, the correct answer is option A.

**Solution 105**

Odds in favorable for student (A) = $\frac{5}{2} = \frac{P(A)}{P(A')} \Rightarrow P(A') = \frac{2}{5}P(A)$

Odds in favorable for student (B) $= \frac{4}{3} = \frac{P(B)}{P(B')} \Rightarrow P(B') = \frac{3}{4}P(B)$

Odds in favorable for student (C) $= \frac{3}{4} = \frac{P(C)}{P(C')} \Rightarrow P(C') = \frac{4}{3}P(C)$

Now, $P(A) + P(A') = 1 \Rightarrow P(A) + \frac{2}{5}P(A) = 1 \Rightarrow P(A) = \frac{5}{7}$

$P(B) + P(B') = 1 \Rightarrow P(B) + \frac{3}{4}P(B) = 1 \Rightarrow P(B) = \frac{4}{7}$

$P(C) + P(C') = 1 \Rightarrow P(C) + \frac{4}{3}P(C) = 1 \Rightarrow P(C) = \frac{3}{7}$

$P(A') = \frac{2}{7}$, $P(B') = \frac{3}{7}$, $P(C') = \frac{4}{7}$

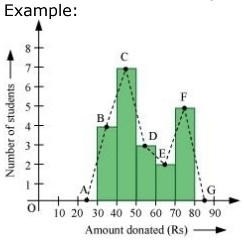
Required probability $= P(A) \times P(B) \times P(C') + P(A) \times P(B') \times P(C) + P(A') \times P(B) \times P(C) + P(A) \times P(B) \times P(C)$

$= \frac{5}{7} \times \frac{4}{7} \times \frac{4}{7} + \frac{5}{7} \times \frac{3}{7} \times \frac{3}{7} + \frac{2}{7} \times \frac{4}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{209}{343}$

Hence, the correct answer is option D.

**Solution 106**

1. In symmetric distribution, values of variables occur at regular frequencies.
Mean = Median (in symmetric disribution)
So, statement 1 is true
2. Range = Maximum value − Minimum value
Thus, statement 2 is also true
3. Sum of areas of rectangles in the histogram is always equal to the total area bounded by frequency polygon and the horizontal axis
Example:




Thus, all the three given statements are correct.
Hence, The correct answer is option D.

**Solution 107**

$\text{Mean of the wrong scores} = \frac{202}{15}$

$\text{Mean of the correct scores} = \frac{200}{15}$

Total students, $n$ = 15 (odd)
With the wrong scores,

$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{term} = \left(\frac{15+1}{2}\right)^{\text{th}} \text{term} = \left(\frac{16}{2}\right)^{\text{th}} \text{term} = 8\text{th term} = 14$

The Median will remain the same even with the error is rectified as the n will remain the same.
Mode(with wrong score) = 16
Mode(with correct score) = 18
So, mean and mode will change
Hence, the correct answer is option D.

**Solution 108**

Given: $\sum x = 130,\ \sum y = 220,\ \sum x^2 = 2288,\ \sum y^2 = 5506$ and $\sum xy = 3467.$

We know that line of regression of $y$ on $x$ is
$y = a + bx$

We solve the normal equations to get the value of $a$ and $b$.

$\sum y = na + b\sum x$

$\Rightarrow 220 = 10a + b(130)$

$\Rightarrow 22 = a + 13b \qquad .....(1)$

Also,

$\sum xy = a\sum x + b\sum x^2$

$\Rightarrow 3467 = a130 + b2288 \qquad .....(2)$

Solving (1) and (2) we get
$a$ = 8.74 and $b$ = 1.02

Thus, the line of regression of $y$ on $x$ will be
$y$ = 8.74 + 1.02$x$

Hence, the correct answer is option B.

**Solution 109**

For Group A :
$\mathrm{CV_A} = \frac{\mathrm{S.D.}}{\mathrm{Mean}} = \frac{10}{22} = 0.4545$
For Group B :
$\mathrm{CV_B} = \frac{12}{23} = 0.522$

Group A is less variable as coefficient of variation of A is less than that of B.

Hence, The correct answer is option D.

**Solution 110**

Class intervals should be exhaustive for grouped frequency distribution.
So, statement 2 is correct.
Class intervals are generally equal in width but this might not be the case always
So, statement 3 is also correct.

Hence, the correct answer is option B.

**Solution 111**

P(effective) = 75% = 0.75 , P(non effective) = 25% = 0.25

P(at least one cured out of 5) = 1 − (None out of 5 is being cured)

$= 1 - \left(\frac{1}{4}\right)^5$
$= 1 - \left(\frac{1}{1024}\right)$
$= \frac{1023}{1024}$

Hence, the correct answer is option C.

**Solution 112**

$\mathrm{P}\left(\mathrm{A}\right) = \frac{3}{5} \Rightarrow \mathrm{P}\left(\mathrm{A'}\right) = 1 - \mathrm{P}\left(\mathrm{A}\right) = \frac{2}{5}$
$\mathrm{P}\left(\mathrm{B}\right) = \frac{3}{10} \Rightarrow \mathrm{P}\left(\mathrm{B'}\right) = 1 - \mathrm{P}\left(\mathrm{B}\right) = \frac{7}{10}$
$\mathrm{P}\left(A|B\right) = \frac{2}{3}$

$\Rightarrow \mathrm{P}\left(A|B\right) = \frac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}} = \frac{2}{3}$

$\Rightarrow \mathrm{P}\left(\mathrm{A \cap B}\right) = \frac{2}{3} \times \frac{3}{10} = \frac{1}{5}$

$\Rightarrow \mathrm{P}\left(\mathrm{A \cup B}\right) = \mathrm{P}\left(\mathrm{A}\right) + \mathrm{P}\left(\mathrm{B}\right) - \mathrm{P}\left(\mathrm{A \cap B}\right)$

$\Rightarrow \mathrm{P}\left(\mathrm{A \cup B}\right) = \frac{3}{5} + \frac{3}{10} - \frac{1}{5} = \frac{7}{10}$

$\Rightarrow \mathrm{P}\left(\mathrm{A \cup B}\right)' = 1 - \frac{7}{10} = \frac{3}{10}$

$\Rightarrow \mathrm{P}\left(\mathrm{A' \cap B'}\right) = \mathrm{P}\left(\mathrm{A \cup B}\right)' = \frac{3}{10}$

$\Rightarrow \mathrm{P}\left(\mathrm{A'|B'}\right) = \frac{\mathrm{P(A' \cap B')}}{\mathrm{P(B')}} = \frac{\frac{3}{10}}{\frac{7}{10}} = \frac{3}{10} \times \frac{10}{7} = \frac{3}{7}$

Hence, the correct answer is option A.

**Solution 113**

Machine will not stop if all 3 parts are working properly simultaneously

P(A) = 0.02 , P(A') = 0.98
P(B) = 0.10 , P(B') = 0.90
P(C) = 0.05 , P(C') = 0.95

P(machine works properly) = $\mathrm{P(A')} \times \mathrm{P(B')} \times \mathrm{P(C')} = 0.98 \times 0.90 \times 0.95 = 0.84$

Hence, the correct answer is option C.

**Solution 114**

Probability that the independent events $A_1$, $A_2$ and $A_3$ occur is

$P\left(A_i\right) = \frac{1}{1+i}$

$P\left(A_1\right) = \frac{1}{1+1} = \frac{1}{2} \Rightarrow P\left(A_1'\right) = \frac{1}{2}$

$P\left(A_2\right) = \frac{1}{1+2} = \frac{1}{3} \Rightarrow P\left(A_2'\right) = \frac{2}{3}$

$P\left(A_3\right) = \frac{1}{1+3} = \frac{1}{4} \Rightarrow P\left(A_3'\right) = \frac{3}{4}$

$P\left(\text{at least one}\right) = 1 - P\left(\text{None of them}\right)$

$= 1 - P\left(A'\right) \times P\left(B'\right) \times P\left(C'\right)$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{4}$

Hence, the correct answer is option C.

**Solution 115**

We know

$\text{var}\,(x+y) = \text{var}\,(x) + \text{var}\,(y) = \sigma_x^2 + \sigma_y^2$

$\text{var}\,(x-y) = \text{var}\,(x) + \text{var}\,(-y) = \text{var}\,(x) + \text{var}\,(y) = \sigma_x^2 + \sigma_y^2$

$\text{cov}\,(x+y, x-y) = \text{cov}\,(x,x) - \text{cov}\,(x,y) + \text{cov}\,(y,x) - \text{cov}\,(y,y)$

$= \text{cov}\,(x,x) - \text{cov}\,(x,y) + \text{cov}\,(x,y) - \text{cov}\,(y,y)$

$= \text{cov}\,(x,x) - \text{cov}\,(y,y)$

$= \text{var}\,(x) - \text{var}\,(y)$

$= \sigma_x^2 - \sigma_y^2$

Now,

$\text{Correlation Coefficient} = \frac{cov(x+y,x-y)}{\sqrt{var(x+y)}\sqrt{var(x-y)}}$

$= \frac{\sigma_x^2 - \sigma_y^2}{\sigma_x^2 + \sigma_y^2}$

Hence, the correct answer is option C.

**Solution 116**

| AGE | Mid Value $x_i$ | Frequency $f_i$ | $f_i x_i$ |
|---|---|---|---|
| 15 - 25 | 20 | 2 | 40 |
| 25 - 35 | 30 | 4 | 120 |
| 35 - 45 | 40 | 6 | 240 |
| 45 - 55 | 50 | 5 | 250 |
| 55 - 65 | 60 | 3 | 180 |
| | | $\Sigma f_i = 20$ | $\Sigma f_i x_i = 830$ |

Mean age = $\frac{\sum f_i x_i}{\sum f_i} = \frac{830}{20} = 41.5$

Hence, the correct answer is option B.

**Solution 117**

$cov\left(x, y\right) = 30$

$var\left(x\right) = 25$

$var\left(y\right) = 144$

$\text{Correlation Coefficient, } r\left(x, y\right) = \frac{cov(x,y)}{\sqrt{var(x) \times var(y)}}$

$\Rightarrow r\left(x, y\right) = \frac{30}{\sqrt{25 \times 144}} = \frac{30}{5 \times 12} = 0.5$

Hence, the correct answer is option B.

**Solution 118**

If a coin is tossed three times, the sample space is
S = {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)}

A = {(TTT)}
B = {(HTT), (THT), (TTH)}
C = {(HHH), (HHT), (HTH), (THH)}

Now, B ∩ C = {∅}

A ∩ (B' ∪ C') = A ∩ (B ∩ C)'
= A ∩ (∅)'
= A ∩ S
= A

Also,
B' ∩ C' = (B ∪ C)' = A

∴ A ∩ (B' ∪ C') = B' ∩ C'

Hence, the correct answer is option D.

**Solution 119**

We have
P(Winning) = P(W) = $\frac{1}{3}$
P(Drawing) = P(D) = $\frac{1}{6}$
Team A scores points 2, 0 and 1 if it wins, losses or draws, respectively.
To score 5 points in the series, the possible options are:
WWD, WDW, DWW

$$\begin{aligned}\therefore \mathrm{P(5\ points)} &= \mathrm{P(WWD)} + \mathrm{P(WDW)} + \mathrm{P(DWW)} \\ &= \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{6}\right) + \left(\frac{1}{3} \times \frac{1}{6} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{1}{6} \times \frac{1}{3}\right) \\ &= \frac{1}{54} + \frac{1}{54} + \frac{1}{54} \\ &= \frac{3}{54} \\ &= \frac{1}{18}\end{aligned}$$

Hence, the correct answer is option D.

**Solution 120**

$\text{We know } \mathrm{P}\left(X = r\right) = {}^{n}C_r\, p^r \left(1 - p\right)^{n-r}$

$\Rightarrow \mathrm{P}\left(X = 4\right) = {}^{6}C_4\, p^4 \left(1 - p\right)^{2}$

$\Rightarrow \mathrm{P}\left(X = 2\right) = {}^{6}C_2\, p^2 \left(1 - p\right)^{4}$

$\text{GIven, } 16\,\mathrm{P}\left(X = 4\right) = \mathrm{P}\left(X = 2\right)$

$\Rightarrow 16 \times {}^{6}C_4\, p^4 \left(1 - p\right)^{2} = {}^{6}C_2\, p^2 \left(1 - p\right)^{4}$

$\Rightarrow 16p^2 = \left(1 - p\right)^2$

$\Rightarrow 16p^2 = 1 + p^2 - 2p$

$\Rightarrow 15p^2 + 2p - 1 = 0$

$\Rightarrow \left(5p - 1\right)\left(3p + 1\right) = 0$

$\Rightarrow p = \frac{1}{5} \text{ or } \frac{-1}{3}$

$\text{As } p \neq -\frac{1}{3}, \ \Rightarrow p = \frac{1}{5}$

Hence, the correct answer is option C.